第十二讲 概统 hw

一、

① $D(x+n)=\begin{cases}0 & x+n<0\\ 1 & x+n\geq 0\end{cases}$

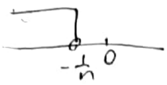
$\downarrow n\to+\infty$

$g(x)=1 \quad -\infty<x<+\infty$ 不满足 $g(-\infty)=0$

$\therefore$ 不是分布函数

②

$D(x+\frac{1}{n})=\begin{cases}0 & x+\frac{1}{n}<0\\ 1 & x+\frac{1}{n}\geq 0\end{cases}$



$\downarrow n\to+\infty$

$g(x)=\begin{cases}0 & x<0\\ 1 & x\geq 0\end{cases}$ 满足单调，有界，右连续

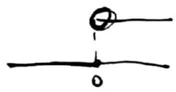
$\therefore$ 是分布函数

③

$D(x-\frac{1}{n})=\begin{cases}0 & x-\frac{1}{n}<0\\ 1 & x-\frac{1}{n}\geq 0\end{cases}$

$\downarrow n\to+\infty$

$g(x)=\begin{cases}0 & x\leq 0\\ 1 & x>0\end{cases}$ 不满足右连续性



$\therefore$ 不是分布函数

2、 $p(x)=\begin{cases}\frac{1}{2} & -1\leq x\leq 1\\ 0 & \text{otherwise}\end{cases}$ $\qquad F(x)=\begin{cases}0 & x<-1\\ \frac{x+1}{2} & -1\leq x<1\\ 1 & x\geq 1\end{cases}$

(a) $Y_n=\frac{X_n}{n}$

$E(\frac{X_n}{n})=\frac{1}{n}E(X_n)=0$

$Var(\frac{X_n}{n})=\frac{1}{n^2}Var(X_n)=\frac{1}{n^2}\cdot\frac{(1+1)^2}{12}=\frac{1}{3}\cdot\frac{1}{n^2}=\frac{1}{3n^2}$

$P(|\frac{X_n}{n}-0|\geq\varepsilon)\leq\frac{\frac{1}{3n^2}}{\varepsilon^2}\to 0$

$\therefore \frac{X_n}{n}\xrightarrow{P}0$

$\therefore Y_n\xrightarrow{P}0$

扫描全能王 创建

(b) $Y_n = (X_n)^n$

$$E\left((X_n)^n\right) = \int_{-1}^{1} x^n \cdot \frac{1}{2}\, dx = \frac{1}{2} \cdot \frac{x^{n+1}}{n+1}\Big|_{-1}^{1} = \frac{1}{2}\left(\frac{1}{n+1} - \frac{(-1)^{n+1}}{n+1}\right) \to 0$$

$$Var(X_n^n) = E(X_n^{2n}) - \cancel{E(X_n^n)^2} = \int_{-1}^{1} x^{2n} \cdot \frac{1}{2}\, dx = \frac{1}{2} \cdot \frac{x^{2n+1}}{2n+1}\Big|_{-1}^{1}$$

$$= \frac{1}{2}\left(\frac{1}{2n+1} - \frac{-1}{2n+1}\right) \to 0$$

$$P(|(X_n)^n - E((X_n)^n)| \geq \varepsilon) \leq \frac{Var(X_n^n)}{\varepsilon^2}$$

在 $n \to \infty$ 时得

$$P(|(X_n)^n - 0| \geq \varepsilon) \leq 0$$

$$\therefore P(|(X_n)^n - 0| \geq \varepsilon) \to 0$$

$$\therefore (X_n)^n \xrightarrow{P} 0$$

$$\therefore Y_n \xrightarrow{P} 0$$

(c) $Y_n = X_1 \cdot X_2 \cdots X_n$

$$E(X_1 X_2 \cdots X_n) = E(X_1) \cdot E(X_2) \cdots E(X_n) = 0 \quad \text{独立}$$

$$Var(X_1 X_2 \cdots X_n) = E(X_1^2 X_2^2 \cdots X_n^2) - \underbrace{E(X_1 X_2 \cdots X_n)^2}_{0}$$

$$= E(X_1^2)\, E(X_2^2) \cdots E(X_n^2)$$

$$\int_{-1}^{1} x^2 \cdot \frac{1}{2}\, dx = \frac{1}{2} \cdot \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{2}\left(\frac{1}{3} - \frac{-1}{3}\right) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$\therefore Var(X_1 X_2 \cdots X_n) = \left(\frac{1}{3}\right)^n$$

$$P(|X_1 X_2 \cdots X_n - E(X_1 X_2 \cdots X_n)| \geq \varepsilon) \leq \frac{Var(X_1 X_2 \cdots X_n)}{\varepsilon^2}$$

$$P(|Y_n - 0| \geq \varepsilon) \leq \frac{(\frac{1}{3})^n}{\varepsilon^2} \to 0 \quad \text{当 } n \to \infty$$

$$\therefore Y_n \xrightarrow{P} 0$$

(d) 当 $x \in [-1, 1)$ 时

$$P(Y_n \leq x) = P(X_1 \leq x) \cdot P(X_2 \leq x) \cdots P(X_n \leq x)$$

$$= \left(\frac{x+1}{2}\right)^n$$

$$P(|Y_n - c| \geq \varepsilon) = P(Y_n - c \geq \varepsilon) + P(Y_n \leq c - \varepsilon) \to 0$$

取 $c=1$ $\quad P(Y_n \geq 1+\varepsilon) = 0$

$$P(Y_n \leq 1-\varepsilon) = \left(\frac{2-\varepsilon}{2}\right)^n \to 0$$

$$\therefore Y_n \xrightarrow{P} 1$$

第十二讲 概统 hw

1、 $P(A)=p$

$\therefore P(X_n=1)=p^2$ $\qquad Var(X_i)=E(X_i^2)-E(X_i)^2=p^2-p^4$

$P(X_n=0)=1-p^2$

$Var(\sum_{i=1}^{n} X_i)=\sum_{i=1}^{n} Var(X_i)+2\sum_{i=1}^{n}\sum_{j=1}^{i-1} Cov(X_i,X_j)$

$Cov(X_i,X_1)$

$Cov(X_i,X_2)$ $\quad i=1,2,3\cdots$

$\vdots$

$Cov(X_i,X_{i-1})$

$\sum_{i=1}^{n} Var(X_i)=np^2-np^4$

X_n $\quad n,n+1$ 日出现

X_{n+1} $\quad n+1,n+2$ 日出现

X_{n+2} $\quad n+2,n+3$ 日出现

$\therefore Cov(X_i,X_j)$ 在 $|j-i|\geq 2$ 时 $=0$

$\therefore \sum_{i=1}^{n}\sum_{j=1}^{i-1} Cov(X_i,X_j)=\sum_{i=1}^{n-1} Cov(X_{i+1},X_i)$

$Cov(X_{i+1},X_i)=E[(X_{i+1}-E(X_{i+1}))(X_i-E(X_i))]$

$=E(X_i\cdot X_{i+1})-p^4=1\cdot p^3-p^4=p^3-p^4$

$\frac{1}{n^2}Var(\sum_{i=1}^{n} X_i)=\frac{1}{n^2}[np^2-np^4+2(n-1)(p^3-p^4)]$

$=\frac{np^2-np^4+2(n-1)p^3-2(n-1)p^4}{n^2}$

$=\frac{p^2-p^4+2p^3-2p^4}{n}+\frac{2p^4-2p^3}{n^2}\to 0$

满足马尔可夫条件

$\therefore \{X_n\}$ 服从大数定律

$x_i = \begin{cases} 1 & \text{第 } i \text{ 人吸烟} \\ 0 & \text{第 } i \text{ 人不吸烟} \end{cases}$

$x_i \sim b(1, f)$

$S_n = \sum_{i=1}^{n} x_i \sim b(n, f)$

$P(|\frac{S_n}{n} - f| \ge \underset{已知}{\varepsilon}) \le \underset{已知}{\delta}$

$f - f^2$

$P(|M_n - f| \ge \varepsilon) \le \frac{f(1-f)}{n\varepsilon^2} \le \frac{1}{4n\varepsilon^2}$

(a) $\varepsilon \to \frac{\varepsilon}{2}$ δ 不变

$\frac{1}{4n\varepsilon^2} \le \delta$

$1 \le 4n\varepsilon^2 \cdot \delta$

$n \ge \frac{1}{4\varepsilon^2 \delta}$

$\therefore$ $\varepsilon \to \frac{\varepsilon}{2}$ n 变为原来的四倍 $n \to 4n$

$\delta \to \frac{\delta}{2}$ n 变为原来的两倍 $n \to 2n$

第十三讲 概统 hw

一、 (a) X_i 为第 i 个数的误差 $\sim U(-0.5, 0.5)$

$$E\left(\sum_{i=1}^{1500} X_i\right) = 0 \qquad Var\left(\sum_{i=1}^{1500} X_i\right) = \sum_{i=1}^{1500} Var(X_i) = \frac{1}{12} \times 1500 = \frac{1500}{12} = 125$$

$$P\left(\left|\sum_{i=1}^{1500} X_i\right| > 15\right) = 1 - P\left(\left|\sum_{i=1}^{1500} X_i\right| \le 15\right) = 1 - P\left(-15 \le \sum_{i=1}^{1500} X_i \le 15\right)$$

$$= 1 - \left(P\left(\sum_{i=1}^{1500} X_i \le 15\right) - P\left(\sum_{i=1}^{1500} X_i \le -15\right)\right)$$

$$= 1 - \Phi\left(\frac{15}{5\sqrt5}\right) + \Phi\left(-\frac{15}{5\sqrt5}\right) = 1 - \Phi\left(\frac{15}{5\sqrt5}\right) + 1 - \Phi\left(\frac{15}{5\sqrt5}\right) = 2 - 2\Phi(1.34)$$

$$= 2 - 2 \times 0.9099$$

$$= 0.1802$$

(b)

$$P\left(\left|\sum_{i=1}^{n} X_i\right| < 10\right) \ge 90\%$$

$$2\Phi\left(\frac{10}{\sqrt{\frac{n}{12}}}\right) - 1 \ge 0.9$$

$$\Phi\left(\frac{20\sqrt3}{\sqrt n}\right) \ge 0.95$$

$$\frac{20\sqrt3}{\sqrt n} \ge 1.645$$

$$n \le \frac{400 \times 3}{1.645^2} \approx 443$$

最多443个数

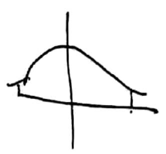
2、

$$X_i = \begin{cases} 1 & \text{第 i 个产品合格} \\ 0 & \text{第 i 个产品不合格} \end{cases}$$

$$Y_n = \sum_{i=1}^{n} X_i \sim b(n, 0.99)$$

$$P(Y_n \ge 100) \ge 0.95$$

$$\therefore P(Y_n < 100) \le 0.05$$



$$\Phi\left(\frac{100 - 0.99n}{\sqrt{0.99 \times n \times 0.01}}\right) \le 0.05$$

$$\frac{100 - 0.99n}{\sqrt{0.99 \times 0.01 \times n}} \le -1.645$$

$$X \ge 102.685$$

$\therefore X_{min} = 103$ ∴ 应装103个此种产品

3、 $1\sim36$ 36个数 18偶 18奇

∵ 轮盘赌公平，所以每一次取偶/奇的概率为$\frac{1}{2}$

$$x_i=\begin{cases}1 & 第i次为奇数\\ 0 & 第i次为偶数\end{cases} \qquad P(x)=\begin{cases}\frac{1}{2} & x=1\\ \frac{1}{2} & x=0\end{cases}$$

$$x_i\sim(1,\tfrac{1}{2})$$

$$S_{100}=\sum_{i=1}^{100}x_i\sim(100,\tfrac{1}{2})$$

求 $P(S_n>55)$

$$=1-P(S_n\le55)=1-P\left(\frac{S_n-100\times\frac{1}{2}}{\sqrt{100\times\frac{1}{2}\times\frac{1}{2}}}\le\frac{55-50}{\sqrt{25}}\right)$$

$$=1-\Phi\left(\frac{5}{5}\right)$$

$$=1-\Phi(1)$$

$$=1-0.8413=0.1587$$