Introduction to Algorithms	Problem Set 1
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1 Problem 3

1.1 Algorithm

We will work to fill a set C with stable, matched pairs $C \subseteq S \times I$ of coordinated departures in which set S holds scheduled students and I is the set of interviewers.

1. For each $s \in S$, have s create a preference list of interviewers where s' earliest interviewer is most preferred and the student's subsequent interviewers decrease in preference (i.e. student s' last interviewer is least preferred by s)

2. For each $i \in I$, have i create a preference list of students where i's earliest student is least preferred and subsequent students in the interviewer's schedule increase in preference

3. Apply the G-S algorithm to build the set of matched pairs C

4. Return C

1.2 Runtime

Assume we are given a valid interview schedule of students and interviewers. Creating a preference list for a student s takes $O(n)$ as we iterate through n interviewers in schedule order. Because we must make a list for all n students, runtime of step 1 is $O(n^2)$. Similarly, creating a preference list for an interviewer i takes $O(n)$ as we iterate through n students in reverse schedule order. Because we must make a list for all n interviewers, runtime of step 2 is $O(n^2)$. It was given in class that the G-S algorithm runs in $O(n^2)$ for sets of size n. Hence, our total running time is $O(n^2 + n^2 + n^2)$ or simply $O(n^2)$.

1.3 Correctness

We will show correctness using a proof by contradiction. Let C denote the set constructed by our algorithm from the set of students S and the set of interviewers I. Let (s_i, i_j), in which $C \subseteq S \times I$, where $s_i \in S$ for $i \in [1, n]$ and $i_j \in I$ for $j \in [1, n]$ denote its elements. Let us call a matching stable if it does not exhibit the following instability

1. Some student s whose schedule has not yet been truncated (and so is still following his/her original schedule) shows up for an interview with an interviewer whos already left for the day.
   There exists two pairs $(s, i), (s', i') \in C$, in which if student s leaves with interviewer i, i misses interviewing student s', whose schedule was not yet truncated.

Lemma 1. *Given any valid schedule, there exists a stable matching that arranges a valid set the coordinated departures of the interviewers and students.*

Proof of Lemma 1. To prove the algorithm will always produce a stable matching, we will prove by contradiction that the stated instability is unattainable.

There exists two pairs $(s, i), (s', i') \in C$, in which when student s leaves with interviewer

i, i misses interviewing student s', whose schedule was not yet truncated. It follows that s' prefers i and i prefers s', which contradicts our original assumption of stable matches $(s, i), (s', i')$. Thus, the instability is impossible.