概统 第五讲 hw

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1、 $E(X)=\int_0^1 (a+bx^2)\cdot x\,dx=\int_0^1 ax+bx^3\,dx= a\frac{x^2}{2}\Big|_0^1+b\frac{x^4}{4}\Big|_0^1= a\times\frac12+b\times\frac14=\frac a2+\frac b4$

$\frac a2+\frac b4=\frac23$

$\int_{-\infty}^{+\infty} p(x)\,dx=1 = \int_{-\infty}^{+\infty} a+bx^2\,dx=\int_0^1 a+bx^2\,dx = a+b\frac13=1$

$$\begin{cases}6a+3b=8\\ 3a+b=3\end{cases} \Rightarrow \begin{cases}b=2\\ a=\frac13\end{cases}$$

2、 $x^2=t$, $x=\sqrt t=t^{\frac12}$, $dx=\frac12 t^{-\frac12}dt=\frac12\frac{1}{\sqrt t}dt$

$p(x)=F'(x)=\cancel{-}e^{-x^2}\cdot\cancel{-}2x=2x\cdot e^{-x^2}\quad x>0$

$E(X)=\int_0^{+\infty}2x\cdot e^{-x^2}\cdot x\,dx=2\int_0^{+\infty}x^2\cdot e^{-x^2}dx=\cancel{2}\int_0^{+\infty}t\cdot e^{-t}\cdot\cancel{\frac12}\frac{1}{\sqrt t}\,dt$

$=\int_0^{+\infty}\sqrt t\cdot e^{-t}\,dt=\Gamma(\frac32)=\frac12\Gamma(\frac12)=\frac12\times\sqrt\pi=\frac{\sqrt\pi}{2}$

$Var(X)=E((X-E(X))^2)=E(X^2-2XE(X)+E(X)^2)=E(X^2)-\underset{\downarrow\ \frac{\pi}{4}}{E^2(X)}$

$E(X^2)=\int_0^{+\infty}x^2\cdot 2x\cdot e^{-x^2}dx \overset{x^2=t}{=} \int_0^{+\infty}t\cdot\cancel{2}\cancel{\sqrt t}\cdot e^{-t}\cancel{\frac{1}{2\sqrt t}}\,dt=\int_0^{+\infty}t\cdot e^{-t}\,dt$

$=\Gamma(2)=1\times\Gamma(1)=1$

$\therefore\ Var(X)=1-\frac{\pi}{4}$

3、 $p(x)=\begin{cases}\frac{1}{b-a} & a<x<b\\ 0 & 其他\end{cases}$ $\qquad \beta_s=\frac{\nu_3}{\nu_2^{\frac32}}=\frac{E(X-E(X))^3}{(Var(X))^{\frac32}}$

$\mu_k=E(X^k)=\int_a^b x^k\frac{1}{b-a}dx=\frac{1}{b-a}\cdot\frac{x^{k+1}}{k+1}\Big|_a^b=\frac{1}{b-a}\left(\frac{b^{k+1}-a^{k+1}}{k+1}\right)=\frac{b^{k+1}-a^{k+1}}{(b-a)(k+1)}$

$\mu_1=E(X)=\frac{b^2-a^2}{2(b-a)}=\frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}=\frac{b+a}{2}$ $\qquad \mu_2=E(X^2)=\frac{b^3-a^3}{(b-a)3}=\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}}=\frac{b^2+ab+a^2}{3}$

$\mu_3=E(X^3)=\frac{b^4-a^4}{(b-a)4}=\frac{(b^2-a^2)(b^2+a^2)}{4(b-a)}=\frac{(b+a)\cancel{(b-a)}(b^2+a^2)}{4\cancel{(b-a)}}=\frac{(b+a)(b^2+a^2)}{4}$

$\mu_4=E(X^4)=\frac{b^5-a^5}{5(b-a)}$

$\nu_1=E(X-E(X))=\int_a^b(x-\frac{a+b}{2})\cdot\frac{1}{b-a}dx=0$

$\nu_2=E(X-\frac{a+b}{2})^2=E(X^2)-(a+b)E(X)+(\frac{a+b}{2})^2=\frac{(b-a)^2}{12}$

$v_3 = E(X-\frac{a+b}{2})^3 = E(X^3) - \frac{a+b}{2}E(X^2) - (a+b)E(X^2) + E(X)\cdot\frac{(a+b)^2}{2} + (\frac{a+b}{2})^2 E(X) - (\frac{a+b}{2})^3$

$=0$

$v_4 = E(X-\frac{a+b}{2})^4 = \frac{\cancel{a^4+b^4}\ (b-a)^4}{80}$

$\beta_s = 0 \qquad \beta_k = \frac{v_4}{v_2^2} - 3 = \frac{\cancel{(b-a)^4}}{80} \times \frac{12^2}{\cancel{(b-a)^4}} - 3 = -1.2$

4、 $F(x_p) = \int_{-\infty}^{x_p} p(x)\,dx = p = 1-\exp\{-(\frac{x_p}{\eta})^m\}$

$\exp\{-(\frac{x_p}{\eta})^m\} = 1-p$

$(\frac{x_p}{\eta})^m = -\ln(1-p)$

$(\frac{x_p}{\eta}) = \sqrt[m]{-\ln(1-p)}$

$x_p = \eta\cdot\sqrt[m]{-\ln(1-p)} = \eta\left[(-\ln(1-p))^{\frac{1}{m}}\right]$

$m=1.5 \quad \eta=1000$时

$x_p = 1000\sqrt[1.5]{-\ln(1-p)}$

$x_{0.1} = 223.08$

$x_{0.5} = 1000\left[(-\ln(0.5))^{\frac{1}{1.5}}\right] = 783.22$

$x_{0.8} = 1000\left[(-\ln(0.2))^{\frac{1}{1.5}}\right] = 1373.36$

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概统第六讲

1. $X<0$ 时 $g(X)=-1=Y$

$X\geq 0$ 时 $g(X)=1=Y$

$\int_{-\infty}^{+\infty}\frac{2}{\pi}\frac{1}{e^{x}+e^{-x}}dx=1$

$\because$ $P(X)$ 为偶函数

$\therefore \int_{0}^{+\infty}\frac{2}{\pi}\frac{1}{e^{x}+e^{-x}}dx=\frac{1}{2}=\int_{-\infty}^{0}\frac{2}{\pi}\frac{1}{e^{x}+e^{-x}}dx$

$P(Y=-1)=P(X<0)=F(0)=\int_{-\infty}^{0}\frac{2}{\pi}\frac{1}{e^{x}+e^{-x}}dx=\frac{1}{2}$

$P(Y=1)=P(X\geq 0)=1-P(X<0)=1-\frac{1}{2}=\frac{1}{2}$

Y	-1	1
P	0.5	0.5

2. $X\sim U(0,1)$

$P(x)=\begin{cases}1 & 0<x<1\\ 0 & 其他\end{cases}$

$Y=\frac{\pi X^2}{4}=g(X)$

$\sqrt{\frac{4Y}{\pi}}=X=h(y)$

$h'(y)=\frac{\frac{2\cdot 4}{\pi}\times\frac{1}{2}}{\sqrt{\frac{4y}{\pi}}}=\frac{2}{\pi}\frac{\sqrt{\pi}}{\sqrt{4y}}=\frac{1}{\sqrt{\pi y}}$

$P_Y(y)=\begin{cases}\frac{1}{\sqrt{y\pi}} & 0<y<\frac{\pi}{4}\\ 0 & 其他\end{cases}$

3. $Y_1=3X$ $\quad X=\frac{Y_1}{3}=h(y)$ $\quad h'(y)=\frac{1}{3}$

$\frac{3}{2}(\frac{y}{3})^2\times\frac{1}{3}$ $\quad \frac{3}{2}\frac{y^2}{9}\times\frac{1}{3}=\frac{y^2}{18}$

(1) $P_{Y_1}(y)=\begin{cases}\frac{y_1^2}{18} & -3<y<3\\ 0 & 其他\end{cases}$

$F_{Y_1}(y)=\begin{cases}1 & y\geq 3\\ \frac{y^3+27}{54} & -3\leq y<3\\ 0 & y<-3\end{cases}$

(2) $Y_2=3-X=g(X)$ $\quad h(y)=3-y$ $\quad h'(y)=-1$

$\frac{3}{2}(3-y)^2\times -1$

$P_{Y_2}(y)=\begin{cases}\frac{3}{2}(y-3)^2 & 2<y<4\\ 0 & 其他\end{cases}$

$F_{Y_2}(y)=\begin{cases}1 & y\geq 4\\ \frac{y^3}{2}-\frac{9y^2}{2}+\frac{27y}{2}-13 & 2\leq y<4\\ 0 & y<2\end{cases}$

(3) ~~$Y_3=X^2=g(X)$ $\quad h(y)=\begin{cases}\sqrt{y}\\ -\sqrt{y}\end{cases}$~~ $\quad 0<Y<1$

$\int_{-\infty}^{\sqrt{y}}P_X(x)dx$

$F_{Y_3}(y)=P_{Y_3}(Y_3\leq y)=P_X(X^2\leq y)=P_X(-\sqrt{y}\leq X\leq\sqrt{y})=F_X(\sqrt{y})-F_X(-\sqrt{y})$

$P_{Y_3}(y)=P_X(\sqrt{y})\frac{1}{2}\frac{1}{\sqrt{y}}-P_X(-\sqrt{y})\cdot -\frac{1}{2\sqrt{y}}=\frac{1}{2\sqrt{y}}(\frac{3}{2}y+\frac{3}{2}y)=\frac{3\sqrt{y}}{2}$

$$\therefore \quad p_{Y_3}(y)=\begin{cases}\dfrac{3\sqrt{y}}{2} & 0<y<1\\ 0 & \text{其他}\end{cases} \qquad F_{Y_3}(y)=\begin{cases}1 & y\geq 1\\ y^{\frac{3}{2}} & 0\leq y<1\\ 0 & y<0\end{cases}$$

4.

X	0	1	2	3	4	5	6	7	8	9
P	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

(1)

Y	0 ($x=0\ 3\ 6\ 9$)	1 ($x=1,4,7$)	2 ($x=2,5,8$)
P	$\frac{1}{10}\times 4$	$\frac{1}{10}\times 3$	$\frac{1}{10}\times 3$

Y	0	1	2
P	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{3}{10}$

(2)

Y	0 ($x=0,4$)	1 ($x=1,3$)	2 ($x=2$)	5 ($x=5,6,7,8,9$)
P	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{2}$