杨昀非

hw 3

第三讲

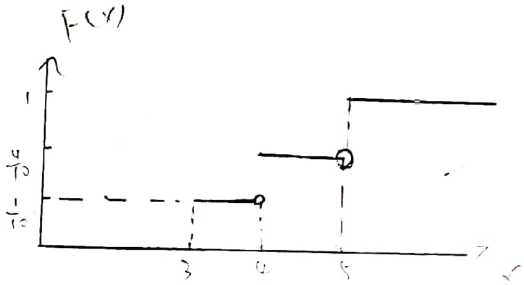
1、 (1) $P(X=3)=\dfrac{1}{C_5^3}=\dfrac{1}{10}$ $\quad P(X=4)=\dfrac{C_3^2}{C_5^3}=\dfrac{3}{10}$

$P(X=5)=\dfrac{C_4^2}{C_5^3}=\dfrac{6}{10}=\dfrac{3}{5}$

X	3	4	5
P	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$

(2)

$$F(x)=\begin{cases}0 & x<3\\ \frac{1}{10} & 3\le x<4\\ \frac{2}{5} & 4\le x<5\\ 1 & x\le 5\end{cases}$$



2、 (1) $F(1)=A$

$\lim\limits_{x\to 1^+} F(x)=1=F(1)=A$

$\therefore A=1$

(2)

$P(0.3<X<0.7)=P(X<0.7)-P(X<0.3)=0.7^2-0.3^2=0.4$

(3) $F(x)=\int_{-\infty}^{+\infty}p(x)\,dx=\begin{cases}0 & x<0\\ Ax^2 & 0\le x<1\\ 1 & x\ge 1\end{cases}$

$p(x)=\dfrac{dF(x)}{dx}=\begin{cases}0 & x<0\\ 2x & 0\le x<1\\ 0 & x\ge 1\end{cases}=\begin{cases}2x & 0<x<1\\ 0 & 其他\end{cases}$

3、 $p(x)=p(-x)$

$\int_{-\infty}^{+\infty}p(x)\,dx=1 \Rightarrow 2\int_0^{+\infty}p(x)\,dx=1$ $\left(\int_0^{+\infty}p(x)\,dx=\frac{1}{2}\right)$

$0.5-\int_0^a p(x)\,dx=\int_0^{+\infty}p(x)\,dx-\int_0^a p(x)\,dx=\int_a^{+\infty}p(x)\,dx$

$F(-a)=\int_{-\infty}^{-a}p(x)\,dx=\int_a^{+\infty}p(x)\,dx$

$F(-a)+F(a)=\int_a^{+\infty}p(x)\,dx+\int_{-\infty}^{a}p(x)\,dx=\int_{-\infty}^{+\infty}p(x)\,dx=1$

$F(-a)=1-F(a)=\int_a^{+\infty}p(x)\,dx=0.5-\int_0^a p(x)\,dx$

(2) $P(|X|<a) = P(-a<x<a) = \int_{-a}^{a} p(x)\,dx = 2\int_0^a p(x)\,dx$

$F(a) = \int_{-\infty}^{a} p(x)\,dx$

$2F(a) - P(|X|<a) = 2\int_{-\infty}^{a} p(x)\,dx - 2\int_0^a p(x)\,dx = 2\left(\int_{-\infty}^{a} p(x)\,dx - \int_0^a p(x)\,dx\right)$

$= 2\int_{-\infty}^{0} p(x)\,dx = \int_{-\infty}^{0} p(x)\,dx + \int_0^{+\infty} p(x)\,dx$

$= \int_{-\infty}^{+\infty} p(x) = 1$

$\therefore 2F(a) - P(|X|<a) = 1$

$\therefore P(|X|<a) = 2F(a) - 1$

(3) $P(|X|>a) = 1 - P(|X|\leq a) = 1 - 2F(a) + 1 = 2 - 2F(a)$

4、 A_1：第一局输　A_2：第一局赢　A_3：第一局平局

B_1：第二局输　B_2：第二局赢　B_3：第三局平局

$P(A_1)=0.6 \quad P(A_2)=0.2 \quad P(A_3)=0.2$

$P(B_1)=0.3 \quad P(B_2)=0.35 \quad P(B_3)=0.35$

$P(X=0) = P(A_1)P(B_1) = 0.6\times0.3 = 0.18$

$P(X=1) = P(A_1)\cdot P(B_3) + P(A_3)P(B_1) = 0.6\times0.35 + 0.2\times0.3 = 0.27$

$P(X=2) = P(A_2)P(B_1) + P(B_2)P(A_1) + P(A_3)P(B_3)$

$= 0.2\times0.3 + 0.35\times0.6 + 0.2\times0.35 = 0.34$

$P(X=3) = P(A_2)P(B_3) + P(B_2)P(A_3) = 0.2\times0.35 + 0.35\times0.2 = 0.14$

$P(X=4) = P(A_2)P(B_2) = 0.2\times0.35 = 0.07$

X	0	1	2	3	4
P	0.18	0.27	0.34	0.14	0.07

5、

(1) $F(x) = \frac{\pi x^2}{\pi r^2} = \left(\frac{x}{r}\right)^2 = \int_0^r p(x)\,dx$

$\therefore p(x) = F'(x) = \frac{2x}{r^2}$

$E(X) = \int_0^r x\,p(x)\,dx = \int_0^r x\cdot\frac{2x}{r^2}\,dx = \frac{2}{r^2}\int_0^r x^2\,dx = \frac{2}{r^2}\,\frac{x^3}{3}\Big|_0^r = \frac{2}{r^2}\cdot\frac{r^3}{3} = \frac{2r}{3}$

$Var(X) = E\left(X-\frac{2r}{3}\right)^2 = E(X^2) - \left(\frac{2r}{3}\right)^2 = \frac{r^2}{2} - \frac{4r^2}{9} = \frac{9r^2-8r^2}{18} = \frac{r^2}{18}$

(2) $F(x) = P(S\leq x) = \begin{cases} 0 & x<0 \\ 1-\frac{t^2}{r^2} & x=0 \\ 1 & x\geq \frac{1}{t} \end{cases}$　S可取0以及$[\frac{1}{t}, +\infty)$上的值

$\therefore$ 不是连续 r.v

第四讲 杨茜雅

1. $P(X(n,p)=k)=\binom{n}{k}p^k(1-p)^{n-k}=C_n^k p^k(1-p)^{n-k}, k=0,1,\dots n$

$P(X(n,p)\le i)=\sum_{k=0}^{i} C_n^k p^k (1-p)^{n-k}, k=0,1,2\dots n$ (i+1 项)

$P(X(n,1-p)\le n-i-1)=\sum_{k=0}^{n-i-1} C_n^k (1-p)^k (p)^{n-k}, k=0,1,\dots n$ (n-i 项)

$C_n^0(1-p)^0(p)^n+C_n^1(1-p)^1(p)^{n-1}+C_n^2(1-p)^2(p)^{n-2}+\cdots C_n^{n-i-1}(1-p)^{n-i-1}(p)^{i+1}$

$C_n^n(p)^n(1-p)^0+C_n^{n-1}(p)^{n-1}(1-p)^1+C_n^{n-2}(p)^{n-2}(1-p)^2-\dots+C_n^{i+1}(p)^{i+1}(1-p)^{n-i-1}$

$=\sum_{k=i+1}^{n} C_n^k (p)^k (1-p)^{n-k}$

$\therefore P(X(n,1-p)\le n-i-1)+P(X(n,p)\le i)=\sum_{k=0}^{i} C_n^k p^k(1-p)^{n-k}+\sum_{k=i+1}^{n} C_n^k(p)^k(1-p)^{n-k}=\sum_{k=0}^{n} C_n^k p^k(1-p)^{n-k}$

$=(p+(1-p))^n=1$

得证

2. $k^2-4\ge 0$

$k^2\ge 4$

$p(k)=\begin{cases}\frac{1}{5}, & 1<x<6\\ 0, & 其他\end{cases}$

$P(k^2\ge 4)=P(k\ge 2)+P(k\le -2)=\int_2^6 \frac{1}{5}dx=\frac{1}{5}(6-2)=\frac{4}{5}$

($P(k\le -2)=0$)

3. $p(x)=\begin{cases}\frac{\frac{1}{2}^2}{\Gamma(2)}x\cdot e^{-0.5x}, & x\ge 0\\ 0 & x<0\end{cases}$

$\Gamma(2)=1\times\Gamma(1)=1$

$p(x)=\begin{cases}\frac{1}{4}x\cdot e^{-\frac{x}{2}}, & x\ge 0\\ 0 & x<0\end{cases}$

$P(X<4)=\int_0^4 \frac{1}{4}x\cdot e^{-\frac{x}{2}}dx=\frac{1}{4}\int_0^4 x\cdot e^{-\frac{x}{2}}dx$

$\int x\cdot e^{-\frac{x}{2}}dx=-2\int x\, d e^{-\frac{x}{2}}=-2(x\cdot e^{-\frac{x}{2}}-\int e^{-\frac{x}{2}}dx)=-2[x\cdot e^{-\frac{x}{2}}-(-2)\cdot e^{-\frac{x}{2}}]$

$=-2[x\cdot e^{-\frac{x}{2}}+2\cdot e^{-\frac{x}{2}}]$

原式 $=\frac{1}{4}\cdot -2(x\cdot e^{-\frac{x}{2}}+2\cdot e^{-\frac{x}{2}})|_0^4=-\frac{1}{2}(4\cdot e^{-2}+2\cdot e^{-2}-2)=-\frac{1}{2}(6\cdot e^{-2}-2)$

$=1-3\cdot e^{-2}=0.5940$

4. $p(x)=\begin{cases}\frac{\Gamma(11)}{\Gamma(2)\Gamma(9)}x(1-x)^8, & 0<x<1\\ 0 & 其他\end{cases}=\begin{cases}90x(1-x)^8, & 0<x<1\\ 0 & 其他\end{cases}$

$\Gamma(11)=10!\quad \Gamma(2)=1\quad \Gamma(9)=8!$

$\frac{\Gamma(11)}{\Gamma(2)\Gamma(9)}=\frac{10!}{8!}=\frac{10\times 9\times 8\dots}{8\times 7\dots}=90$

$P(X<0.1)=\int_0^{0.1} 90x(1-x)^8\,dx = 90\int_0^{0.1} x(1-x)^8\,dx$ 令 $1-x=t$ $x=1-t$

$=-90\int_1^{0.9}(1-t)t^8\,dt$

$=90\int_{0.9}^{1}(1-t)t^8\,dt = 90\int_{0.9}^{1} t^8-t^9\,dt = 90\left[\frac{t^9}{9}\Big|_{0.9}^{1} - \frac{t^{10}}{10}\Big|_{0.9}^{1}\right]$

$=90\left[\frac{1}{9}-\frac{0.9^9}{9} - \left[\frac{1}{10}-\frac{0.9^{10}}{10}\right]\right]=0.2639$

5、 $n=5$时 设事件A：凯尔特人队赢

(a) X为A出现的次数

$X\sim b(n,p)$

$P(X=k)=C_n^k p^k(1-p)^{n-k}$

$1-F(X\le 2) = 1-\sum_{i=0}^{2}P(X=i) = 1-C_5^0p^0(1-p)^5 - C_5^1p^1(1-p)^4 - C_5^2p^2(1-p)^3$

$=1-(1-p)^5-5p(1-p)^4-10p^2(1-p)^3$

$n=3$时

$1-F(X\le 1) = 1-\sum_{i=0}^{1}P(X=i) = 1-C_3^0p^0(1-p)^3 - C_3^1p^1(1-p)^2 = 1-(1-p)^3-3p(1-p)^2$

$\cancel{1}-(1-p)^5-5p(1-p)^4-10p^2(1-p)^3 > \cancel{1}-(1-p)^3-3p(1-p)^2$

$(1-p)^5+5p(1-p)^4+10p^2(1-p)^3 < (1-p)^3+3p(1-p)^2$

$(1-p)^3+5p(1-p)^2+10p^2(1-p) < 1-p+3p = 1+2p$

$\cancel{1+2p}+3p^2-6p^3 < \cancel{1+2p}$

$\cancel{3}p^2<\cancel{6}p^3$

$1<2p$

$p>\frac{1}{2}$

∴当对凯尔特人来说$n=5$好于$n=3$时 p值应大于$\frac{1}{2}$

(b)

$1-F(X\le k) = 1-\sum_{i=0}^{k}P(X=i)$ $1-F(X\le k-1) = 1-\sum_{i=0}^{k-1}P(X=i)$

$\cancel{1}-\sum_{i=0}^{k}P(X=i) > \cancel{1}-\sum_{i=0}^{k-1}P(X=i)$

$\sum_{i=0}^{k}P(X=i) < \sum_{i=0}^{k-1}P(X=i)$

$C_{2k+1}^0p^0(1-p)^{2k+1}+C_{2k+1}^1p^1(1-p)^{2k}+C_{2k+1}^2p^2(1-p)^{2k-1}+\cdots C_{2k+1}^kp^k(1-p)^{k+1} < C_{2k-1}^0p^0(1-p)^{2k-1}+C_{2k-1}^1$

$p^1(1-p)^{2k-2}+C_{2k-1}^2p^2(1-p)^{2k-3}+\cdots C_{2k-1}^{k-1}p^{k-1}\cancel{(1-p)^k}$

$k=1$ $n=3$ $n=1$

$1-F(X\le 1)=1-C_3^0p^0(1-p)^3-C_3^1p^1(1-p)^2$

$1-F(X=0)=1-p$

6、

杨茜雅

$E(X)=\mu=10$

$\sqrt{Var(X)}=10=\sqrt{\sigma^2}=\sigma$

$p(x)=\frac{1}{10\sqrt{2\pi}}e^{-\frac{(x-10)^2}{200}}$ $\quad -\infty<x<\infty$

$F(59)=P(X\le 59)=\Phi(\frac{59-10}{10})=\Phi(4.9)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{4.9}e^{-\frac{t^2}{2}}dt$

$\int_{-\infty}^{4.9}e^{-\frac{t^2}{2}}dt=t\cdot e^{-\frac{t^2}{2}}\Big|_{-\infty}^{4.9}-\int_{-\infty}^{4.9}t\,de^{-\frac{t^2}{2}}$ $=0.9^6 5208$

$=4.9\cdot e^{-\frac{4.9^2}{2}}-\int_{-\infty}^{4.9}t\cdot e^{-\frac{t^2}{2}}\cdot -\frac{2t}{2}dt$

$=4.9\cdot e^{-\frac{4.9^2}{2}}+\int_{-\infty}^{4.9}e^{-\frac{t^2}{2}}\cdot t^2dt$