习题10　10215501435　杨茜雅　DaSE 算法

一、

1. $\nabla f(x,y) = (50x, 2y)$

∴ 在 $(=(0.6, 4)$ 处的梯度向量为 $(30, 8)$

2、在 $(0.6, 4)$ 处的梯度向量为 $(30, 8)$

标准化后 $\left(\frac{15}{\sqrt{241}}, \frac{4}{\sqrt{241}}\right)$

3、$x^{(1)} = x^{(0)} - 0.5\times(50\times0.6, 2\times4) = (0.6, 4) - 0.5\times(30, 8)^T$

$= (0.6, 4) - (15, 4) = (-14.4, 0)$

4、$x^{(1)} = x^{(0)} - 0.5\times(50x, 2y)$

$= (1, 0) - 0.5\times(50, 0)$

$= (1, 0) - (25, 0) = (-24, 0)$

4、

令 $e_{ui}^2 = (r_{ui} - \hat{r}_{ui})^2 = (r_{ui} - p_u^T q_i)^2$

则原式 $= \min\limits_{P^*, Q^*} J(R; P, Q) = \frac{1}{2}\left[\sum_{(u,i)\in K} e_{ui}^2 + \lambda(\|P\|_F^2 + \|Q\|_F^2)\right]$

$$\frac{\partial J(R; P, Q)}{\partial p_{uj}} = -\sum_{i:(u,i)\in K} e_{ui} q_{ji} + \lambda \cdot \cancel{\frac{1}{2}} \cancel{2} p_{uj}$$

$$\frac{\partial J(R; P, Q)}{\partial q_{ji}} = -\sum_{u:(u,i)\in K} e_{ui} p_{uj} + \lambda \cdot \cancel{\frac{1}{2}} \cdot \cancel{2} q_{ji}$$

因此在第 t+1 次迭代中，参数 p_{ij} 和 q_{ji} 的更新公式为

$$p_{uj}^{(t+1)} \leftarrow p_{uj}^{(t)} + \epsilon\left(\sum_{i:(u,i)\in K} e_{ui}^{(t)} q_{ji}^{(t)} - \lambda p_{uj}^{(t)}\right)$$

$$q_{ji}^{(t+1)} \leftarrow q_{ji}^{(t)} + \epsilon\left(\sum_{u:(u,i)\in K} e_{ui}^{(t)} p_{uj}^{(t)} - \lambda q_{ji}^{(t)}\right)$$

其中，$e_{ui}^{(t)} = r_{ui} - \sum_{j=1}^{k} p_{uj}^{(t)} q_{ji}^{(t)}$

写成矩阵形式

$$P^{(t+1)} \leftarrow (1-\epsilon\lambda)P^{(t)} + \epsilon E^{(t)} Q^{(t)}$$

$$Q^{(t+1)} \leftarrow (1-\epsilon\lambda)Q^{(t)} + \epsilon E^{(t)T} P^{(t)}$$

5. $J(R;P,Q) = \frac{1}{2}\sum_{(u,i)\in K}(r_{ui} - p_u^T q_i)^2$

(1) $\alpha f(P) + (1-\alpha)f(X)$

$= \alpha \cdot \frac{1}{2}\sum (r - p_u^T q_i)^2 + (1-\alpha)\cdot\frac{1}{2}\sum(r - X^T q_i)^2$

$= \alpha \cdot \cancel{\frac{1}{2}}\sum[r^2 - 2rp_u^T q_i + (p_u^T q_i)^2] + \cancel{\frac{1}{2}}(1-\alpha)\sum[r^2 - 2rX^T q_i + (X^T q_i)^2]$

$f(\alpha P + (1-\alpha)X) = \cancel{\frac{1}{2}}\sum(r - (\alpha p_u^T + (1-\alpha)X^T)q_i)^2$

$\alpha[\sum \cancel{r^2} - \sum 2r\cancel{p_u^T}q_i + \sum(p_u^T q_i)^2] + \sum\cancel{r^2} - \sum 2\cancel{rX^T}q_i + \sum\cancel{(X^T q_i)^2}$

右 $-\alpha[\sum\cancel{r^2} - \cancel{\sum 2rX^T q_i} + \cancel{\sum(X^T q_i)^2}]$

左 $\sum(\cancel{r^2} - 2r(\cancel{\alpha p_u^T} + (1-\alpha)X^T)q_i + (\alpha p_u^T q_i + (1-\alpha)X^T q_i)^2$

$= \sum \cancel{-2r(1-\alpha)X^T q_i} + \alpha^2 p_u^T q_i^2 + 2\alpha(1-\alpha)p_u^T q_i X^T q_i + \cancel{(1-\alpha)^2} X^T q_i^2$

$\cancel{1-2\alpha+\alpha^2}$

$\therefore$ 右: $\alpha(\sum(p_u^T q_i)^2)$ $\quad \cancel{2(\alpha-\alpha^2)}$

左: $\alpha^2(\sum(p_u^T q_i)^2) + \sum 2\alpha(1-\alpha)p_u^T q_i X^T q_i + (\alpha^2-\alpha)(\sum(X^T q_i)^2)$

左$-$右 $= (\alpha^2-\alpha)(\sum(p_u^T q_i)^2 + \sum(X^T q_i)^2 - 2\sum p_u^T q_i X^T q_i)$

$= \underbrace{(\alpha^2-\alpha)}_{\le 0}\underbrace{(\sum(p_u^T q_i - X^T q_i)^2)}_{\ge 0}$

$\therefore$ 左$-$右 $\le 0 \Rightarrow$ 左 $\le$ 右

$\therefore f(\alpha P + (1-\alpha)X) \le \alpha f(P) + (1-\alpha)f(X)$

$\therefore J(R;P,Q)$ 是关于 P 的凸函数 (Q 同理)

Date

(2) $J(R;P,Q) = \frac{1}{2}\sum_{(u,i)\in K} (r_{ui} - p_u^T q_i)^2$

$\because e_{ui}^2 = (r_{ui} - \hat{r}_{ui})^2 = (r_{ui} - \sum_{j=1}^{k} p_{uj} q_{ji})^2$

$\therefore J(R;P,Q) = \frac{1}{2}\sum_{(u,i)\in K} e_{ui}^2$

$$\frac{\partial J(R;P,Q)}{\partial p_{uj}} = \frac{\partial \frac{1}{2}\sum_{(u,i)\in K} e_{ui}^2}{\partial p_{uj}} = -\sum_{i:(u,i)\in K} e_{ui}\, q_{ji}$$

$$\frac{\partial J(R;P,Q)}{\partial q_{ji}} = \frac{\partial \frac{1}{2}\sum_{(u,i)\in K} e_{ui}^2}{\partial q_{ji}} = -\sum_{u:(u,i)\in K} e_{ui}\, p_{uj}$$

(3)

$\therefore p_{uj}^{(t+1)} \leftarrow p_{uj}^{(t)} + \epsilon \sum_{i:(u,i)\in K} e_{ui}^{(t)} q_{ji}^{(t)}$

$q_{ji}^{(t+1)} \leftarrow q_{ji}^{(t)} + \epsilon \sum_{u:(u,i)\in K} e_{ui}^{(t)} p_{uj}^{(t)}$