d. Assume universal U that takes in M

U(M) = yes if there exists s st M accepts x -> 2

Construct U’ st U’ solves the HP

U’(M,x) --- decides if M halts on x

Now construct input and feed into U’ -> will give YES or NO

Construct M’ st U(M’) says YES then M halts on x

If M halts on x, then the language of M’ contains a string 2 in it

M’ takes in y

Check y = 2y’

YES: simulate M on x and if halts then accept

NO: loop

M’ accepts y with 2 iff M halts on x

If M’ accepts y with 2, then y = 2#y’ AND M halts on x

If M halts on x then 2y’ = y, hence M’ accepts y with 2

There fore in the language of this TM there exists and input with a 2 by construction hence U’ solves the halting problem

The only way for M’ to accept some string is if we ran the HP and accepted

Want U(M’) = HP(M,x)

U says YES: there exists y with 2 st M’ accepts y

Assume

U(M)

c. P accepts null string; Takes in machine M and decides if empty tring in language of L(M)

Assume there exists a TM U takes input i that solves P(i)

Construct a TM U’ st U’ solves the HP

U: takes M

Outputs YES: if null string in language of M (L(M))

Outputs NO: otherwise

Now, want to construct U’

HP takes (M,x) and decides if M halts on x

U’ takes (M,x)

Construct M’ feeds in U

U: decides if empty string in L(M)

The YES/NO answer out of M that maps to HP

Output YES/NO

U(M’) = YES iff HP(M,x) is YES

1. Construct input y that is concat x and x
2. Given a Turing machine M, and an input string x of length n, does M accept x in T(n) = 2n steps?

Construct from M and x a new machine U with input y. Construct y by concatenating x#x. We can simulate M on input y with a universal machine for 2n steps (as x is length n) and accept or reject depending on whether M has halted by that time.

We build a Turing machine U, that given the encode of M

1. Given a Turing machine M, and an input string x of length n, does M accept x in T(n) = 2n steps?
2. Given a Turing machine M, does M accept the empty input (that is all tape positions blank, accept for the left end marker.)
3. Given a Turing machine M, does M accept at least one input x, that has a 2 in some position?
4. Given a Turing machine M, and input x, does M accept input x, without writing any symbols, or leaving the initial segment of the tape where input x is written.