概统 21 & 22

1、 求 $P(\lambda=1.5 \mid X=3)$

$P(\lambda=1.8 \mid X=3)$

$$P(X=3) = P(X=3 \mid \lambda=1.5) \times P(\lambda=1.5) + P(X=3 \mid \lambda=1.8) \times P(\lambda=1.8)$$

$$= \frac{1.5^3}{3!} e^{-1.5} \times 0.45 + \frac{1.8^3}{3!} e^{-1.8} \times 0.55$$

$$= \frac{\frac{243}{160} e^{-1.5} + \frac{8019}{2500} e^{-1.8}}{6}$$

$$P(\lambda=1.5 \mid X=3) = \frac{P(X=3 \mid \lambda=1.5) \times P(\lambda=1.5)}{P(X=3)} = \frac{\frac{243}{160} e^{-1.5}}{P(X=3)}$$

$$= 0.38992$$

$$P(\lambda=1.8 \mid X=3) = \frac{P(X=3 \mid \lambda=1.8) \times P(\lambda=1.8)}{P(X=3)} = \frac{\frac{8019}{2500} e^{-1.8}}{P(X=3)} = 0.610077$$

2、 $N(\mu, \sigma^2)$ 证 $\frac{1}{\sigma^2}$ 的分布

$\sigma^2 \sim$ 倒伽玛分布 $\quad \frac{1}{\sigma^2} \sim$ 伽玛分布 $Ga(\alpha, \lambda)$

Y.U 函数的分布

$$P\left(\frac{1}{\sigma^2}\right) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \left(\frac{1}{\sigma^2}\right)^{\alpha-1} \cdot e^{-\frac{\lambda}{\sigma^2}} \qquad \frac{1}{\sigma^2} \geq 0 \qquad \frac{1}{\sigma^2} \sim Ga(\alpha, \lambda)$$

$$\pi(\sigma^2) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \left(\frac{1}{\sigma^2}\right)^{\alpha+1} e^{-\frac{\lambda}{\sigma^2}} \qquad \sigma^2 \geq 0 \qquad \sigma^2 \sim IG(\alpha, \lambda)$$

$$\pi(\sigma^2 \mid x_1 \cdots x_n) = \frac{P(x_1 \cdots x_n \mid \sigma^2)\, \pi(\sigma^2)}{\int_0^{+\infty} P(x_1 \cdots x_n \mid \sigma^2)\, \pi(\sigma^2)\, d\sigma^2}$$

$P(x_1 \cdots x_n \mid \sigma^2) \qquad x_1 \cdots x_n$ iid $N(\mu, \sigma^2)$

$$= (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{\sum_{i=1}^n (x_i-\mu)^2}{2\sigma^2}\right\}$$

$$\frac{(2\pi\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{\sum_{i=1}^n (x_i-\mu)^2}{2\sigma^2}\right\} \cancel{\frac{\lambda^\alpha}{\Gamma(\alpha)}} \left(\frac{1}{\sigma^2}\right)^{\alpha+1} e^{-\frac{\lambda}{\sigma^2}}}{\int_0^{+\infty} (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{\sum_{i=1}^n (x_i-\mu)^2}{2\sigma^2}\right\} \cancel{\frac{\lambda^\alpha}{\Gamma(\alpha)}} \left(\frac{1}{\sigma^2}\right)^{\alpha+1} e^{-\frac{\lambda}{\sigma^2}}\, d\sigma^2}$$

$$= \frac{(\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right\} (\sigma^2)^{-1-\alpha} e^{-\frac{\lambda}{\sigma^2}}}{\int_0^{+\infty} (\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right\} (\sigma^2)^{-1-\alpha} e^{-\frac{\lambda}{\sigma^2}} d\sigma^2}$$

$$= \frac{(\sigma^2)^{-\frac{n}{2}-1-\alpha} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+2\lambda\right]\right\}}{\int_0^{+\infty} (\sigma^2)^{-\frac{n}{2}-1-\alpha} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+2\lambda\right]\right\} d\sigma^2}$$

欲证 $\pi(\sigma^2 | x_1 \cdots x_n)$ 是倒伽玛分布

$$= \frac{\left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+\alpha+1} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+2\lambda\right]\right\}}{\underline{\int_0^{+\infty} (\sigma^2)^{-\frac{n}{2}-1-\alpha} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+2\lambda\right]\right\} d\sigma^2}}$$

$\downarrow$

$$\frac{1}{\sigma^2} = x \qquad \sigma^2 = \frac{1}{x} \qquad d\sigma^2 = -\frac{1}{x^2} dx$$

$$\int_0^{+\infty} x^{\frac{n}{2}+1+\alpha} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+2\lambda\right]x\right\} \cdot \frac{-1}{x^2} dx$$

$$= \int_0^{+\infty} x^{\frac{n}{2}+\alpha-1} \exp\left\{-\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}+\lambda\right]x\right\} dx$$

$$= \frac{\Gamma(\frac{n}{2}+\alpha)}{\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}+\lambda\right]^{\frac{n}{2}+\alpha}} \underbrace{\int_0^{+\infty} \frac{\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}+\lambda\right]^{\frac{n}{2}+\alpha}}{\Gamma(\frac{n}{2}+\alpha)} x^{\frac{n}{2}+\alpha-1} \exp\left\{-\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}+\lambda\right]x\right\} dx}_{1}$$

$$\therefore \pi(\sigma^2 | x_1 \sim x_n) = \frac{\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}+\lambda\right]^{\frac{n}{2}+\alpha}}{\Gamma(\frac{n}{2}+\alpha)} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+\alpha+1} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu)^2+\lambda\right]\right\}$$

$$\sigma^2 | x_1 \cdots x_n \sim IG\left(\frac{n}{2}+\alpha,\ \lambda+\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}\right)$$

$\therefore$ σ^2 的共轭先验分布是倒伽玛分布

3、

c 已知

(a) $\theta \sim$ 帕雷托分布

$\therefore \pi(\theta) = \frac{k\theta_{min}^{k}}{\theta^{k+1}} I\{\theta > \theta_{min}\}$

$$P(\theta|x_1\cdots x_n) = \frac{P(x_1\cdots x_n|\theta)\pi(\theta)}{\int_{\theta_{min}}^{+\infty} P(x_1\cdots x_n|\theta)\pi(\theta)\,d\theta} = \frac{\cancel{c^n(\prod_{i=1}^{n} x_i)^{c-1}}\,\theta^{-nc} I\{x_{(n)}\le\theta\}\frac{k\theta_{min}^{k}}{\theta^{k+1}} I\{\theta>\theta_{min}\}}{\cancel{c^n(\prod_{i=1}^{n} x_i)^{c-1}}\int_{\theta_{min}}^{+\infty}\theta^{-nc} I\{x_{(n)}\le\theta\}\frac{k\theta_{min}^{k}}{\theta^{k+1}} I\{\theta>\theta_{min}\}\,d\theta}$$

$$= \frac{\theta^{-nc} I\{x_{(n)}\le\theta\}\cancel{\frac{k\theta_{min}^{k}}{\theta^{k+1}}} I\{\theta>\theta_{min}\}}{\int_{\theta_{min}}^{+\infty}\theta^{-nc} I\{x_{(n)}\le\theta\}\cancel{\frac{k\theta_{min}^{k}}{\theta^{k+1}}}\cancel{I\{\theta>\theta_{min}\}}\,d\theta} = \frac{\theta^{-nc} I\{x_{(n)}\le\theta \text{且} \theta>\theta_{min}\}\,\theta^{-k-1}}{\int_{\mu}^{+\infty}\theta^{-nc}\theta^{-k-1}\,d\theta}$$

$\mu = \max\{\theta_{min}, x_{(n)}\}$

$$= \frac{\theta^{-nc-k-1} I\{\theta\ge\mu\}(nc+k)\mu^{nc+k}}{} = \frac{(nc+k)\mu^{nc+k} I\{\theta\ge\mu\}}{\theta^{nc+k+1}}$$

$\therefore p(\theta|x_1\cdots x_n)$ 也属于帕雷托分布 $\therefore \theta$ 的共轭先验分布为帕雷托分布

(b) $c \sim Ga(\alpha,\lambda)$

$\pi(c) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} c^{\alpha-1} e^{-\lambda c}, \quad c\ge 0$

$$P(c|x_1\cdots x_n) = \frac{P(x_1\cdots x_n|\theta)\pi(\theta)}{\int_0^{+\infty} P(x_1\cdots x_n|\theta)\pi(\theta)\,dc} = \frac{c^n(\prod_{i=1}^{n} x_i)^{c-1}\theta^{-nc}\cancel{I\{0\le x\le\theta\}}\cancel{\frac{\lambda^{\alpha}}{\Gamma(\alpha)}} c^{\alpha-1}e^{-\lambda c}}{\int_0^{+\infty} c^n(\prod_{i=1}^{n} x_i)^{c-1}\theta^{-nc}\cancel{I\{0\le x\le\theta\}}\cancel{\frac{\lambda^{\alpha}}{\Gamma(\alpha)}} c^{\alpha-1}e^{-\lambda c}\,dc}$$

$$= \frac{c^{n+\alpha-1}(\prod_{i=1}^{n} x_i)^{c\cancel{-1}}\theta^{-nc}e^{-\lambda c}}{\int_0^{+\infty} c^{n+\alpha-1}(\prod_{i=1}^{n} x_i)^{c\cancel{-1}}\theta^{-nc}e^{-\lambda c}\,dc} = \frac{c^{n+\alpha-1}(\prod_{i=1}^{n}\frac{x_i}{\theta})^{c}e^{-\lambda c}}{\int_0^{+\infty} c^{n+\alpha-1}(\prod_{i=1}^{n}\frac{x_i}{\theta})^{c}e^{-\lambda c}\,dc}$$

$$(\prod_{i=1}^{n}\frac{x_i}{\theta})^{c} = e^{c[\sum_{i=1}^{n}\ln x_i - \ln\theta]}$$

$$= \frac{c^{n+\alpha-1}e^{-c[\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i)]}}{\int_0^{+\infty} c^{n+\alpha-1}e^{-c[\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i)]}\,dc} \quad \text{凑 } Ga \text{ 分布}$$

$$= \frac{\frac{c^{n+\alpha-1}}{\Gamma(n+\alpha)}(\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i))^{n+\alpha}e^{-c[\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i)]}}{\underbrace{\int_0^{+\infty}\frac{c^{n+\alpha-1}}{\Gamma(n+\alpha)}(\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i))^{n+\alpha}e^{-c[\lambda+\sum_{i=1}^{n}(\ln\theta-\ln x_i)]}\,dc}_{1}}$$

$$= \frac{(\lambda + \sum_{i=1}^{n}(\ln\theta - \ln x_i))^{n+\alpha}}{\Gamma(n+\alpha)} c^{n+\alpha-1} e^{-c[\lambda + \sum_{i=1}^{n}(\ln\theta - \ln x_i)]}$$

$\therefore c \mid x_1 \cdots x_n \sim Ga(n+\alpha, \lambda + \sum_{i=1}^{n}(\ln\theta - \ln x_i)^{n+\alpha})$

$\therefore$ c 的共轭先验分布为伽玛分布

4、$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad \lambda \geq 0$

$$P(x_1 \cdots x_n \mid \lambda) = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!}$$

$$P(\lambda \mid x_1 \cdots x_n) = \frac{\frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\cancel{\prod_{i=1}^{n} x_i!}} \cancel{\frac{\beta^\alpha}{\Gamma(\alpha)}} \lambda^{\alpha-1} e^{-\beta\lambda}}{\int_0^{+\infty} \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\cancel{\prod_{i=1}^{n} x_i!}} \cancel{\frac{\beta^\alpha}{\Gamma(\alpha)}} \lambda^{\alpha-1} e^{-\beta\lambda} d\lambda} = \frac{\lambda^{\sum_{i=1}^{n} x_i + \alpha - 1} e^{-\lambda(n+\beta)}}{\int_0^{+\infty} \lambda^{\sum_{i=1}^{n} x_i + \alpha - 1} e^{-\lambda(n+\beta)} d\lambda}$$

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$$= \frac{\frac{(n+\beta)^{\sum_{i=1}^{n} x_i + \alpha}}{\Gamma(\sum_{i=1}^{n} x_i + \alpha)} \lambda^{\sum_{i=1}^{n} x_i + \alpha - 1} e^{-\lambda(n+\beta)}}{\underbrace{\int_0^{+\infty} \frac{(n+\beta)^{\sum_{i=1}^{n} x_i + \alpha}}{\Gamma(\sum_{i=1}^{n} x_i + \alpha)} \lambda^{\sum_{i=1}^{n} x_i + \alpha - 1} e^{-\lambda(n+\beta)} d\lambda}_{1}}$$

$\therefore \lambda \mid x_1 \cdots x_n \sim Ga(\sum_{i=1}^{n} x_i + \alpha, n+\beta)$

$\therefore$ λ 的后验分布为 $Ga(\sum_{i=1}^{n} x_i + \alpha, \beta + n)$

(b)

$$\hat{\lambda}_1 = \frac{\sum_{i=1}^{n} x_i + \alpha}{\beta + n} \quad \text{(后验分布求期望)}$$

(c)

$$P(x_1 \cdots x_n) = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\underbrace{\prod_{i=1}^{n} x_i!}_{\text{(与}\lambda\text{无关)}}} \propto L(\lambda)$$

$L(\lambda) = \lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}$

$\ln L(\lambda) = -n\lambda + \sum_{i=1}^{n} x_i \ln\lambda$

$$\frac{\ln L(\lambda)}{d\lambda} = -n + \frac{\sum_{i=1}^{n} x_i}{\lambda} = 0$$

$$\hat{\lambda}_2 = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{n\bar{x}}{n} = \bar{x}$$

(5) $\sim Be(a,b)$

$$\pi(p)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}p^{a-1}(1-p)^{b-1},\quad 0<p<1$$

$$P(X_1\cdots X_n|p)=\prod_{i=1}^{n}\binom{x_i-1}{k-1}p^{nk}(1-p)^{\sum_{i=1}^{n}x_i-nk}$$

$$P(p|X_1\cdots X_n)=\frac{\cancel{\prod_{i=1}^{n}\binom{x_i-1}{k-1}}p^{nk}(1-p)^{\sum_{i=1}^{n}x_i-nk}\cancel{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}p^{a-1}(1-p)^{b-1}}{\int_0^1\cancel{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}p^{a-1}(1-p)^{b-1}\cancel{\prod_{i=1}^{n}\binom{x_i-1}{k-1}}p^{nk}(1-p)^{\sum_{i=1}^{n}x_i-nk}dp}$$

$$=\frac{p^{nk+a-1}(1-p)^{\sum_{i=1}^{n}x_i-nk+b-1}}{\int_0^1 p^{nk+a-1}(1-p)^{\sum_{i=1}^{n}x_i-nk+b-1}dp}=\frac{\frac{\Gamma(nk+a+\sum_{i=1}^{n}x_i-nk+b)}{\Gamma(nk+a)\Gamma(\sum_{i=1}^{n}x_i-nk+b)}p^{nk+a-1}(1-p)^{\sum_{i=1}^{n}x_i-nk+b-1}}{\text{凑}Be\ \underbrace{\int_0^1\frac{\Gamma(nk+a+\sum_{i=1}^{n}x_i-nk+b)}{\Gamma(nk+a)\Gamma(\sum_{i=1}^{n}x_i-nk+b)}p^{nk+a-1}(1-p)^{\sum_{i=1}^{n}x_i-nk+b-1}dp}_{1}}$$

$\therefore\ p|X_1\cdots X_n\sim Be(nk+a,\ \sum_{i=1}^{n}x_i-nk+b)$

$\therefore$ P的后验分布为 $Be(nk+a,\sum_{i=1}^{n}x_i-nk+b)$

$\therefore$ P的共轭先验分布族为Be分布族

一、

1、 $\left[\bar{x} - z_{0.975} \cdot \frac{3}{\sqrt{10}},\ \bar{x} + z_{0.975} \frac{3}{\sqrt{10}}\right]$

2、 $\left[\bar{x} - z_{0.957} \cdot \frac{3}{\sqrt{30}},\ \bar{x} + z_{0.955} \frac{3}{\sqrt{30}}\right]$

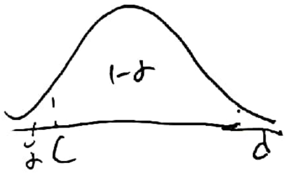
3、 $\left[\bar{x} - t_{0.975}(9) \frac{s}{\sqrt{10}},\ \bar{x} + t_{0.975}(9) \frac{s}{\sqrt{10}}\right]$

4、 $\left[\bar{x} - t_{0.975}(9) \frac{s}{\sqrt{30}},\ \bar{x} + t_{0.975}(9) \frac{s}{\sqrt{30}}\right]$

2.

$$u = \frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$$

$$P\left(c \le \frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \le d\right) = 0.95$$



$$c = z_{\frac{\alpha}{2}} \qquad d = z_{1-\frac{\alpha}{2}}$$

$$z_{\frac{\alpha}{2}} \le \frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \le z_{1-\frac{\alpha}{2}}$$

$$\frac{\sigma z_{\frac{\alpha}{2}}}{\sqrt{n}} \le \bar{x}-\mu \le \frac{\sigma z_{1-\frac{\alpha}{2}}}{\sqrt{n}}$$

$$\underline{\bar{x} - \frac{\sigma z_{1-\frac{\alpha}{2}}}{\sqrt{n}}} \le \mu \le \underline{\bar{x} - \frac{\sigma z_{\frac{\alpha}{2}}}{\sqrt{n}}}$$

$$\frac{-\sigma z_{\frac{\alpha}{2}}}{\sqrt{n}} + \frac{\sigma z_{1-\frac{\alpha}{2}}}{\sqrt{n}} \le k$$

$$\cancel{2\sigma} z_{1-\frac{\alpha}{2}} \le \frac{\sqrt{n}\,k}{\sigma 2}$$

$$\frac{\sqrt{n}\,k}{2\sigma} \ge \underline{z_{0.975}} = 1.96$$

$$\therefore n \ge \frac{(1.96)^2 4\sigma^2}{k^2} = 15.3664\left(\frac{\sigma}{k}\right)^2$$

3.

(a)

$n_1 = 10 \quad \bar{x} = 1.64 \quad S_x = 0.2$

$n_2 = 10 \quad \bar{y} = 1.62 \quad S_y = 0.1$

$$\left[\frac{S_x^2}{S_y^2}\cdot\frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)},\ \frac{S_x^2}{S_y^2}\frac{1}{F_{\frac{\alpha}{2}}(n_1-1,n_2-1)}\right]$$

$$\left[\frac{4}{F_{0.975}(9,9)},\ 4\frac{1}{F_{0.025}(9,9)}\right]$$

$$\left[\frac{4}{4.03},\ \underline{4\times 4.03}\right] = [0.9926,\ 16.12]$$

(b)

对 σ_1^2 和 σ_2^2 无信息

$$S_0^2 = \frac{S_x^2}{10} + \frac{S_y^2}{10} = \frac{1}{200}$$

$$S_0 = \frac{\sqrt{2}}{20}$$

$$l = \frac{\left(\frac{1}{200}\right)^2}{\frac{(0.2)^4}{10^2 \times 9} + \frac{(0.1)^4}{10^2 \times 9}} = \frac{225}{17}$$

$$t_{0.975}(13.2353) = 2.1565$$

$$\left[\bar{x} - \bar{y} - \frac{\sqrt{2}}{20} \times 2.1565, \ \bar{x} - \bar{y} + \frac{\sqrt{2}}{20} \times 2.1565\right]$$

$$= \left[1.64 - 1.62 - \frac{\sqrt{2}}{20} \times 2.1565, 1.64 - 1.62 + \frac{\sqrt{2}}{20} \times 2.1565\right]$$

$$= [-0.1325, 0.1725]$$

4、

(a) $P_1(x) = \frac{n!}{(n-1)!}(1-F(x))^{n-1}p(x) = n(e^{-x+\theta})^{n-1}e^{-(x-\theta)}, \quad x>\theta$

$Y = X_{(1)} - \theta$

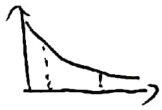
$P(y) = n(e^{-y})^{n-1}e^{-y}|1| = ne^{-yn+y}e^{-y} = ne^{-yn} \quad y>0$

∴ Y的分布与θ无关，即 $X_{(1)}-\theta$ 分布与θ无关

$Y = X_{(1)}-\theta \sim Exp(n)$

(b)

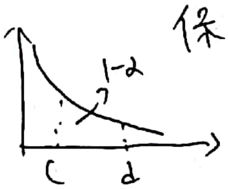
$G = X_{(1)}-\theta$ 分布已知，未知para只有θ，与样本$X_{(1)}$有关



$P(c \le \underbrace{X_{(1)}-\theta}_{Y} \le d) = 1-\alpha = \int_c^d ne^{-yn}\,dy$

$-\int_c^d e^{-yn}\,d\,ny = -e^{-yn}\Big|_c^d = -(e^{-dn}-e^{-cn}) = e^{-cn}-e^{-dn}$

$e^{-cn}-e^{-dn} = 1-\alpha$ （跟据密度函数图可以看出，自向右移，为了保证 $F(d)-F(c)=1-\alpha$，c d之间距离会增加）



∴ 取c=0 $\quad \cancel{1-}e^{-dn} = \cancel{1-}\alpha$

$e^{-dn} = \alpha$

$-dn = \ln\alpha$

$d = \frac{\ln\alpha}{-n}$

$Y \quad [0, \frac{\ln\alpha}{-n}] \Rightarrow 0 \le X_{(1)}-\theta \le \frac{\ln\alpha}{-n}$

∴ $-\frac{\ln\alpha}{-n}+X_{(1)} \le \theta \le X_{(1)}$

$X_{(1)}+\frac{\ln\alpha}{n} \le \theta \le X_{(1)}$

∴ 区间 $[X_{(1)}+\frac{\ln\alpha}{n}, X_{(1)}]$

5、

(a) $\bar{Y} = \dfrac{\ln 0.5 + \ln 1.25 + \ln 0.8 + \ln 2}{4} = 0$

$\bar{Y} \sim N(\mu, \frac{1}{4})$

$\dfrac{\bar{Y} - \mu}{\sqrt{\frac{1}{4}}} \sim N(0, 1)$

$P\left(c \le \dfrac{\bar{Y} - \mu}{\frac{1}{2}} \le d\right) = 0.95$

$c = z_{0.025}$ $\quad d = z_{1-0.025} = z_{0.975} = 1.96$

$-1.96 \le -2\mu \le 1.96$

$-0.98 \le \mu \le 0.98$

$[-0.98, 0.98]$

(b)

$Y = \ln X \sim N(\mu, 1)$

$X \sim$ lognormal $\quad p(x) = \dfrac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\dfrac{(\ln x - \mu)^2}{2\sigma^2}\right\}$

$E(X) = e^{\mu + \frac{1}{2}}$ $\quad \underline{\uparrow \text{ as } \mu \uparrow}$

$-0.98 \le \mu \le 0.98$

$e^{-0.98 + \frac{1}{2}} \le e^{\mu + \frac{1}{2}} \le e^{0.98 + \frac{1}{2}}$

$0.6188 \le E(X) \le 4.3929$

$[0.6188, 4.3929]$