Weekly Exercise 6

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Exercise 1

Solving an equation of 3rd degree

By *Vieta's formula* it is possible to solve an equation of 3rd degree.

Let the original equation be

$$ax^3 + bx^2 + cx = 0, \qquad \text{where } a \neq 0 \tag{1}$$

Doing $x \leftarrow t - \dfrac{b}{3a}$ (*Tschirnhaus transformation*) we get for *the equation to be solved*:

$$t^3 + pt + q = 0 \tag{2}$$

By using *Vieta's formula*

$$t = w - \frac{p}{3w}, \qquad \text{where } w \neq 0, \tag{3}$$

we get the equation

$$w^3 + q - \frac{p^3}{27w^3} = 0, \tag{4}$$

By multiplying both sides by w^3 we get *an equation of 6th degree* but actually it is 2nd degree of w^3:

$$w^6 + qw^3 - \frac{p^3}{27} = 0, \tag{5}$$

Let us solve this for w^3. Let w_1, w_2, and w_3 w^3 be the *the cubic roots.*

> All real numbers, except 0, have exactly one *real* cubic root, and two complex conjugates, and all non-zero complex numbers have three distinct complex cubic roots.
> For example:

$$\sqrt[3]{0} = 0 \tag{6a}$$

$$\sqrt[3]{8} = \begin{cases} 2 \\ -1 - i\sqrt[2]{3} \\ -1 + i\sqrt[2]{3} \end{cases} \tag{6b}$$

$$\sqrt[3]{-27i} = \begin{cases} 3i \\ \frac{3\sqrt{3}}{2} - \frac{3}{2}i \\ -\frac{3\sqrt{3}}{2} - \frac{3}{2}i \end{cases} \tag{6c}$$

Now the roots of equation (2) are

$$t_1 = w - \frac{p_1}{3w_1}, \tag{7a}$$
$$t_2 = w - \frac{p_2}{3w_2}, \text{ and} \tag{7b}$$
$$t_3 = w - \frac{p_3}{3w_3} \;\; . \tag{7c}$$

Exercise 2

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \tag{8}$$

The adjungated matrix is computed as follows:

$$\mathsf{adj}(A) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{21} & a_{21} \\ a_{32} & a_{33} \end{vmatrix} \\ & & \\ - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{32} & a_{33} \end{vmatrix} \\ & & \\ + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix} \tag{9}$$

Thus

$$\mathsf{adj} \begin{pmatrix} -3 & 2 & -5 \\ -1 & 0 & -2 \\ 3 & -4 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 18 & -4 \\ -5 & 12 & -1 \\ 4 & -6 & 2 \end{pmatrix} \tag{10}$$

Exercise 3

$$\begin{tikzcd}
A \arrow[rr, dash] \arrow[dr, dash] \arrow[dd, dash] & & B \arrow[dr, dash] \arrow[dd, dash] & \\
& E \arrow[rr, dash] \arrow[dd, dash] & & F \arrow[dd, dash] \\
C \arrow[rr, dash] \arrow[dr, dash] & & D \arrow[dr, dash] & \\
& G \arrow[rr, dash] & & H
\end{tikzcd}$$

Exercise 4

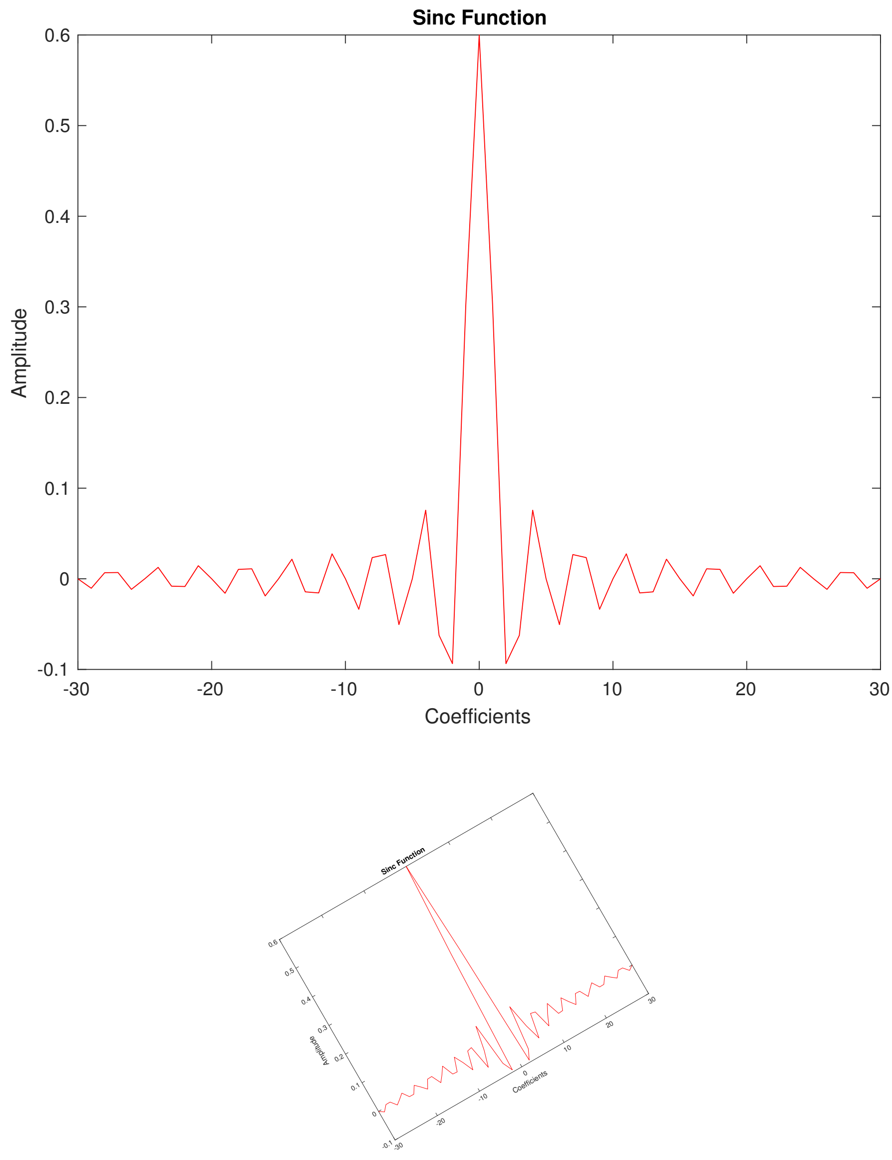

Figure 1: “interesting”curve

Exercise 5

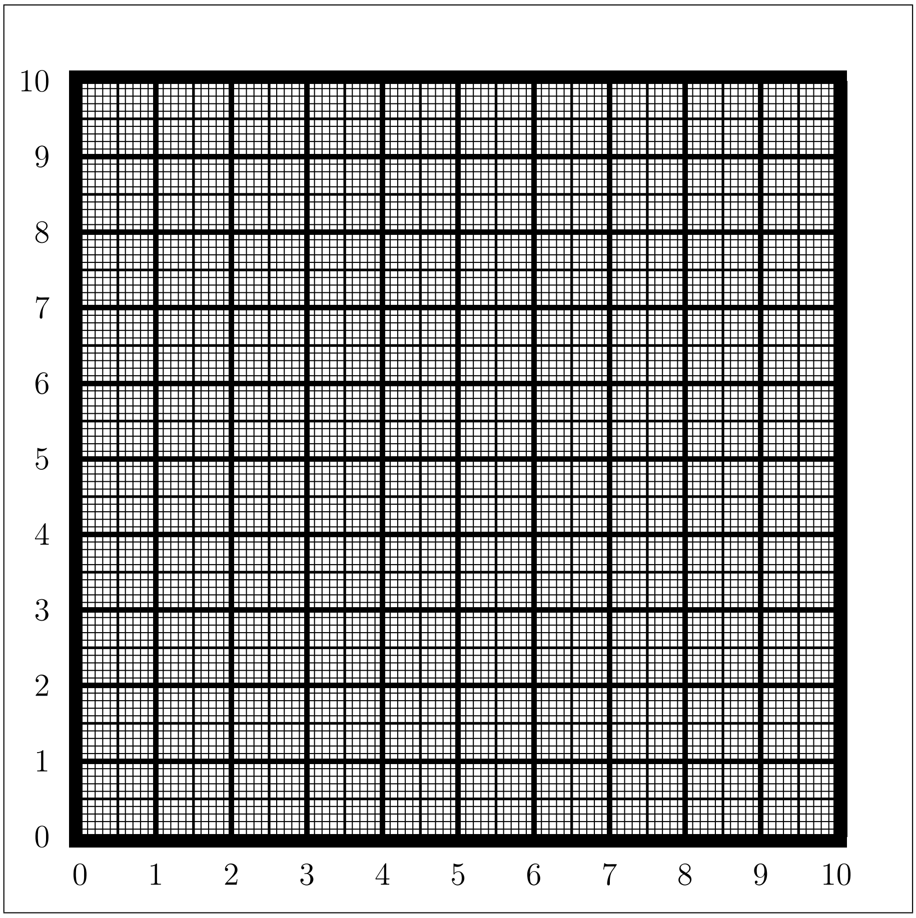

Figure 2: mm grid