机器学习理论作业4

—10215501435 杨茜雅

李航 机器学习 P213 习题10.2

解答思路

1. 列出前向算法

2. 根据前向概率和后向概率，列出单个状态概率的计算公式

3. 自编程实现用前向后向概率计算 $P(i_4=q_3|O,\lambda)$

给定模型 λ 和观测 O，在时刻 t 处状态 q_i 的概率：

$$P(i_t=q_i|O,\pi)=\frac{\alpha_t(i)\beta_t(i)}{P(O|\lambda)}=\frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{N}\alpha_t(j)\beta_t(j)}$$

故由 $$P(i_4=q_3|O,\pi)=\frac{\alpha_4(3)\beta_4(3)}{\sum_{j=1}\alpha_4(j)\beta_4(j)}$$

(1) 计算 α_i（前向）

$$\alpha_1(1)=\pi_1 b_1(O_1)=0.2\times0.5=0.1$$

$$\alpha_1(2)=\pi_2 b_2(O_1)=0.3\times0.4=0.12$$

$$\alpha_1(3)=\pi_3 b_3(O_1)=0.5\times0.7=0.35$$

$$\alpha_2(1)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i1}\right]b_1(O_2)=\left[\alpha_1(1)a_{11}+\alpha_1(2)a_{21}+\alpha_1(3)a_{31}\right]\cdot0.5$$
$$=0.156\times0.5=0.078$$

$$\alpha_2(2)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i2}\right]b_2(O_2)=0.14\times0.6=0.084$$

$$\alpha_2(3)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i3}\right]b_3(O_2)=0.274\times0.3=0.0822$$

$$\alpha_3(1)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i1}\right]b_1(O_3)=0.08064\times0.5=0.04032$$

$$\alpha_3(2)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i2}\right]b_2(O_3)=0.06624\times0.4=0.026496$$

$$\alpha_3(3)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i3}\right]b_3(O_3)=0.09732\times0.7=0.068124$$

$$\alpha_4(1)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_1(O_4)=0.0417336\times0.5=0.0208668$$

$$\alpha_4(2)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_2(O_4)=0.0309048\times0.4=0.01236192$$

$$\alpha_4(3)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_3(O_4)=0.0623016\times0.7=0.04361112$$

(2) 计算 β（后向）

$$A=\begin{pmatrix}0.5 & 0.1 & 0.4\\ 0.3 & 0.5 & 0.2\\ 0.2 & 0.2 & 0.6\end{pmatrix}\quad B=\begin{pmatrix}0.5 & 0.5\\ 0.4 & 0.6\\ 0.7 & 0.3\end{pmatrix}\quad \pi=(0.2,0.3,0.5)^T$$

$\beta_8(i)=1 \qquad i=1、2、3$

$$\beta_7(1)=\sum_{j=1}^{3}a_{1j}b_j(O_8)\beta_8(j)=0.5\times0.5\times1+0.1\times0.6\times1+0.4\times0.3\times1=0.43$$

$$\beta_7(2)=\sum_{j=1}^{3}a_{2j}b_j(O_8)\beta_8(j)=0.3\times0.5\times1+0.5\times0.6\times1+0.2\times0.3\times1=0.51$$

$$\beta_7(3)=\sum_{j=1}^{3}a_{3j}b_j(O_8)\beta_8(j)=0.2\times0.5\times1+0.2\times0.6\times1+0.6\times0.3\times1=0.4$$

$\beta_6(1)=\sum_{j=1}^{3}a_{1j}b_j(O_7)\beta_7(j)=0.1861$ （计算过程同上）

$$\beta_6(2)=\sum_{j=1}^{3}a_{2j}b_j(O_7)\beta_7(j)=0.2415$$

$$\beta_6(3)=\sum_{j=1}^{3}a_{3j}b_j(O_7)\beta_7(j)=0.1762$$

$$\beta_5(1)=\sum_{j=1}^{3}a_{1j}b_j(O_6)\beta_6(j)=0.105521$$

$$\beta_5(2)=\sum_{j=1}^{3}a_{2j}b_j(O_6)\beta_6(j)=0.100883$$

$$\beta_5(3)=\sum_{j=1}^{3}a_{3j}b_j(O_6)\beta_6(j)=0.111934$$

$$\beta_4(1)=\sum_{j=1}^{3}a_{1j}b_j(O_5)\beta_5(j)=0.04586531$$

$$\beta_4(2)=\sum_{j=1}^{3}a_{2j}b_j(O_5)\beta_5(j)=0.05280909$$

$$\beta_4(3)=\sum_{j=1}^{3}a_{3j}b_j(O_5)\beta_5(j)=0.04280618$$

$$\therefore P(i_4=q_3|O,\pi)=\frac{0.04361112\times0.04280618}{\sum_{j=1}^{3}\alpha_4(j)\beta_4(j)}$$

$$\alpha_4(1)\beta_4(1)+\alpha_4(2)\beta_4(2)+\alpha_4(3)\beta_4(3)$$

$$\therefore P(i_4=q_3|O,\pi)=0.5369518$$