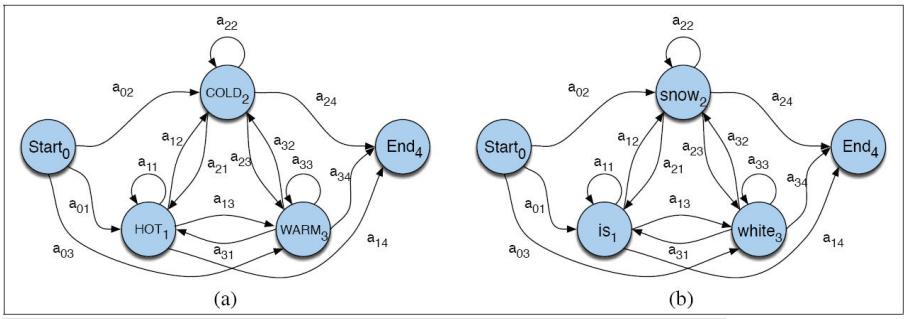
HMMs → MEMMs (→CRFs)

- Algorithms for HMMs
- MaxEnt classifier
- MEMMs

Markov chains

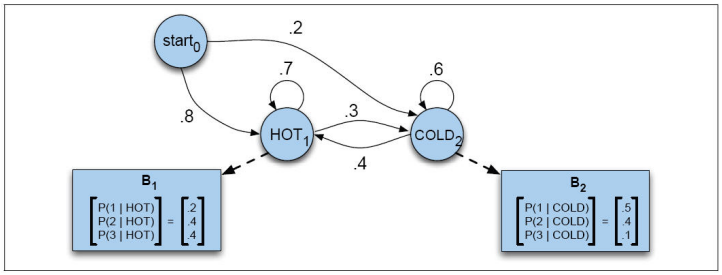


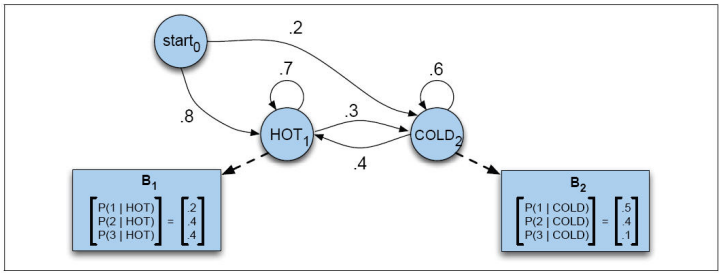
$Q = q_1 q_2 \ldots q_N$	a set of N **states**
$A = a_{01} a_{02} \ldots a_{n1} \ldots a_{nn}$	a **transition probability matrix** A, each a_{ij} representing the probability of moving from state i to state j, s.t. $\sum_{j=1}^{n} a_{ij} = 1 \quad \forall i$
q_0, q_F	a special **start state** and **end (final) state** that are not associated with observations

Assign probability to an unambiguous sequence

Hidden Markov model

- Useful when we want to determine the probability of a sequence of events that is not directly observable, e.g. POS tagging **or temperature (given # of ice cream cones eaten)**





$Q = q_1 q_2 \ldots q_N$	a set of N **states**
$A = a_{11} a_{12} \ldots a_{n1} \ldots a_{nn}$	a **transition probability matrix** A, each a_{ij} representing the probability of moving from state i to state j, s.t. $\sum_{j=1}^{n} a_{ij} = 1 \quad \forall i$
$O = o_1 o_2 \ldots o_T$	a sequence of T **observations**, each one drawn from a vocabulary $V = v_1, v_2, ..., v_V$
$B = b_i(o_t)$	a sequence of **observation likelihoods**, also called **emission probabilities**, each expressing the probability of an observation o_t being generated from a state i
q_0, q_F	a special **start state** and **end (final) state** that are not associated with observations, together with transition probabilities $a_{01} a_{02} \ldots a_{0n}$ out of the start state and $a_{1F} a_{2F} \ldots a_{nF}$ into the end state

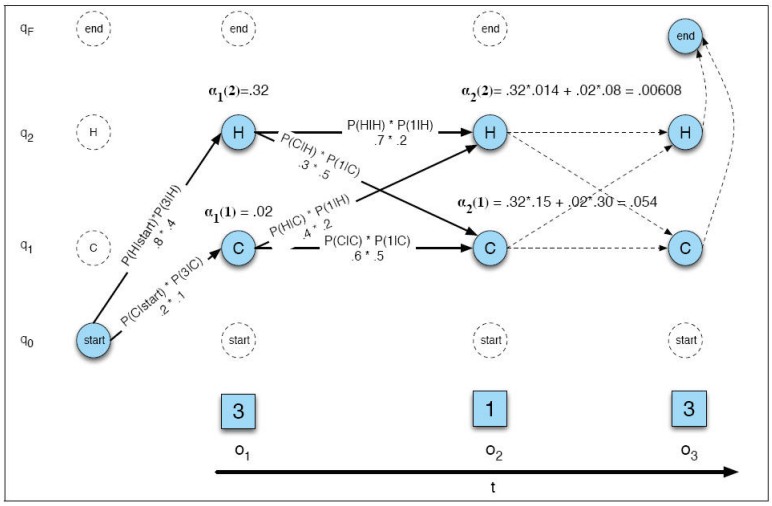
Forward algorithm

Viterbi

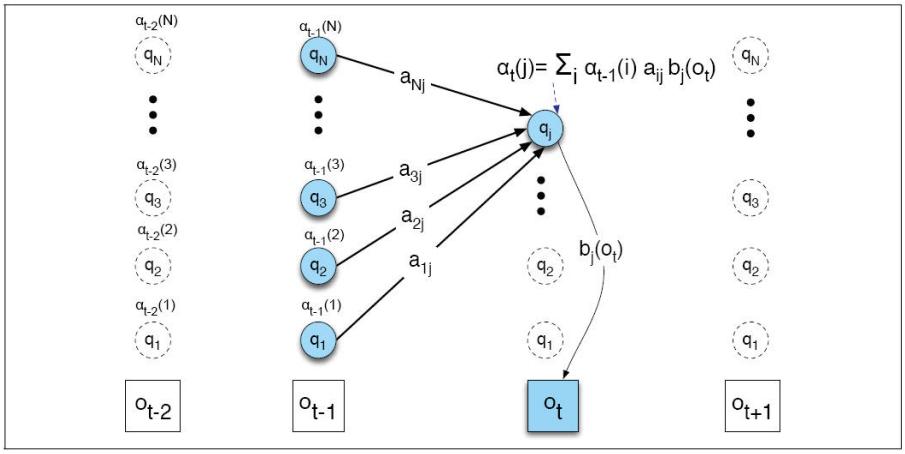
Problem 1 (Likelihood):	Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O\|\lambda)$.
Problem 2 (Decoding):	Given an observation sequence O and an HMM $\lambda = (A, B)$, discover the best hidden state sequence Q.
Problem 3 (Learning):	Given an observation sequence O and the set of states in the HMM, learn the HMM parameters A and B.

Forward-backward algorithm / EM

Forward algorithm



Different from forward pass of Viterbi



function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward*[$N+2,T$]
for each state s **from** 1 **to** N **do** ; initialization step
  $forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F, T]$

Forward algorithm

Viterbi

Problem 1 (Likelihood):	Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O\|\lambda)$.
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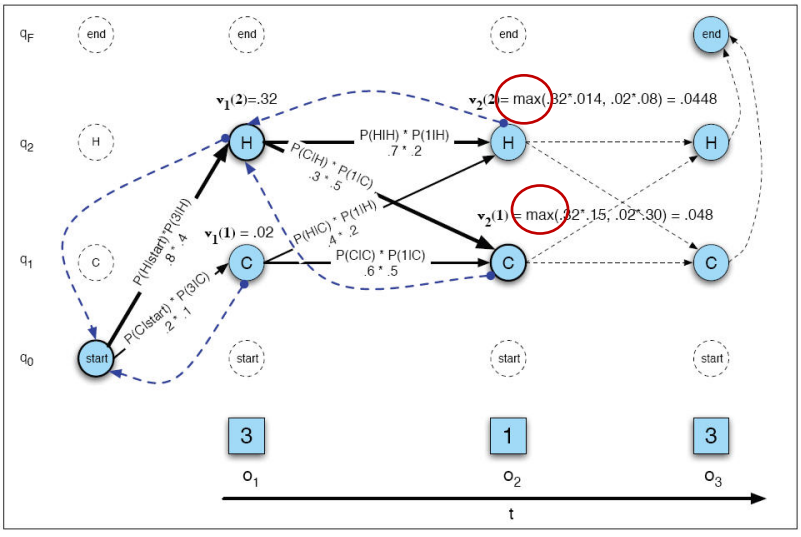
Forward-backward algorithm / EM

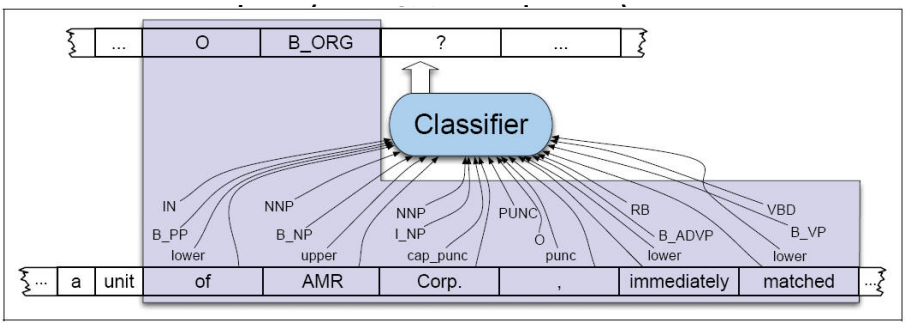
Not covering this

Viterbi



Feature extraction

- We'd like to be able to include lots of features as in classification-based



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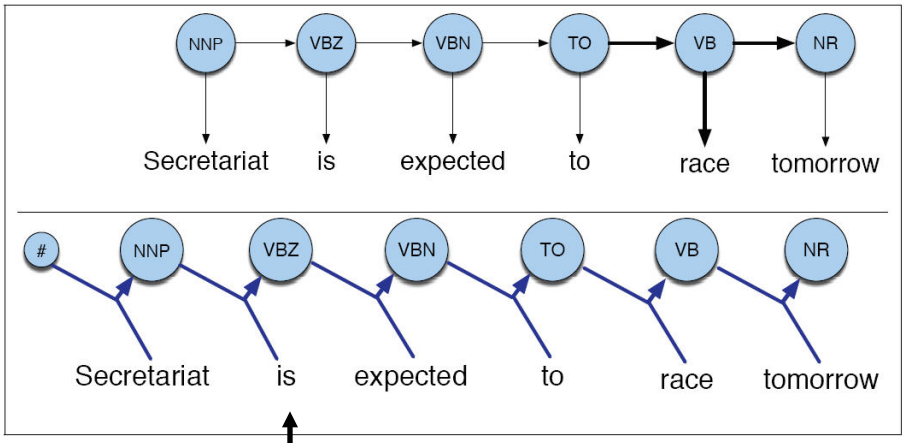
Not possible with HMMs



Maximum entropy Markov model (MEMM)

Solution

- Combine classifier AND Viterbi
- Use MaxEnt classifier
  - Multinomial logistic regression

HMMs → MEMMs (→CRFs)

- Algorithms for HMMs
- MaxEnt classifier
- MEMMs

MaxEnt

- From the class of **exponential** or **log-linear** classifiers
  - Extracts features from the input
  - Combines them linearly
  - Uses the sum as an exponent

c class
x input
w weight
f feature
Z normalization factor

$$p(c|x) = \frac{1}{Z} \exp\left(\sum_i w_i f_i\right)$$

MaxEnt

- A bit of a simplification
- Features *f* and weights *w* depend on the class (so f_i is really $f_i(c,x)$)
- Features are **indicator functions**, return 0 or 1

$$p(c|x) = \frac{\exp\left(\sum_{i=0}^{N} w_{ci} f_i\right)}{\sum_{c' \in C} \exp\left(\sum_{i=0}^{N} w_{c'i} f_i\right)} \leftarrow Z$$

(# of features; # of classes)

Features

- Should **race** be a VB or an NN?

Secretariat/NNP is/BEZ expected/VBN to/TO **race**/?? tomorrow/ADV

$$f_1(c, x) = \begin{cases} 1 & \text{if } word_i = \text{“race”} \ \& \ c = \text{NN} \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(c, x) = \begin{cases} 1 & \text{if } t_{i-1} = \text{TO} \ \& \ c = \text{VB} \\ 0 & \text{otherwise} \end{cases}$$

MaxEnt

- Provides a probability distribution over the classes
- For classification, choose highest
  - $argmax_{c \text{ in } C}$
- In sequence tagging, use the distribution
- See textbook for how to LEARN weights

HMMs → MEMMs (→CRFs)

- Algorithms for HMMs
- MaxEnt classifier
- MEMMs

HMM: find best possible tag sequence
highest probability given the word in the sentence

HMM vs. MEMM

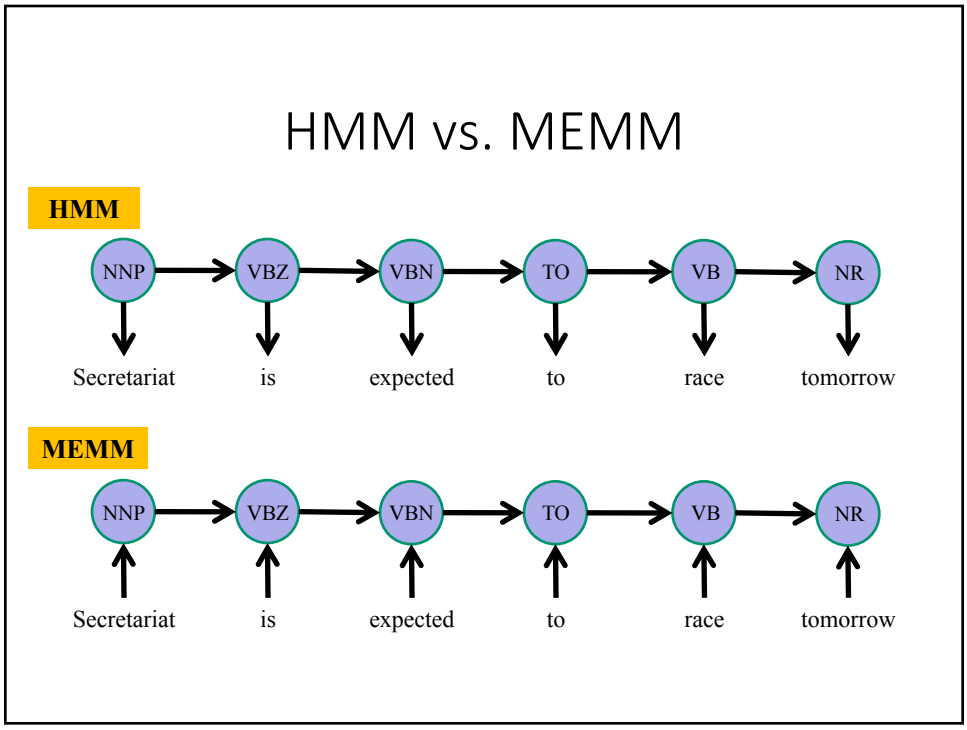
HMM

$$\hat{T} = \operatorname{argmax}_T P(T|W)$$
$$= \operatorname{argmax}_T P(W|T)\,P(T)$$
$$= \operatorname{argmax}_T \prod_i P(word_i|tag_i) \prod_i P(tag_i|tag_{i-1})$$

MEMM

$$\hat{T} = \operatorname{argmax}_T P(T|W)$$
$$= \operatorname{argmax}_T \prod_i P(tag_i|word_i, tag_{i-1})$$

Go through all possible tag sequences w/o exponential time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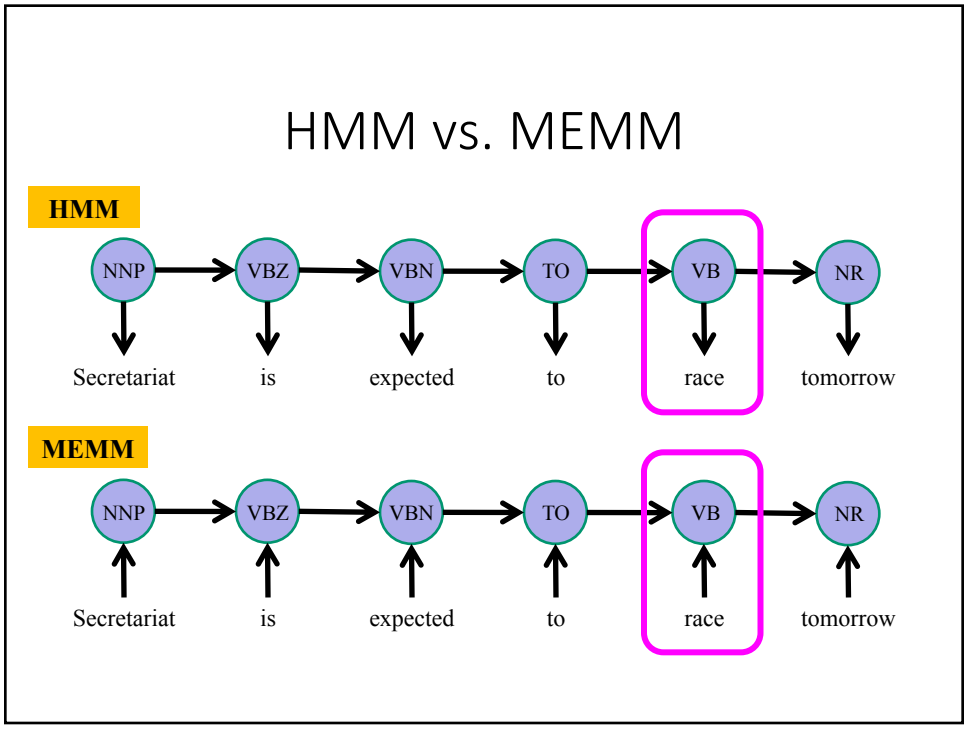
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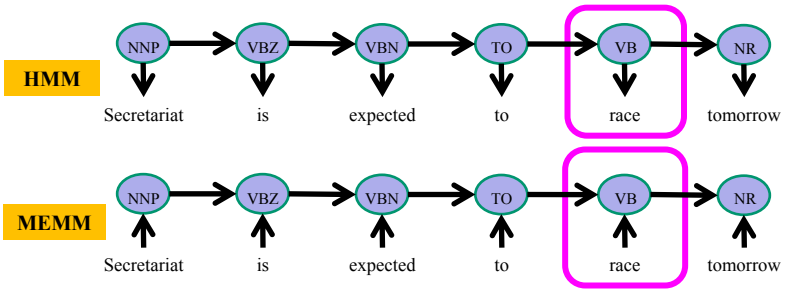
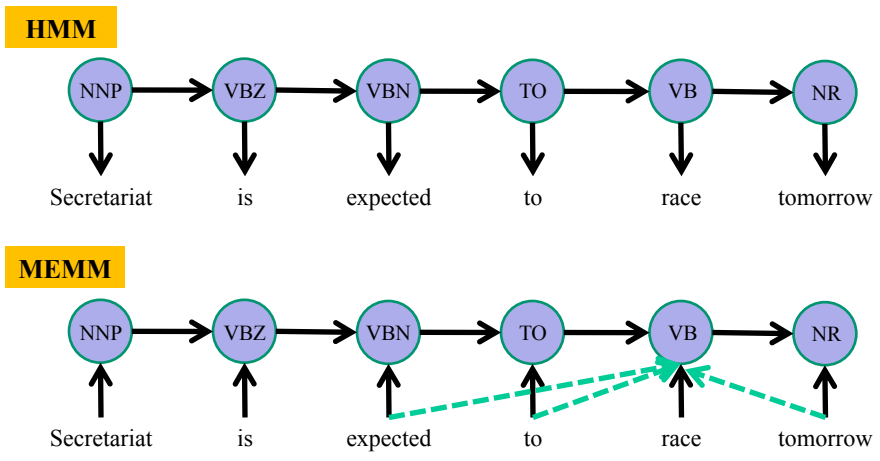


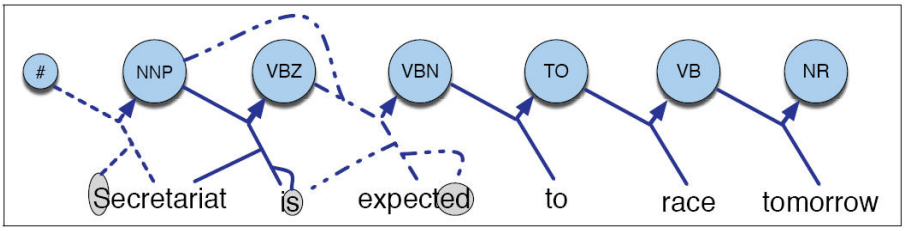
HMM	MEMM
"Generative" models ➔ joint probability **p(words, tags)** ➔"generate" input (in addition to tags) ➔ but we need to predict tags, not words!	"Discriminative" or "Conditional" models ➔ conditional probability **p(tags | words)** ➔"condition" on input ➔ Focusing only on predicting tags
Probability of each slice = emission * transition = p(word_i | tag_i) * p(tag_i | tag_i-1) = ➔ Cannot incorporate long distance features	Probability of each slice = p(tag_i | tag_i-1, word_i) or p(tag_i | tag_i-1, all words) ➔ Can incorporate long distance features

HMM vs. MEMM

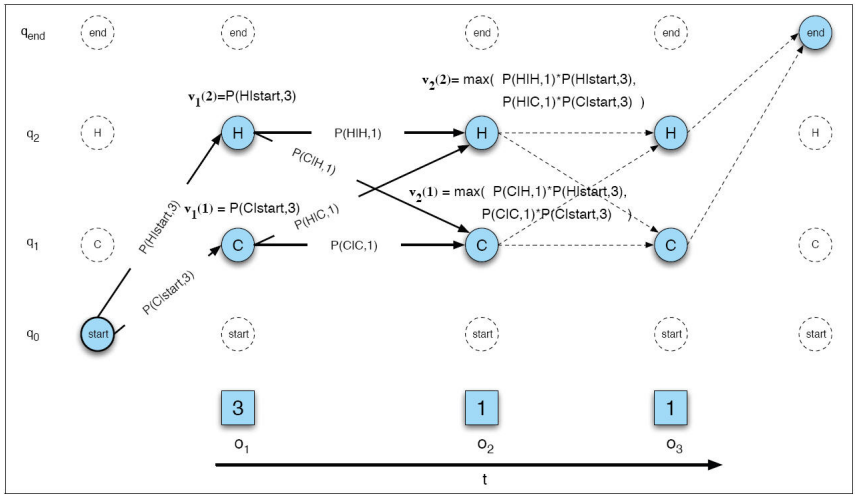


MEMM's

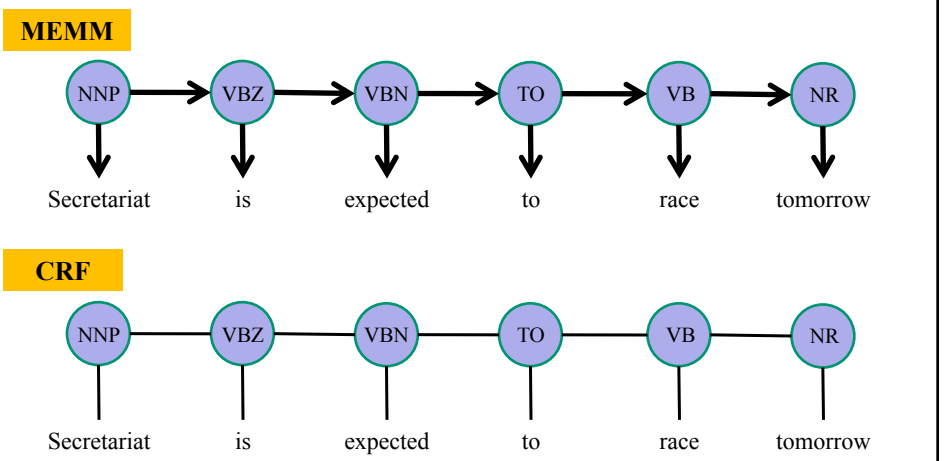
- Can condition on **many** features of the input

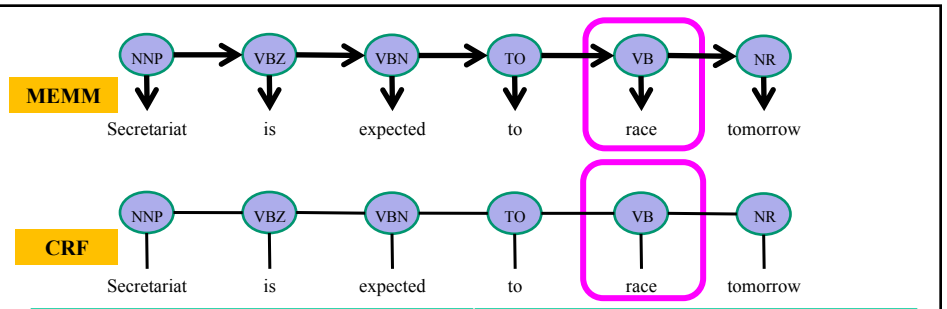


Inference (Viterbi)



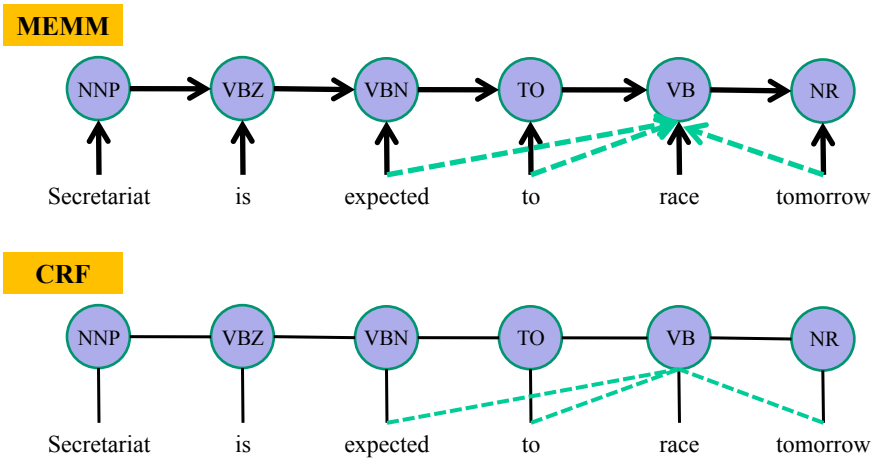
MEMM vs. CRF





MEMM	CRF
Directed graphical model	Undirected graphical model
"Discriminative" or "Conditional" models ➔ conditional probability **p(tags | words)**	
Probability is defined for each slice = P (tag_i | tag_i-1, word_i) or p (tag_i | tag_i-1, all words)	Instead of probability, ***potential (energy function)*** is defined for each slice = ϕ (tag_i, tag_i-1) * ϕ (tag_i, word_i) or ϕ(tag_i, tag_i-1, all words) * ϕ (tag_i, all words)
➔ Can incorporate long distance features	

MEMM vs. CRF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