概统 第十七讲

1、 $X_i \quad (i=1,2\cdots n)$ 独立同分布 $\sim Ge(\theta)$

$\therefore \quad T=\sum_{i=1}^{n} X_i \sim Nb(n,\theta)$

$P(X_1=x_1,\ X_2=x_2 \cdots \ X_n=x_n \mid T=t)$

$$= \frac{P(X_1=x_1,\ X_2=x_2 \cdots X_n=t-\sum_{i=1}^{n-1}x_i)}{P(T=t)}$$

$$= \frac{\prod_{i=1}^{n-1} P(X_i=x_i)\cdot P(X_n=t-\sum_{i=1}^{n-1}x_i)}{\binom{t-1}{n-1}\theta^{n}(1-\theta)^{t-n}}$$

$$= \frac{(1-\theta)^{\sum_{i=1}^{n-1}x_i-(n-1)}\cdot\cancel{\theta^{n-1}}\cdot(1-\theta)^{t-\sum_{i=1}^{n-1}x_i-1}\cdot\cancel{\theta}}{\binom{t-1}{n-1}\cancel{\theta^{n}}(1-\theta)^{t-n}}$$

$$= \frac{(1-\theta)^{\cancel{\sum_{i=1}^{n-1}x_i}-(n-1)+t-\cancel{\sum_{i=1}^{n-1}x_i}-1}}{\binom{t-1}{n-1}(1-\theta)^{t-n}}$$

$$= \frac{\cancel{(1-\theta)^{-n+1+t-1}}}{\binom{t-1}{n-1}\cancel{(1-\theta)^{t-n}}} = \frac{1}{\binom{t-1}{n-1}}$$

与θ无关，所以$T=\sum_{i=1}^{n}x_i$是

充分统计量

2、 $P_{(\theta_1,\theta_2)}(x) = \begin{cases} \frac{1}{\theta_2-\theta_1} & \theta_1 < x < \theta_2 \\ 0 & \text{otherwise} \end{cases}$

$$P_{(\theta_1,\theta_2)}(x_1, x_2 \cdots x_n) = \begin{cases} \left(\frac{1}{\theta_2-\theta_1}\right)^n & \theta_1 < x_i < \theta_2 \quad i=1,2,\cdots n \\ 0 & \text{otherwise} \end{cases}$$

$\theta_1 < x_i < \theta_2 \Rightarrow \theta_1 < x_1, x_2 \cdots x_n < \theta_2 \Rightarrow \theta_1 < X_{(1)} < X_{(n)} < \theta_2$

$\therefore$ 令 $t_1 = X_{(1)} \quad t_2 = X_{(n)}$

则 $g(t_1, t_2, \theta_1, \theta_2) = \left(\frac{1}{\theta_2-\theta_1}\right)^n I\{\theta_1 < t_1 < t_2 < \theta_2\}$

令 $h(x_1, x_2 \cdots x_n) = 1$

$\therefore P(\theta_1, \theta_2)(x_1, x_2 \cdots x_n) = g(t_1, t_2, \theta_1, \theta_2)\, h(x_1, x_2 \cdots x_n)$

$\therefore$ $T = (t_1, t_2) = (X_{(1)}, X_{(n)})$ 为参数 (θ_1, θ_2) 的充分统计量

(3)

$P_{(\mu,\sigma_1^2,\sigma_2^2)}(x_1, x_2 \cdots y_1, y_2 \cdots y_m)$

$$= \frac{1}{(2\pi)^{\frac{n}{2}}\sigma_1^n} \exp\left\{-\frac{\sum_{i=1}^n (x_i-\mu)^2}{2\sigma_1^2}\right\} \cdot \frac{1}{(2\pi)^{\frac{m}{2}}\sigma_2^m} \exp\left\{-\frac{\sum_{i=1}^m (y_i-\mu)^2}{2\sigma_2^2}\right\}$$

$$= (2\pi)^{-(\frac{n}{2}+\frac{m}{2})} \cdot \sigma_1^{-n} \cdot \sigma_2^{-m} \exp\left\{-\left(\frac{\sum_{i=1}^n (x_i-\mu)^2}{2\sigma_1^2} + \frac{\sum_{i=1}^m (y_i-\mu)^2}{2\sigma_2^2}\right)\right\} \quad \begin{matrix} -\infty < x_i < +\infty \\ -\infty < y_i < +\infty \end{matrix}$$

$$= 2\pi^{-\frac{1}{2}(n+m)} \cdot \sigma_1^{-n} \cdot \sigma_2^{-m} \exp\left\{-\left(\frac{\sum_{i=1}^n x_i^2 - 2\mu \overset{n\bar{x}}{\uparrow}{\sum_{i=1}^n x_i} + n\mu^2}{2\sigma_1^2} + \frac{\sum_{i=1}^m y_i^2 - 2\mu \overset{m\bar{y}}{\uparrow}{\sum_{i=1}^m y_i} + m\mu^2}{2\sigma_2^2}\right)\right\}$$

$$= 2\pi^{-\frac{1}{2}(n+m)} \sigma_1^{-n} \cdot \sigma_2^{-m} \exp\left\{-\frac{1}{2}\left[\left(\frac{n}{2\sigma_1^2} + \frac{m}{2\sigma_2^2}\right)\mu^2 - \left(\frac{n\bar{x}}{\sigma_1^2} + \frac{m\bar{y}}{\sigma_2^2}\right)\mu + \frac{\sum_{i=1}^n x_i^2}{2\sigma_1^2} + \frac{\sum_{i=1}^m y_i^2}{2\sigma_2^2}\right]\right\}$$

令 $T = (t_1, t_2, t_3, t_4) = (\bar{x}, \bar{y}, \sum_{i=1}^n x_i^2, \sum_{i=1}^m y_i^2)$

则 $g(t, \mu, \sigma_1^2, \sigma_2^2) = 2\pi^{-\frac{1}{2}(n+m)} \sigma_1^{-n} \cdot \sigma_2^{-m} \exp\left\{-\frac{1}{2}\left[\left(\frac{n}{2\sigma_1^2} + \frac{m}{2\sigma_2^2}\right)\mu^2 - \left(\frac{nt_1}{\sigma_1^2} + \frac{mt_2}{\sigma_2^2}\right)\mu + \frac{t_3}{2\sigma_1^2} + \frac{t_4}{2\sigma_2^2}\right]\right\}$

$h(x_1, x_2 \cdots x_n, y_1 \cdots y_m) = 1$

$\therefore P_{(\mu,\sigma_1^2,\sigma_2^2)}(x_1, x_2 \cdots x_n, y_1 \cdots y_m) = g(t, \mu, \sigma_1^2, \sigma_2^2)\, h(x_1, x_2 \cdots x_n, y_1 \cdots y_m)$

$\therefore T = (t_1, t_2, t_3, t_4) = (\bar{x}, \bar{y}, \sum_{i=1}^n x_i^2, \sum_{i=1}^m y_i^2)$ 为 $(\mu, \sigma_1^2, \sigma_2^2)$ 的充分统计量