概统 hw 23

7.1 假设检验

1、

(a) $\alpha = P(H_1\text{真} \mid H_0\text{真})$ $\longrightarrow$ $X_1, X_2 \cdots X_n \overset{iid}{\sim} N(2,1)$

$$\Downarrow$$

$\bar{X} \sim N(2, \frac{1}{n})$

$\frac{\bar{X}-2}{\sqrt{\frac{1}{n}}} \sim N(0,1)$

$= P(\bar{X} \geq 2.6 \mid H_0\text{真})$

$= P\left(\frac{\bar{x}-2}{\sqrt{\frac{1}{20}}} \geq \frac{2.6-2}{\sqrt{\frac{1}{20}}}\right)$

$= 1 - \Phi\left(\frac{6\sqrt{5}}{5}\right) = 1 - \Phi(2.68) = 1 - 0.9963 = 0.0037$

$\beta = P(H_0\text{真} \mid H_1\text{真}) = P(\bar{X} < 2.6 \mid \mu = 3) = P\left(\frac{\bar{X}-3}{\sqrt{\frac{1}{20}}} < \frac{2.6-3}{\sqrt{\frac{1}{20}}}\right)$

$= \Phi\left(\frac{2.6-3}{\sqrt{\frac{1}{20}}}\right) = \Phi\left(-\frac{4\sqrt{5}}{5}\right) = \Phi(-1.79) = 1 - \Phi(1.79) = 1 - 0.9633 = 0.0367$

(b)

$\beta = P(\bar{X} < 2.6 \mid \mu = 3) = \Phi\left(\frac{2.6-3}{\sqrt{\frac{1}{n}}}\right) = 1 - \Phi\left(\frac{0.4}{\sqrt{\frac{1}{n}}}\right) \leq 0.01$

$\Phi\left(\frac{0.4}{\sqrt{\frac{1}{n}}}\right) \geq 0.99$

$\therefore \frac{0.4}{\sqrt{\frac{1}{n}}} \geq 2.33$

$\sqrt{\frac{1}{n}} \leq \frac{0.4}{2.33}$

$\frac{1}{n} \leq \left(\frac{0.4}{2.33}\right)^2$

$\left(\frac{2.33}{0.4}\right)^2 \leq n$

$33.93 \leq n$

$\therefore$ n 至少取 34

(c)

$\alpha = P(H_1\text{真} \mid H_0\text{真}) = P(\bar{X} \geq 2.6 \mid \mu = 2) = 1 - \Phi\left(\frac{2.6-2}{\sqrt{\frac{1}{n}}}\right) = 1 - \Phi\left(\frac{0.6}{\sqrt{\frac{1}{n}}}\right)$

$= 1 - \Phi(0.6\sqrt{n})$ 在 $n \to \infty$ 时 $\Phi(0.6\sqrt{n}) \to 1$ $1 - \Phi(0.6\sqrt{n}) \to 0 \Rightarrow \alpha \to 0$

$\beta = P(H_0\text{真} \mid H_1\text{真}) = P(\bar{X} < 2.6 \mid \mu = 3) = \Phi\left(\frac{2.6-3}{\sqrt{\frac{1}{n}}}\right) = 1 - \Phi(0.4\sqrt{n})$

在 $n \to \infty$ 时 $\Phi(0.4\sqrt{n}) \to 1$ $1 - \Phi(0.4\sqrt{n}) \to 0$ $\beta \to 0$

2、

$$\alpha = P(H_1 \mid H_0\text{为真}) = P(X_{(n)} \le 2.5 \mid \theta \ge 3)$$

$$P(x) = \begin{cases} \frac{1}{\theta} & 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \int_0^x \frac{1}{\theta}\,dx = \frac{1}{\theta}x$$

$$P_n(x) = n\left(\frac{x}{\theta}\right)^{n-1}\frac{1}{\theta} = \frac{n x^{n-1}}{\theta^n}$$

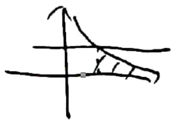
$$\alpha = \int_0^{2.5} \frac{n x^{n-1}}{\theta^n}\,dx$$ 对 $\forall\,\theta \ge 3$ 取 α 最大值

则 θ 取 $\min = 3$

$$= \frac{1}{\theta^n} x^n \Big|_0^{2.5}$$

$$= \frac{2.5^n}{3^n}$$

$$\therefore\ \alpha_{max} = \frac{2.5^n}{3^n}$$



$$\left(\frac{2.5}{3}\right)^n \le 0.05$$

$$n \ln\frac{2.5}{3} \le \ln 0.05$$

$$n \ge 16.431$$

$\therefore$ n 至少应取 17

3、

$$\sum_{i=1}^{20} X_i \sim b(20, p)$$

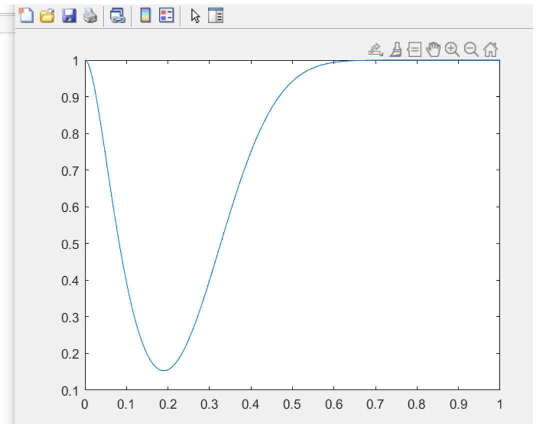
(a) $$g(p) = P\left(\sum_{i=1}^{20} X_i \ge 7\right) + P\left(\sum_{i=1}^{20} X_i \le 1\right) = (1-p)^{20} + p(1-p)^{19} + \sum_{i=7}^{20} C_{20}^{i} p^i (1-p)^{20-i}$$

~~$g(0)=1$ $g(0.1)=0.3941$ $g(0.2)=0.1559$ $g(0.3)=0.3996$~~

p	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$g(p)$	1	0.3941	0.1559	0.3996	0.7505	0.9424	0.9935	0.9997

p	0.8	0.9	1
$g(p)$	1	1	1

```
>> x=0:0.1:1;
y=1-binocdf(6,20,x)+binocdf(1,20,x);
plot(x,y);
>> x=0:0.001:1;
y=1-binocdf(6,20,x)+binocdf(1,20,x);
plot(x,y);
>>
```



(b) $$\beta = 1 - g(0.05) = 0.2641$$

```
>> y=binocdf(6,20,0.05)-binocdf(1,20,0.05)

y =

    0.2641

>>
```

hw 7.2

(1)

$X \sim N(\mu, \sigma^2)$ $\mu = 100kg$ $\sigma = 1.2kg$

$\because \sigma$ 不变 $\therefore X \sim N(\mu, 1.2^2)$

$\therefore H_0: \mu = 100kg$ vs $H_1: \mu \neq 100kg$

$\bar{x} \sim N(\mu, \frac{1.2^2}{9})$

$u = \frac{\bar{x}-\mu}{\frac{2}{5}} = \frac{\bar{x}-100}{\frac{2}{5}}$ $\bar{x} = \frac{4499}{45} = 99.98$ $u_0 = -0.005$

$W = \{|u| \geq z_{0.975}\} \Rightarrow u \geq 1.96$ $u_0 < u$

∴ 不落在拒绝域内

∴ 可以接受这一天的工作是正常的

```
>> y=normcdf(0.005, 0, 1)

y =

    0.5020
```

P值 书 P325 表 7.2.1

$P = 2(1-\Phi(|u_0|)) = 2(1-\Phi(0.005)) = 2(1-0.5020) = 0.996$

2、

$H_0: \mu \leq 1.2$ vs $H_1: \mu > 1.2$

$t = \frac{\bar{x}-\mu_0}{\sqrt{\frac{s^2}{n}}} \sim t(n-1)$

$s = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(x_i-0.97)^2} = 0.3302$

$= \frac{\bar{x}-1.2}{\sqrt{\frac{0.3302^2}{10}}} = \frac{0.97-1.2}{\sqrt{\frac{0.3302^2}{10}}} = -2.2027$

$W = \{t \geq t_{0.9}(9)\} = \{t \geq 1.3830\}$ $-2.2027 < 1.3830$

∴ 不落在拒绝域内

∴ 接受原假设 $H_0: \mu \leq 1.2$

$t \sim t(9)$

P值 $P = P(t \geq -2.2027) = 1 - P(t \leq -2.2027) = 0.9725$

```
>> y=1-tcdf(-2.2027, 9)

y =

    0.9725
```

3、两样本u检验

$X_1 \cdots X_n \sim N(\mu_1, \sigma_1^2)$ $\quad \bar{X} \sim N(\mu_1, \frac{\sigma_1^2}{n})$

$Y_1 \cdots Y_m \sim N(\mu_2, \sigma_2^2)$ $\quad \bar{Y} \sim N(\mu_2, \frac{\sigma_2^2}{m})$

$$\alpha = P(H_1 | H_0) = P(\bar{x} > 2\bar{y} \mid \mu_1 = 2\mu_2)$$

$$\Downarrow$$

$$\bar{x} \sim N(2\mu_2, \frac{\sigma_1^2}{n})$$

$$\bar{y} \sim N(\mu_2, \frac{\sigma_2^2}{m})$$

$$\bar{x} - 2\bar{y} \sim N(0, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m})$$

$$\underbrace{u = \frac{\bar{x} - 2\bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1)}_{\text{检验统计量}}$$

$$\alpha = P(\bar{x} - 2\bar{y} > c) \le \alpha$$

$$P(\underbrace{\frac{\bar{x} - 2\bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}}_{u} > \frac{c}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}) \le \alpha$$

$$1 - \Phi(\frac{c}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}) \le \alpha$$

$$W = \{u > z_{1-\alpha}\} = \{u > z_{0.95}\}$$

P值: $p = 1 - \Phi(u) = 1 - \Phi\left(\frac{\bar{x} - 2\bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}\right)$

4、

$X \sim N(\mu_1, \sigma^2)$ $\quad Y \sim N(\mu_2, \sigma^2)$

$\alpha = 0.05$ $\quad H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \ne \mu_2$

$$t = \frac{\bar{x} - \bar{y}}{S_w \sqrt{\frac{1}{m} + \frac{1}{n}}} \qquad W = \{|t| \ge t_{1-\frac{\alpha}{2}}(m+n-2)\}$$

$$= \{|t| \ge t_{0.975}(13+8-2)\}$$

$$t = \frac{80.021 - 79.979}{0.0269\sqrt{\frac{1}{13} + \frac{1}{8}}} = 3.31 \qquad = \{|t| \ge \underbrace{t_{0.975}(19)}_{2.0930}\}$$

$$S_w^2 = \frac{\sum_{i=1}^{13}(x_i - 80.021)^2 + \sum_{i=1}^{8}(y_i - 79.979)^2}{13+8-2}$$

$$= \frac{6.893 \times 10^{-3} + 6.888 \times 10^{-3}}{19}$$

$\because 3.31 > 2.0930$

$\therefore$ 落在拒绝域内

$\therefore$ 拒绝原假设 $\qquad t \sim t(19)$

P值: $P(|t| \ge 3.31) = 1 - (-3.31 \le t \le 3.31)$

$= 0.0037$

```
>> y=1-tcdf(3.31,19)+tcdf(-3.31,19)

y =

    0.0037
```

5.

$X \sim N(\mu_1, \sigma_1^2)$

$Y \sim N(\mu_2, \sigma_2^2)$

$X-Y \sim N(\mu_1-\mu_2, \sigma_1^2+\sigma_2^2)$

$d \sim N(\underbrace{\mu_1-\mu_2}_{\mu}, \sigma_1^2+\sigma_2^2)$

$H_0: \mu=0 \text{ vs } H_1: \mu \neq 0$

$$t=\frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.41375}{\frac{0.321}{\sqrt{8}}} \qquad s_d=\sqrt{\frac{1}{7}\sum_{i=1}^{8}(d_i-0.41375)^2}=0.321$$

$$=3.646$$

$$W=\{|\underset{\substack{\| \\ 3.646}}{t}| \geq \underset{\substack{\| \\ 2.3646}}{t_{0.975}(7)}\}$$

```
>> y=1-tcdf(3.646,7)+tcdf(-3.646,7)

y =

    0.0082
```

落在拒绝域内

∴ 拒绝原假设，接受备择假设"有显著差异"

$t \sim t(7)$

P值 $P=P(|t| \geq |3.646|)=1-(-3.646<t \leq 3.646)=0.0082$

6、

(1) $H_0: \sigma_1^2=\sigma_2^2 \text{ vs } H_1: \sigma_1^2 \neq \sigma_2^2$

$$F=\frac{S_x^2}{S_y^2} \qquad S_x^2=\frac{1}{5}\sum_{i=1}^{6}(x_i-\bar{x})^2 \qquad \bar{x}=0.141$$

$$S_y^2=\frac{1}{5}\sum_{i=1}^{5}(y_i-\bar{y})^2 \qquad \bar{y}=0.1385$$

$$S_x=\sqrt{S_x^2}=0.0028$$

$$S_y=\sqrt{S_y^2}=0.0027$$

$$W=\{F \leq F_{\frac{\alpha}{2}}(m-1,n-1)\} \cup \{F \geq F_{1-\frac{\alpha}{2}}(m-1,n-1)\}$$

$$=\{F \leq F_{0.025}(5,5)\} \cup \{F \geq F_{0.975}(\underset{\substack{\| \\ 7.15}}{5,5})\}$$

$$F=\frac{784}{729}=1.075 \qquad F_{0.025}(5,5) = \frac{1}{F_{0.975}(5,5)}=0.140$$

$$F_{0.975}(5,5) > F=1.075 > F_{0.025}(5,5)=0.14$$

∵ 不落在拒绝域内

∴ 接受原假设

P值：$P=2\min\{P(F \leq \underset{1.075}{F_0}), P(F \geq \underset{1.075}{F_0})\}$

$F \sim F(5,5)$

$P(F \le 1.075) = 0.5307$

$P(F \ge 1.075) = 1 - 0.5307 = 0.4693$

$\therefore \quad P = 2 \times 0.4693 = 0.9386$

```
>> y=fcdf(1.075, 5, 5)

y =

    0.5307

>> y=1-fcdf(1.075, 5, 5)

y =

    0.4693
```

(2)

$X \sim N(\mu_1, \sigma^2) \qquad Y \sim N(\mu_2, \sigma^2)$

$H_0: \mu_1 = \mu_2 \quad vs \quad H_1: \mu_1 \ne \mu_2$

$$t = \frac{\bar{x} - \bar{y}}{S_w \sqrt{\frac{1}{m} + \frac{1}{n}}}$$

$\bar{x} = 0.1407 \quad \bar{y} = 0.1385$

$m = n = 6$

$$S_w = \sqrt{\frac{5S_x^2 + 5S_y^2}{10}} = 0.00275$$

$$t = \frac{0.1407 - 0.1385}{0.00275\sqrt{\frac{1}{6}+\frac{1}{6}}} = 1.3856$$

$$W = \{ |t| \ge \underbrace{t_{0.975}(10)}_{=\,2.2281} \}$$

$1.3856 < 2.2281$

$\therefore$ 不落在拒绝域内

$\therefore$ 接受原假设，可认为 $\mu_1 = \mu_2$

P值：$p = P(|T| \ge 1.3856) \qquad T \sim t(10)$

$= 1 - P(-1.3856 \le T \le 1.3856)$

$= 0.1960$

```
>> y=1-tcdf(1.3856, 10)+tcdf(-1.3856, 10)

y =

    0.1960
```

hw-24

$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(X-\mu)^2}{2\sigma^2}\}$

1、$p(x_1 \cdots x_n) = (2\pi\sigma^2)^{-\frac{n}{2}} \cdot \exp\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\}$

$H_0: \sigma^2 = \sigma_0^2 \quad vs \quad H_1: \sigma^2 \neq \sigma_0^2$

$$\Lambda = \frac{\sup_{\theta\in\Theta} p(x_1 \cdots x_n)}{\sup_{\theta\in\Theta_0} p(x_1 \cdots x_n)} = \frac{(2\pi S_x^2)^{-\frac{n}{2}} \exp\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2S_x^2}\}}{(2\pi\sigma_0^2)^{-\frac{n}{2}} \exp\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma_0^2}\}} \qquad S_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$= \frac{(\frac{S_*}{\sigma_0})^{-n} \exp\{(-\frac{n}{2})\}}{\exp\{\frac{-nS_*^2}{2\sigma_0^2}\}} = (\frac{S_*}{\sigma_0})^{-n} \exp\{-\frac{n}{2} + \frac{nS_*^2}{2\sigma_0^2}\}$$

$\underline{-n\ln\frac{S_*}{\sigma_0} + -\frac{n}{2} + \frac{nS_*^2}{2\sigma_0^2}}$ 取对数

再求导，$-\cancel{n}\frac{1}{S_*} + \frac{\cancel{2n}S_*}{\cancel{2}\sigma_0^2} = 0$

$$\frac{S_*}{\sigma_0^2} = \frac{1}{S_*}$$

$$S_*^2 = \sigma_0^2 \Rightarrow S_* = \sigma_0$$

在 $S_* < \sigma_0$ 时 Λ 递减；$S_* > \sigma_0$ 时 Λ 递增

$\therefore \quad (\frac{S_*}{\sigma_0})^2 \uparrow$ 时 Λ 递增 $\Rightarrow n(\frac{S_*}{\sigma_0})^2 \uparrow$ 时，Λ 递增

$$\frac{nS_*^2}{\sigma_0^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sigma_0^2} \sim \chi^2(n)$$

$$\therefore \{\Lambda \geq C\} \Leftrightarrow \{\underbrace{\frac{nS_*^2}{\sigma_0^2}}_{\chi^2(n)} \geq C_1\} + \{\frac{nS_*^2}{\sigma_0^2} \leq C_2\}$$

```
>> y=1-chi2cdf(2.8,5)

y =

    0.7308
```

2、离散

$H_0: \quad P(X=i) = \frac{1}{6} \quad i=1,2,3,4,5,6 \qquad \alpha = 0.05$

$W = \{\chi^2 \geq \underbrace{\chi^2_{0.95}(5)}_{=11.0705}\}$

$2.8 < 11.0705$

$\therefore$ 不落在拒绝域

可以认为骰子均匀

书P333表7.2-3

P值：$p = P(\chi^2 \geq \chi_0^2) = P(\chi^2_{(5)} \geq 2.8) = 0.7308$

$$\chi^2 = \sum_{i=1}^{6}\frac{(x_i-10)^2}{10} = \frac{(7-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(12-10)^2}{10} + \frac{(11-10)^2}{10} + \frac{(9-10)^2}{10} + \frac{(13-10)^2}{10} = \frac{14}{5} = 2.8$$