概统 hw 7-8

1、

$2\times2\times2$ 种

8

(a)

X_1	X_2	X_3	P
1	0	0	0.1632
1	1	0	0.0932
1	1	1	0.0350
1	0	1	0.0932
0	1	0	0.1632
0	0	0	0.1958
0	0	1	0.1632
0	1	1	0.0932

$$P(X_1=1, X_2=0, X_3=0) = P(X_1=0, X_2=1, X_3=0)$$
$$= P(X_1=0, X_2=0, X_3=1) = \frac{5}{13}\times\frac{8}{12}\times\frac{7}{11} = \frac{70}{429}$$
$$= 0.1632$$

$$P(X_1=0, X_2=0, X_3=0) = \frac{C_8^3}{C_{13}^3} = \frac{28}{143} = 0.1958$$

$$P(X_1=1, X_2=1, X_3=1) = \frac{C_5^3}{C_{13}^3} = \frac{5}{143} = 0.035$$

$$P(X_1=0, X_2=1, X_3=1) = P(X_1=1, X_2=0, X_3=1) =$$
$$P(X_1=1, X_2=1, X_3=0) = \frac{5}{13}\times\frac{4}{12}\times\frac{8}{11} = \frac{40}{429} = 0.0932$$

(b)

X_1 \ X_2	0	1
1	0.2564	0.1282
0	0.3590	0.2564

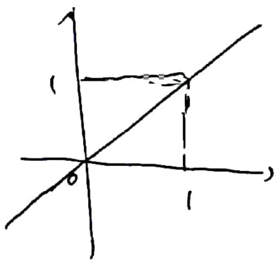
$$P(X_1=1, X_2=0) = \frac{70}{429}+\frac{40}{429} = \frac{10}{39} = 0.2564$$

$$P(X_1=1, X_2=1) = \frac{40}{429}+\frac{5}{143} = \frac{5}{39} = 0.1282$$

$$P(X_1=0, X_2=0) = \frac{28}{143}+\frac{70}{429} = \frac{14}{39} = 0.3590$$

$$P(X_1=0, X_2=1) = \frac{70}{429}+\frac{40}{429} = \frac{10}{39} = 0.2564$$

2、



$$P(X>0.5, Y>0.5) = \int_{0.5}^{\infty}\int_{0.5}^{+\infty} 6(1-y)\,dx\,dy$$
$$= \int_{0.5}^{1}\int_{0.5}^{y} 6(1-y)\,dx\,dy$$
$$= 6\int_{0.5}^{1} (y-0.5) - y(y-0.5)\,dy$$
$$= 6\int_{0.5}^{1} y-0.5-y^2+0.5y\,dy = 6\int_{0.5}^{1} -y^2+1.5y-0.5\,dy$$
$$= 6\cdot\left(-\frac{y^3}{3}\Big|_{0.5}^{1} + 1.5\frac{y^2}{2}\Big|_{0.5}^{1} - 0.5y\Big|_{0.5}^{1}\right)$$
$$= 6\left(-\frac{1}{3}+\frac{0.5^3}{3}+\frac{1.5}{2}-1.5\frac{0.5^2}{2}-0.5\times0.5\right)$$
$$= 6\times\frac{1}{48} = \frac{1}{8}$$

(b) $P(X<0.5)=\int_0^{0.5}\int_x^1 6(1-y)\,dy\,dx = 6\int_0^{0.5}dx\int_x^1 1-y\,dy = 6\int_0^{0.5}(1-x)-\frac{y^2}{2}\Big|_x^1\,dx$

$=6\int_0^{0.5}1-x-\frac{1}{2}+\frac{x^2}{2}\,dx = 6\int_0^{0.5}\frac{x^2}{2}-x+\frac{1}{2}\,dx = 6\left(\frac{x^3}{6}\Big|_0^{0.5}-\frac{x^2}{2}\Big|_0^{0.5}+\frac{1}{2}x\Big|_0^{0.5}\right)$

$=6\left(\frac{0.5^3}{6}-\frac{0.5^2}{2}+\frac{1}{2}\times 0.5\right)=\frac{7}{8}$

$P(Y<0.5)=\int_0^{0.5}dx\int_x^{0.5}6(1-y)\,dy$

$=6\int_0^{0.5}dx\int_x^{0.5}1-y\,dy$

$=6\int_0^{0.5}(0.5-x)-\frac{y^2}{2}\Big|_x^{0.5}\,dx$

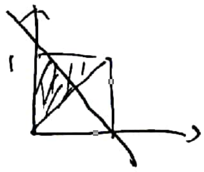
$=6\int_0^{0.5}0.5-x-\frac{0.5^2}{2}+\frac{x^2}{2}\,dx$

$=6\int_0^{0.5}\frac{x^2}{2}-x-\frac{0.5^2}{2}+0.5\,dx$

$=6\left(\frac{1}{2}\times\frac{x^3}{3}\Big|_0^{0.5}-\frac{x^2}{2}\Big|_0^{0.5}+\left(0.5-\frac{0.5^2}{2}\right)\times 0.5\right)$

$=6\left(\frac{1}{2}\times\frac{0.5^3}{3}-\frac{0.5^2}{2}+\frac{3}{16}\right)=\frac{1}{2}$

(c) $Y<1-X$



$P(X+Y<1)=\int_0^{0.5}dx\int_x^{1-x}6(1-y)\,dy$

$=6\int_0^{0.5}(1-2x)-\frac{y^2}{2}\Big|_x^{1-x}\,dx$

$=6\int_0^{0.5}1-2x-\frac{(1-x)^2}{2}+\frac{x^2}{2}\,dx$

$=6\int_0^{0.5}\frac{1}{2}-x\,dx = 6\left(\frac{1}{2}x\Big|_0^{0.5}-\frac{x^2}{2}\Big|_0^{0.5}\right)$

$=6\left(\frac{1}{2}\times 0.5-\frac{0.5^2}{2}\right)=\frac{3}{4}$

3、 $P(X=x_2, Y=y_1)=\frac{1}{9}$ $\quad P(X=x_2, Y=y_3)=\frac{1}{3}$

$=P(X=x_2)\cdot P(Y=y_1)$ $\quad =P(X=x_2)\cdot P(Y=y_3)$

$3(a+\frac{1}{9})=c+\frac{1}{3}$

$3a+\cancel{\frac{1}{3}}=c+\cancel{\frac{1}{3}}$

$a+b+c=1-\frac{2}{9}-\frac{1}{3}=\frac{4}{9}$

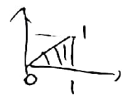
$P(X=x_1, Y=y_1)=P(X=x_1)\cdot P(Y=y_1)=(a+c+\frac{1}{9})\cdot(a+\frac{1}{9})=(4a+\frac{1}{9})(a+\frac{1}{9})=a$

$4a^2+\frac{5}{9}a+\frac{1}{81}=a \Rightarrow a=\frac{1}{18}\quad c=\frac{1}{6}\quad b=\frac{2}{9}$

4.

(1) $P_X(x) = \int_0^x 3x\,dy = 3\cdot x\cdot x = 3x^2$

$0 \le y < x \le 1$



$P_Y(y) = \int_y^1 3x\,dx = 3\cdot\frac{x^2}{2}\Big|_y^1 = \frac{3}{2}(1-y^2)$

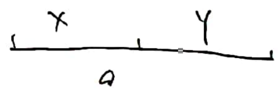
$\therefore P_X(x) = \begin{cases} 3x^2 & 0<x<1 \\ 0 & \text{其他} \end{cases}$ $\quad P_Y(y) = \begin{cases} \frac{3}{2}(1-y^2) & 0<y<1 \\ 0 & \text{其他} \end{cases}$

(2) ~~在~~

$P_X(x)\cdot P_Y(y) = \frac{9}{2}x^2(1-y^2) \ne P(x,y) = 3x$

$\therefore$ X与Y不独立

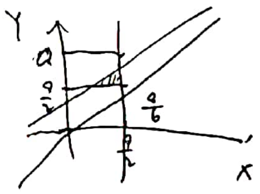
5.



$X \sim U(0, \frac{a}{2})$

$Y \sim U(\frac{a}{2}, a)$ 且相互独立

$P(x,y) = P_X(x)P_Y(y) = \begin{cases} \frac{2}{a}\times\frac{2}{a} = \frac{4}{a^2} & 0<x<\frac{a}{2},\ \frac{a}{2}<y<a \\ 0 & \text{其他} \end{cases}$



$P(|X-Y|<\frac{a}{3}) = \int_{\frac{a}{6}}^{\frac{a}{2}} dx \int_{\frac{a}{2}}^{\frac{a}{3}+x} \frac{4}{a^2}\,dy$

$X-Y<\frac{a}{3}$

$Y > X-\frac{a}{3}$

$Y-X<\frac{a}{3}$

$Y<\frac{a}{3}+X$

$= \int_{\frac{a}{6}}^{\frac{a}{2}} \cancel{dx}\ \frac{4}{a^2}(x-\frac{1}{6}a)\,dx$

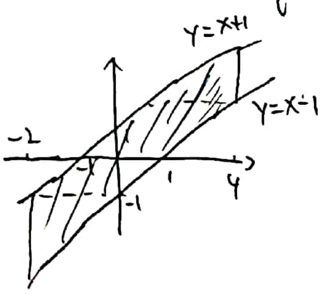
$= \frac{4}{a^2}\int_{\frac{a}{6}}^{\frac{a}{2}} x-\frac{1}{6}a\,dx$

$= \frac{4}{a^2}\left(\frac{x^2}{2}\Big|_{\frac{a}{6}}^{\frac{a}{2}} - \frac{1}{6}a(\frac{a}{3})\right)$

$= \frac{4}{a^2}(\frac{1}{18}a^2) = \frac{2}{9}$

6. $P(x,y) = \begin{cases} \frac{1}{12}, & -2\le x\le 4,\ -1\le y-x\le 1 \\ 0 & \text{其他} \end{cases}$

$x-1\le y\le x+1$



$\int_{-2}^{4} dx \int_{x-1}^{x+1} k\,dy$

$\int_{-2}^{4} k\cdot(\cancel{x}+1-\cancel{x}+1)\,dx = 1$

$2k\cdot 6 = 1$

$k = \frac{1}{12}$

$-3\le y\le -1$

$P_Y(y) = \int_{-2}^{y+1} \frac{1}{12}\,dx = \frac{y+3}{12}$

$-1\le y\le 3$

$P_Y(y) = \int_{y-1}^{y+1} \frac{1}{12}\,dx = \frac{1}{6}$

$3\le y\le 5$

$P_Y(y) = \int_{y-1}^{4} \frac{1}{12}\,dy = \frac{5-y}{12}$

$P_X(x) = \int_{x-1}^{x+1} \frac{1}{12}\,dy = \frac{1}{12}\times 2 = \frac{1}{6}$

$P_Y(y) = \begin{cases} \frac{y+3}{12} & -3\le y\le -1 \\ \frac{1}{6} & -1\le y\le 3 \\ \frac{5-y}{12} & 3\le y\le 5 \end{cases}$

$P_X(x) = \begin{cases} \frac{1}{6} & -2\le x\le 4 \\ 0 & \text{其他} \end{cases}$ $\quad E(X) = \int_{-2}^{4} x\frac{1}{6}\,dx = 1$

$E(Y) = 3$

$$E(Y) = \int_{-\infty}^{+\infty} y \cdot p_Y(y)\, dy = \int_{-3}^{-1} y \cdot \frac{y+3}{12}\, dy + \int_{-1}^{3} y \cdot \frac{1}{6}\, dy + \int_{3}^{5} y \cdot \frac{5-y}{12}\, dy$$

$$= \frac{31}{18} + \frac{1}{6} \times 4 + \frac{11}{18} = 3$$

(b) $E(100X + 200Y) = E(100X) + E(200Y) = 100E(X) + 200E(Y) = 100 \times 1 + 200 \times 3$

$= 700$

第八讲概统

1、 有n颗骰子，设有 $X_1, X_2 \cdots X_n$，n个随机变量，X_i = 第i颗骰子出现的点数

X_i 的分布列

X_i	1	2	3	4	5	6
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\therefore E(X_i) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$

$E(\sum_{i=1}^{n} X_i) = \frac{7}{2} \times n = \frac{7n}{2}$

$Var(X_i) = E(X-E(X))^2 = E(X^2) - E(X)^2 \quad \frac{49}{4}$

$E(X_i^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6}$

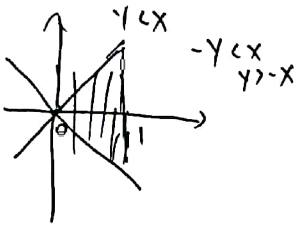
$\therefore Var(X_i) = \frac{35}{12}$

$Var(\sum_{i=1}^{n} X_i) = \frac{35n}{12}$

2、

$E(X) = \int_{-\infty}^{+\infty} x P_X(x)\,dx = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x \cdot 1\, dy\, dx = \int_0^1 \int_{-x}^{x} x\, dy\, dx = \int_0^1 2x^2 dx$

$= 2 \cdot \frac{x^3}{3}\Big|_0^1 = \frac{2}{3}$



$E(Y) = \int_{-x}^{x}\int_0^1 y\, dx\, dy = \int_{-x}^{x} y\, dy = \frac{y^2}{2}\Big|_{-x}^{x} = \frac{x^2}{2} - \frac{x^2}{2} = 0$

$Cov(X,Y) = E(XY) - \cancel{E(X)E(Y)} = E(XY)$

$P_X(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{else} \end{cases}$ $\quad P_Y(y) = \begin{cases} 1-y & 0<y<1 \\ 1+y & -1<y<0 \\ 0 & \text{else} \end{cases}$

$E(XY) = \int_0^1 dx \int_{-x}^{x} xy\, dy = \int_0^1 x \cdot \frac{y^2}{2}\Big|_{-x}^{x} dx = \int_0^1 x \cdot (\frac{x^2}{2} - \frac{x^2}{2})\, dx = 0$

$\therefore Cov(X,Y) = 0$

(3)

$Cov(X,Y) = Cov(X, X^2) = E(X^3) - E(X) \cdot E(X^2) = 0 - 0 = 0$

$E(X) = \int_{-\infty}^{+\infty} \underset{奇}{x}\, \underset{偶}{p(x)}\, dx = 0 = E(X^3)$ $\quad \therefore X$ 与 $Y = X^2$ 不相关

$\because Y = X^2 \therefore$ 显然 Y 与 X 不独立

4.

$$\rho(R,S) = \frac{Cov(R.S)}{\sqrt{Var(R)}\cdot\sqrt{Var(S)}} = \frac{Cov(W+X,X+Y)}{\sqrt{Var(W+X)}\cdot\sqrt{Var(X+Y)}} = \frac{Cov(W+X,X+Y)}{\sqrt{1+1}\ \sqrt{1+1}} = \frac{Cov(W+X,X+Y)}{2}$$

$$= \frac{Cov(W,X+Y)+Cov(X,X+Y)}{2} = \frac{Cov(Y,W)+Cov(X,W)+Cov(X,X)+Cov(Y,X)}{2}$$

$$= \frac{E((Y-E(Y))\cdot(W-E(W)))+E((X-E(X))(W-E(W)))+Var(X)+E((Y-E(Y))\cdot(X-E(X)))}{2}$$

$$= \frac{E(Y\cdot W)+E(X\cdot W)+1+E(Y\cdot X)}{2} = \frac{0+0+1+0}{2} = \frac{1}{2}$$

$$\rho(R,T) = \frac{Cov(R.T)}{\sqrt{Var(R)}\cdot\sqrt{Var(S)}} = \frac{Cov(W+X,Y+Z)}{\sqrt{1+1}\cdot\sqrt{1+1}} = \frac{Cov(W,Y+Z)+Cov(X,Y+Z)}{2}$$

$$= \frac{Cov(Y,W)+Cov(Z,W)+Cov(Y,X)+Cov(Z,X)}{2}$$

$$= \frac{E(Y\cdot W)+E(Z\cdot W)+E(Y\cdot X)+E(Z\cdot X)}{2} = 0$$