数学基础 5th

1、 $\begin{cases} 2x_1+4x_2=5 \\ 4x_1+5x_2=3 \end{cases}$

$$\begin{pmatrix} 2 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$A \quad x = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$A = LU = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 0 & -3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & -3 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 0 & -3 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} -3 & -4 \\ 0 & 2 \end{pmatrix}}{-6} = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} \\ 0 & -\frac{1}{3} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}}{1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} \\ 0 & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{5}{6} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{13}{6} \\ \frac{7}{3} \end{pmatrix}$$

习题 2

LU分解的前提

(1) 方阵　　(2) 可逆　　(3) 对于 $A \in R^{n\times n}$，前n阶顺序主子式≠0

例：$A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$　前1阶顺序主子式为0

分解

$$LU = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} = A = \begin{pmatrix} u_{11} & u_{12} \\ u_{11}l_{21} & l_{21}u_{12}+u_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$$

$\therefore u_{11}=0$，$u_{11}l_{21}=2$　不可能

∴对 A 的行重新排列

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$$A=\begin{pmatrix}0&1\\2&1\end{pmatrix}$$

$A^{(0)}$ $\quad a_{11}^{(0)}=0$ $\quad\therefore$ 与 $a_{21}^{(0)}\neq 0$ 的第2行交换

$$\underset{L_1}{\begin{pmatrix}1&0\\0&1\end{pmatrix}}\underset{(P_1)}{\begin{pmatrix}0&1\\1&0\end{pmatrix}}\underset{A}{\begin{pmatrix}0&1\\2&1\end{pmatrix}}\xRightarrow{R_1\leftrightarrow R_2}\underset{U}{\begin{pmatrix}2&1\\0&1\end{pmatrix}}$$

$$A=L_1^{-1}P_1^{-1}U=\begin{pmatrix}1&0\\0&1\end{pmatrix}\frac{\begin{pmatrix}0&-1\\-1&0\end{pmatrix}}{-1}\begin{pmatrix}2&1\\0&1\end{pmatrix}=\begin{pmatrix}0&1\\2&1\end{pmatrix}$$

$$P_1A=\begin{pmatrix}2&1\\0&1\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}2&1\\0&1\end{pmatrix}=\underbrace{\begin{pmatrix}1&0\\0&1\end{pmatrix}}_{\text{下三角(单位)}}\underbrace{\begin{pmatrix}2&1\\0&1\end{pmatrix}}_{\text{上三角}}$$

$\therefore$ 我们只要对A的行重新排列，就能对重排后的矩阵进行LU分解

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习题3：

$$A=\begin{pmatrix}1&2&2\\2&1&2\\1&2&1\end{pmatrix}\qquad X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\qquad b=\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$a_1\quad a_2\quad a_3$$

$$b_1=a_1=\begin{pmatrix}1\\2\\1\end{pmatrix}\Rightarrow\begin{pmatrix}\frac{1}{\sqrt6}\\\frac{2}{\sqrt6}\\\frac{1}{\sqrt6}\end{pmatrix}$$

$$b_2=a_2-\frac{(a_2,b_1)}{(b_1,b_1)}b_1=\begin{pmatrix}2\\1\\2\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}1\\-1\\1\end{pmatrix}\Rightarrow\begin{pmatrix}\frac{1}{\sqrt3}\\\frac{-1}{\sqrt3}\\\frac{1}{\sqrt3}\end{pmatrix}$$

$$b_3=a_3-\frac{(a_3,b_1)}{(b_1,b_1)}b_1-\frac{(a_3,b_2)}{(b_2,b_2)}b_2=\begin{pmatrix}2\\2\\1\end{pmatrix}-\frac{7}{6}\begin{pmatrix}1\\2\\1\end{pmatrix}-\frac{1}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}\frac12\\0\\-\frac12\end{pmatrix}$$

$$\Downarrow$$

$$\sqrt2\begin{pmatrix}\frac12\\0\\-\frac12\end{pmatrix}$$

$$\begin{pmatrix}\frac{1}{\sqrt6}\\\frac{2}{\sqrt6}\\\frac{1}{\sqrt6}\end{pmatrix}\quad\begin{pmatrix}\frac{1}{\sqrt3}\\-\frac{1}{\sqrt3}\\\frac{1}{\sqrt3}\end{pmatrix}\quad\begin{pmatrix}\frac{\sqrt2}{2}\\0\\-\frac{\sqrt2}{2}\end{pmatrix}$$

$$\Downarrow$$

$$Q=(q_1,q_2,q_3)=\begin{pmatrix}\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&\frac{\sqrt2}{2}\\\frac{2}{\sqrt6}&\frac{-1}{\sqrt3}&0\\\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&\frac{-\sqrt2}{2}\end{pmatrix}$$ 正交矩阵Q

$$b_1=\sqrt6 q_1$$
$$b_2=\sqrt3 q_2$$
$$b_3=\frac{q_3}{\sqrt2}$$

$$a_1=b_1=\sqrt6 q_1$$
$$a_2=b_2+b_1=\sqrt6 q_1+\sqrt3 q_2$$
$$a_3=\frac76 b_1+\frac13 b_2+b_3=\frac76\sqrt6 q_1+\frac13\sqrt3 q_2+\frac{1}{\sqrt2}q_3$$

$$R=\begin{pmatrix}\sqrt6&\sqrt6&\frac{7}{\sqrt6}\\0&\sqrt3&\frac{1}{\sqrt3}\\0&0&\frac{1}{\sqrt2}\end{pmatrix}\qquad\therefore A=QR=\begin{pmatrix}\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&\frac{\sqrt2}{2}\\\frac{2}{\sqrt6}&-\frac{1}{\sqrt3}&0\\\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&\frac{-\sqrt2}{2}\end{pmatrix}\begin{pmatrix}\sqrt6&\sqrt6&\frac{7}{\sqrt6}\\0&\sqrt3&\frac{1}{\sqrt3}\\0&0&\frac{1}{\sqrt2}\end{pmatrix}$$

$$Ax=b\qquad QRx=b$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&\frac{\sqrt2}{2}\\\frac{2}{\sqrt6}&-\frac{1}{\sqrt3}&0\\\frac{1}{\sqrt6}&\frac{1}{\sqrt3}&-\frac{\sqrt2}{2}\end{pmatrix}^{-1}\begin{pmatrix}\sqrt6&\sqrt6&\frac{7}{\sqrt6}\\0&\sqrt3&\frac{1}{\sqrt3}\\0&0&\frac{1}{\sqrt2}\end{pmatrix}^{-1}\begin{pmatrix}1\\2\\3\end{pmatrix}$$

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