1 Problem 1

Claim 1. *SJF produces an optimal schedule σ.*

Proof. We will prove SJF produces an optimal schedule by using proof by contradiction. Given a schedule $\sigma =< J_1, J_2, ..., J_n >$ such that $c_i < c_k$ for all $1 \leq i < k \leq n$, the average response time

$$R_\sigma = c_1 + (c_1 + c_2) + ... + \sum_{k=1}^{i} c_k + \sum_{k=1}^{i+1} c_k + ... + \sum_{k=1}^{n} c_k$$

.

We assume that schedule $\sigma$is suboptimal. Hence, there exists a schedule σ' that is optimal in which for $1 \leq i < j \leq n$, $c_i > c_j$. Note, for this proof we assume for $1 \leq i \leq n$, $c_i > c_{i+1}$. Thus, σ' is constructed as follows:

$$\sigma' =< J_1, J_2, ..., J_{i+1}, J_i, ..., J_n >$$

,

with average response time,

$$\begin{aligned} R_{\sigma'} &= c_1 + (c_1 + c_2) + ... + \sum_{k=1}^{i-1} c_k + c_{i+1} + \sum_{k=1}^{i-1} c_k + c_{i+1} + c_i + ... + \sum_{k=1}^{n} c_k \\ &= c_1 + (c_1 + c_2) + ... + \sum_{k=1}^{i-1} c_k + c_{i+1} + \sum_{k=1}^{i} c_i + ... + \sum_{k=1}^{n} c_k. \end{aligned}$$

Thus, $R_\sigma < R_{\sigma'}$ as $c_i < c_{i+1}$. However, this implies that σ' is suboptimal compared to σin terms of average response time. Thus, by proof by contradiction, SJF produces an optimal schedule σ. Hence, swapping any two jobs, yields a new schedule σ' whose average response time is no better than that of σ.