杨茜雅 10245501435　　概率论 hw2 0310

$P(B|A)=1$

$P(B|\bar{A})=0.5$

1、A：原球为白球　则 $\bar{A}$：原球为black　B：取出白球

$P(A)=0.5$

$P(\bar{A})=0.5$

求 $P(A|B)=\dfrac{P(AB)\Rightarrow P(B|A)P(A)}{P(A)P(B|A)+P(\bar{A})P(B|\bar{A})}=\dfrac{0.5\times1}{0.5\times1+0.5\times0.5}=\dfrac{2}{3}$

2. A_i：第i天无雨　$P(A_1)=1$　$P(A_{i+1}|A_i)=p$　$P(A_{i+1}|\bar{A_i})=1-p$

$$P(A_n)=P(A_n|A_{n-1})P(A_{n-1})+P(A_n|\overline{A_{n-1}})P(\overline{A_{n-1}})$$

$$=p\cdot P(A_{n-1})+(1-p)(1-P(A_{n-1}))$$

$$=p\cdot P(A_{n-1})+1-p-(1-p)P(A_{n-1})=P(A_n)$$

$$P(A_n)=1-p+p\cdot P(A_{n-1})-(1-p)P(A_{n-1})$$

$$=1-p+P(A_{n-1})(p-1+p)=1-p+(2p-1)P(A_{n-1})$$

$$[P(A_n)-x]=(2p-1)[P(A_{n-1})-x]$$

$$x-(2p-1)x=1-p=x-2px+x=x(2-2p)$$

$$x=\frac{1-p}{2-2p}=\frac{1}{2}$$

$P(A_n)-\frac{1}{2}=(2p-1)[P(A_{n-1})-\frac{1}{2}]$　等比 $q=2p-1$

$$P(A_1)-\frac{1}{2}=\frac{1}{2}$$

$$\therefore P(A_n)-\frac{1}{2}=\frac{1}{2}(2p-1)^{n-1}$$

$P(A_n)=\frac{1}{2}(2p-1)^{n-1}+\frac{1}{2}$　$n\in N^*$ 且 $n\geq2$

3、$P(B|A)=\dfrac{P(AB)}{P(A)}\Longrightarrow$

$$P(A+B)=P(A)+P(B)-P(AB)$$

$$P(AB)=P(A)+P(B)-P(A+B)$$

$$=P(A)+[1-P(\bar{B})]-P(A+B)$$

$\therefore P(B|A)=\dfrac{P(AB)}{P(A)}\geq\dfrac{P(A)-P(\bar{B})}{P(A)}=1-\dfrac{P(\bar{B})}{P(A)}$

$$P(AB)=P(A)-P(\bar{B})+1-P(A+B)$$

$P(A+B)\leq1$　$\therefore 1-P(A+B)\geq0$

$$\therefore P(AB)\geq P(A)-P(\bar{B})$$

4、A：甲赢　$\bar{A}$：乙赢

三局两胜　B_i：甲在第i局赢

$$P(A)=P(B_1)\cdot P(B_2)\cdot P(\bar{B_3})+P(B_1)\cdot P(\bar{B_2})\cdot P(B_3)+P(\bar{B_1})\cdot P(B_2)\cdot P(B_3)+P(B_1)P(B_2)P(B_3)$$

$$=0.6\times0.6\times0.4\times3+0.6^3=0.648$$

五局三胜 P　$0.6^5+0.6^4\times0.4\times5+0.6^3\times0.4^2\times C_5^2=0.68256$

5.

~~$P(A\cap B|B)=$~~ 证 $P(A\cap B|B)=\frac{P(AB)}{P(B)}$

$P(A\cap B)=P(AB)=P(B)P(A|B)$

$P(A|B)=\frac{P(AB)}{P(B)}$

$P(B)\cdot P(A\cap B|B)=P(A\cap B)=P(AB)$

6. A: 0 $P(A)=p$ $\bar{A}$: 1 $P(\bar{A})=1-p$

① B: 0 错 $P(B)=\varepsilon_0$ $\bar{B}$: 1 错 $P(\bar{B})=\varepsilon_1$

C_k: 第k个符号正接 $\bar{C}_k$: 第k个符号错接

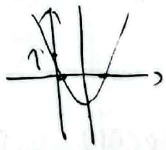
$$P(C_k)=P(A)P(\bar{B}|A)+P(\bar{A})P(\bar{B}|\bar{A})$$
$$=p\times(1-\varepsilon_0)+(1-p)(1-\varepsilon_1)$$

② 1011

$$P(1011)=(1-\varepsilon_1)^3\times(1-\varepsilon_0)$$

③ $$P(0正)=(1-\varepsilon_0)^3+3\times(1-\varepsilon_0)^2\varepsilon_0$$

④ $$P(0正)=3\varepsilon_0^2-3\varepsilon_0+1-\varepsilon_0^3+3\varepsilon_0(1-2\varepsilon_0+\varepsilon_0^2)$$
$$=\cancel{3\varepsilon_0^2}-\cancel{3\varepsilon_0}+1-\cancel{\varepsilon_0^3}+\cancel{3\varepsilon_0}-\cancel{6\varepsilon_0^2}+\cancel{3\varepsilon_0^3}$$
$$=\cancel{1-\varepsilon_0^3}\quad 2\varepsilon_0^3-3\varepsilon_0^2+1$$
$$6\varepsilon_0^2-6\varepsilon_0+1=0$$
$$\varepsilon_0=\frac{3\pm\sqrt{3}}{6}$$



$\cancel{\varepsilon_0}=$

取 $\varepsilon_0=\frac{3-\sqrt{3}}{6}$

⑤ 设 $P(A_n)$: 接收到的符号为n的概率

$P(B_n)$ 发出为n的概率

$p\cdot\varepsilon_0^2(1-\varepsilon_1)$

$$P(B_0|A_{101})=\frac{P(B_0)P(A_{101}|B_0)}{P(A_{101}|B_0)\cdot P(B_0)+P(A_{101}|\bar{B}_0)P(\bar{B}_0)}=\frac{p\cdot\varepsilon_0^2(1-\varepsilon_0)}{\varepsilon_0^2(1-\varepsilon_0)\cdot p+(1-\varepsilon_1)^2\varepsilon_1(1-p)}$$

$\varepsilon_0^2(1-\varepsilon_0)\cdot p$ $(1-\varepsilon_1)^2\varepsilon_1\cdot(1-p)$