Your homework submissions need to be typeset (hand-drawn figures are OK). See the course web page for suggestions on typing formulas. Solution to each question needs to be uploaded to CMS as a separate pdf file. **Homework submitted using late days may not be graded before the upcoming prelim (on April 14th).**

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time. The running time must be bounded by a polynomial function of the input size.

If A and B are two decision problems, proving A $\leq$ B often just consists of a polynomial-time algorithm that takes an instance of A and outputs an instance of B. To show that such a reduction is correct, you must prove two things. First, that it transforms YES instances of A to YES instances of B. Second, that it transforms NO instances of A to NO instances of B.

The list of problems we proved NP-complete in class so far are SAT, 3-SAT, INDEPENDENT SET, CLIQUE, VERTEX-COVER, HAMILTONIAN CYCLE, SUBSET SUM. You can use in your proof that these problems are NP-complete. You can also use HAMILTONIAN PATH: where the input is a directed graph G with two specified nodes s and t, and the question is whether there is a simple path (without repeated nodes) in G from s to t going through each node of the graph. Our reduction to prove that HAMILTONIAN CYCLE is NP-complete, in fact, also proves that HAMILTONIAN PATH is NP-complete.

(1) *(10 points)* You are traveling in a remote part of the world. The possible travel is described by a directed graph $G = (V, E)$. You start at node s, need to get to node t, and then get back home to s. You are planning to take a scenic, but rather long path from t back to s (which doesn't go through other nodes of G). You do not need to visit all the nodes other than s and t. However, there is a truly severe gas-shortage. On each edge $e = (v, w)$ you are given, $g(v, w)$ the amount of gas needed to get from v to w (in gallons), and in each node v you can pick up at most $g(v)$ gallons of gas. You start with $g(s)$, the gas available at node s, and your car can hold W gallons of gas at-a-time. The REMOTE TRAVELER'S PROBLEM is defined by a directed graph G, nodes s and t, and numbers $g(v, w)$ and $g(v)$ for all edges and nodes in the graph G. The problem is to decide if there is a way to travel from s through the graph to t and then go directly back to s using edge (t, s) while never running out of gas. We will require (to simplify the problem) that the traveler cannot visit each node more than once).

Show that the REMOTE TRAVELER'S PROBLEM is NP-complete.

(2) *(10 points)* A monotone SAT formula is a SAT formula with no negated variables. So, for example,

$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_4)$$

is a monotone formula. A monotone formula is easy to satisfy, we can simply set all variables to be *true*. In the MONOTONE SAT problem we are given a monotone formula and an integer k, and ask if there is a way to satisfy formula Φ with setting at most k variables true (and the rest of the variables false).

Show that the MONOTONE SAT problem is NP-complete.