METHODS IN PHYSICS

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Project 3

The Compton Scattering

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Contents

1. Introduction

The wave-particle duality of the nature of light has been broiling topic of debate amongst philosophers and scientist since the long before the birth of quantum mechanics. "Aristotle was one of the first to publicly hypothesize about the nature of light, proposing that light is a disturbance in the element aether (i.e. a wave-like phenomenon). On the other hand, Democritus – the original atomist – argued that all things in the universe, including light, are composed of indivisible sub-components (light being form of solar atoms)" [1].

At the beginning of the 11th Century, the Arabic scientist Alhazen wrote the first comprehensive treatise on optics, describing refraction, reflection, and the operation of a pinhole lens via rays of light travelling from the point of emission to the eye. He asserted that these rays were composed of particles of light [1].

The modern duality about the nature of light comes from the combined hypotheses of Newton and Huygens. Newton, with his invention of the reflecting telescope and experimental investigation of color when white light passed through a prism, argued that light is made up of particles which he called 'corpuscles'. However, this experimental analysis did not provide any information about the particle nature of light.

Huygens, however, thought light as a wave. He demonstrated how waves might interfere to form a wavefront, propagating in a straight line. He argued that known properties of light could be understood by wave-like nature of light.

The duality nature of light was proven formally by the work of Max Planck who suggested that energy carried by electromagnetic waves could only be released in "packets" of energy and Albert Einstein's photoelectric effect advancing Planck's hypothesis. In 1922, an American Physicist, Arthur H. Compton provided further evidence for duality nature of light through his work, widely known as Compton effect.

The Compton Effect is the quantum theory of the scattering of electromagnetic waves by a charged particle in which a portion of the energy of the electromagnetic wave is given to the charged particle in an elastic, relativistic collision but inelastic scattering. Compton scattering was discovered in 1922 while conducting research on scattering of X-rays by light elements. He won Nobel Prize in Physics for this in 1927.

2. Theoretical Background on Compton Scattering

Compton scattering involves the scattering of photons by charged particles where both energy and momentum are transferred to the charged particles while the photon moves off with a reduced energy and change of momentum. The process in inelastic scattering because the incoming photon exits with reduced energy whereas the collision is elastic because the total energy of the system remains the same. In most of the cases, the charged particles is considered to be an electron at rest and photon is usually considered to be an energetic photon such as an X-ray photon or gamma ray photon. In our case, the incoming photon would be gamma rays from Americium, ^{241}Am radioactive source. The scatterer used in this experiment is Aluminum plate because it has relatively higher density of free electrons.

2.1 Derivation of Compton wavelength and other energy equations

We will consider the relativistic mechanics in our theory of Compton scattering mainly for two reasons. First, the process involves the scattering of massless photons, and second, the energy transferred to the electron is in the range of electron's rest energy. Consequently, the expression for energy and momentum of photon are expressed in relativistic form. Like any other classical case of collision, we will consider the conservation of momentum and conservation of relativistic energy to develop the theory of Compton scattering.

Relativistic form for momentum is,

$$\boldsymbol{p} = \gamma m \boldsymbol{v} \tag{1}$$

where, m is rest mass and $\boldsymbol{\gamma}$ is the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

The expression for relativistic energy is,

$$E^2 = E_0^2 + (pc)^2 \tag{3}$$

where, c is the speed of light in vacuum, E_0 is the rest mass energy and is given by,

$$E_0 = mc^2 \tag{4}$$

For gamma ray with zero mass the relativistic energy becomes,

$$E = pc \tag{5}$$

The energy of the gamma ray can also be expressed using photon theory as,

$$E = hf = \frac{hc}{\lambda} \tag{6}$$

where, f is the frequency of the gamma ray, λ is the wavelength. Now, we can visualize the photon-matter interaction using the picture below. This gives a similar idea to classical collision, which helps us understand the interaction vividly.

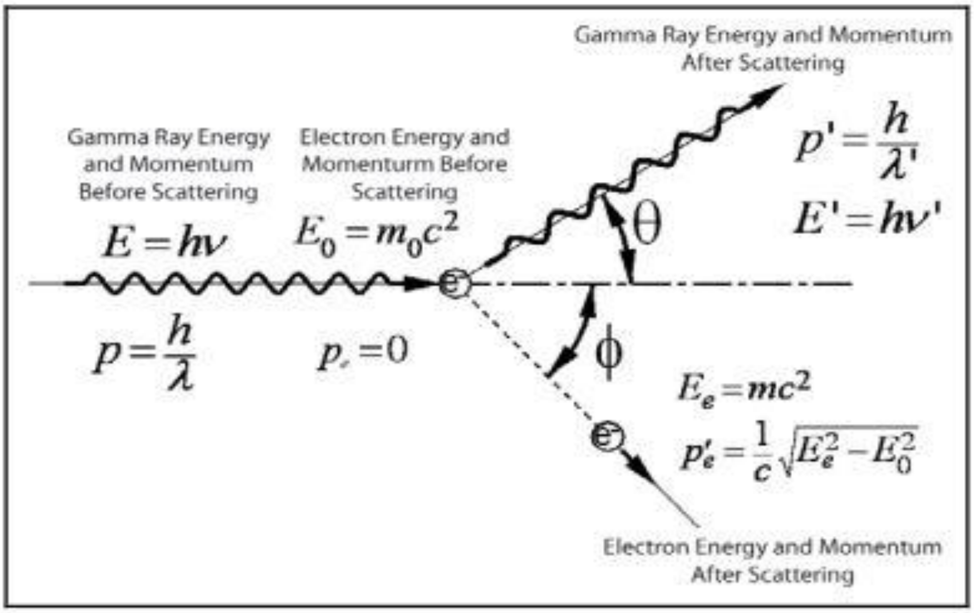


Figure 1: *Compton scattering diagram showing the before and after conditions of Collison between a gamma ray with certain momentum and energy and electron at rest with rest mass energy [2].*

We can now use the first condition of our scattering that is the conservation of relativistic energy.

Before condition: Gamma ray with photon energy and electron with rest mass energy.

After condition: Gamma ray with reduced energy and electron with total energy.

Therefore,

$$hf + E_0 = hf' + E_e \tag{7}$$

where, again E_0 is the rest mass energy of the electron, f', is the new frequency of the gamma ray after the collision and E_e is the total energy of the electron. Using equation (3), we can form a relationship between the total energy of an electron and momentum of the electron,

$$E_e^2 = E_0^2 + (p_e'c)^2 \tag{8}$$

and

$$E_e = \sqrt{E_0^2 + (p_e'c)^2} \tag{9}$$

where, $\boldsymbol{p_e}'$ is the momentum of the electron after collision.

Now, using equation (9) in equation (7), we get

$$hf + E_0 = hf' + \sqrt{E_0^2 + (p_e'c)^2} \tag{10}$$

Using equation (5) for energy expression of gamma ray, we get

$$pc + E_0 = p'c + \sqrt{E_0^2 + (p_e'c)^2} \tag{12}$$

After some rearrangement,

$$p^2 + p'^2 - 2pp' + \frac{2(p-p')E_0}{c} = p_e'^2 \tag{13}$$

This expression is a result derived from using conservation of relativistic energy.

We will now invoke second condition of collision theory, i.e. conservation of momentum. We will use our figure above to get some directional momentum after collision.

Initial momentum: In x-direction is $\boldsymbol{p}$ and in y-direction it is zero.

Final momentum: In x-direction is $p'cos\theta + p_e'cos\phi$ and in y-direction, it is $p'sin\theta - p_e'sin\phi$.

We can now compare our momenta to get,

$$\boldsymbol{p} = p'cos\theta + p_e'cos\phi \tag{14}$$

and

$$0 = p'sin\theta - p_e'sin\phi \tag{15}$$

After some manipulation we get,

$$p^2 + p'^2 - 2pp'cos\theta = p_e'^2 \tag{16}$$

Equation (16) is a result from our second condition, that is, conservation of momentum. We can directly see from this equation, the relation between the angle of scattering and the momentum of the electron and gamma ray after and before the collision.

Now, we have everything that we need to derive the final equation of Compton scattering.

Combining equations (13) and (16), we get

$$\frac{1}{p'} - \frac{1}{p} = \frac{c}{E_0}(1 - cos\theta) \tag{17}$$

Finally, using de Broglie relations $p = \frac{h}{\lambda}$, we get the general form of Compton scattering.

$$\frac{1}{\lambda\prime} - \frac{1}{\lambda} = \frac{h}{mc}(1 - cos\theta) \tag{18}$$

This equation is the entirety of Compton scattering. It basically tells us the relation between the wavelength of gamma ray before and after the collision and its dependence on the angle of scattering.

Similarly, we can express equation (17) in terms of energy of the scattered photon and rest mass energy of an electron. Using the fact that, *p = E/c*,

$$\frac{1}{E\prime} - \frac{1}{E} = \frac{1}{E_0}(1 - cos\theta) \tag{19}$$

If we rearrange equation (19) in a bit elegant way, then we get this direct relationship between the energy of scattered photon and incident photon.

$$E' = \frac{E}{\frac{E}{mc^2}(1-cos\theta)+1} \tag{20}$$

From equation (20), we see that the energy of scattered photon lies in the range of maximum value when cos is -1 and minimum value when cos is 1. So, we can expect that our experimental result would give us the energy of scattered photon in this range.

$$(E'_{max}, E'_{min}) = (E, \frac{E}{\frac{2E}{mc^2}+1}) \tag{21}$$

We can also find an expression for kinetic energy of a scattered electron and scattered energy of the gamma ray.

Kinetic energy of an electron is given by,

$$K.E = E_e - E_0 \tag{22}$$

So, if we again invoke the law of conservation of Energy then we get,

$$K.E = E - E' \tag{23}$$

and substituting equation (20) in equation (23), we get

$$K.E = \frac{E}{\frac{E_0}{E(1-cos\theta)}+1} \tag{24}$$

where, E_0 is the rest mass energy of the electron. This equation explicitly shows the dependency of kinetic energy of scattered electron on the angle of scattering and the energy of an incident gamma ray.

using condition (21) we can also find the range of kinetic energy of an scattered electron,

$$(K.E_{min}, K.E_{max}) = \left(0, \frac{E}{\frac{E_0}{2E}+1}\right) \qquad (25)$$

With equation (20) and (24), we can observe the change in energy of scattered photon and gain in Kinetic energy of the scattered electron. If we plot a graph of these energies against the angle of scattering then we can easily understand the Compton scattering phenomenon.

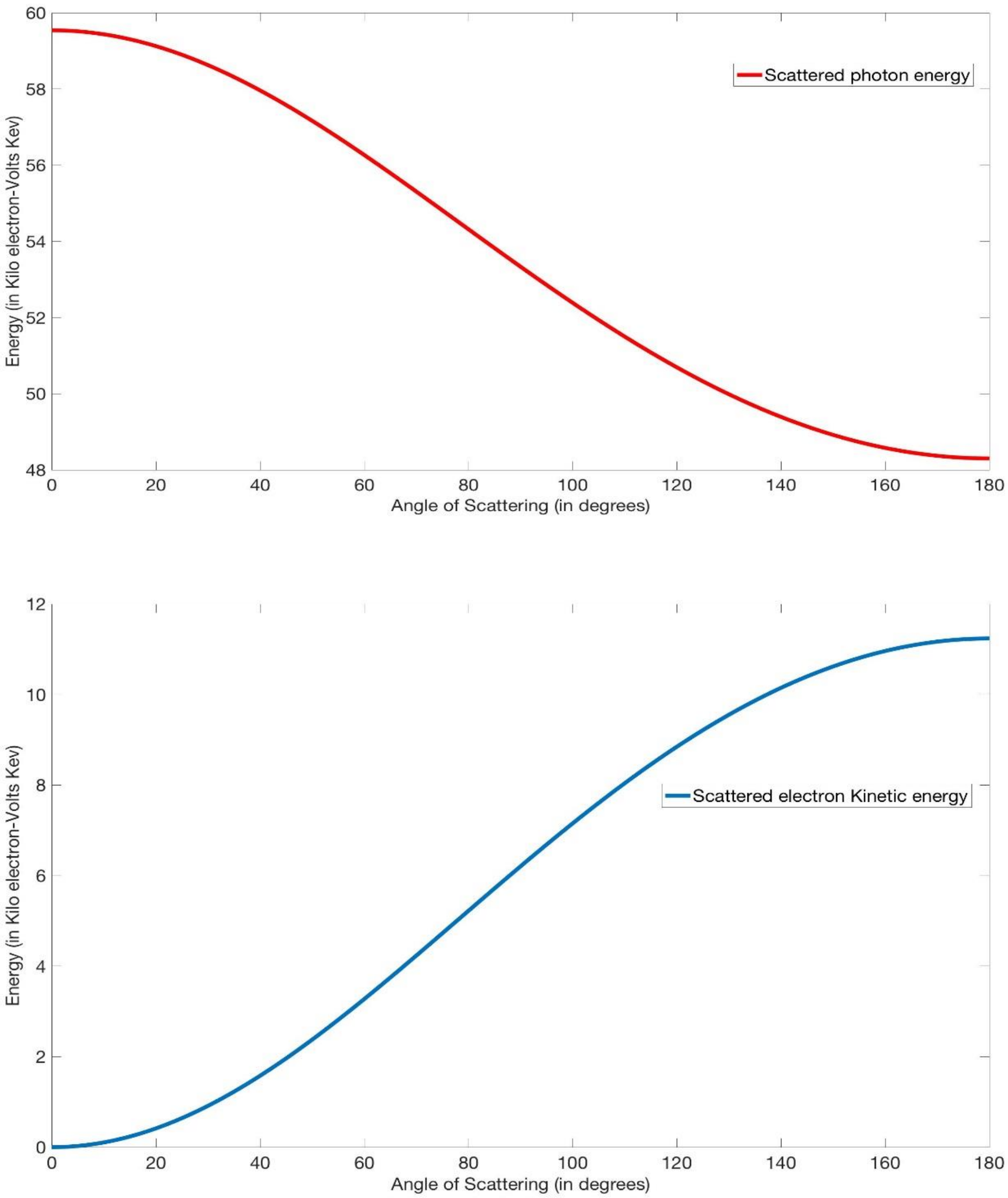


Figure 2: *Scattered photon energy (top) and scattered electron energy (bottom) as a function of angle of scattering [11].*

The figure above clearly shows that the maximum value of scattered photon energy is at an angle 0 and minimum value at angle 180. Whereas, the maximum kinetic energy of scattered electron is at an angle 180 and minimum at an angle 0.

2.2 Matter-wave interaction

Compton scattering is the motive of our experiment, however it is necessary to understand the interactions of high energy, electromagnetic photon radiation with materials in general. Gamma rays are high-energy photons emitted from radioactive sources. When they interact with matter, they generally interact with matters in three primary ways on the basis of their absorbed energies by the materials.

These three primary ways are: 1) photoelectric effect – low energy photon interaction, 2) Compton scattering – medium/high energy photon interaction, and pair production – high-energy photon interaction. In addition to these primary processes, there are several lesser ways such as X-ray production and Bremsstrahlung. The Compton Effect is studied with the measurement of a gamma ray energy spectrum using a scintillator, photomultiplier tube, and multichannel analyser.

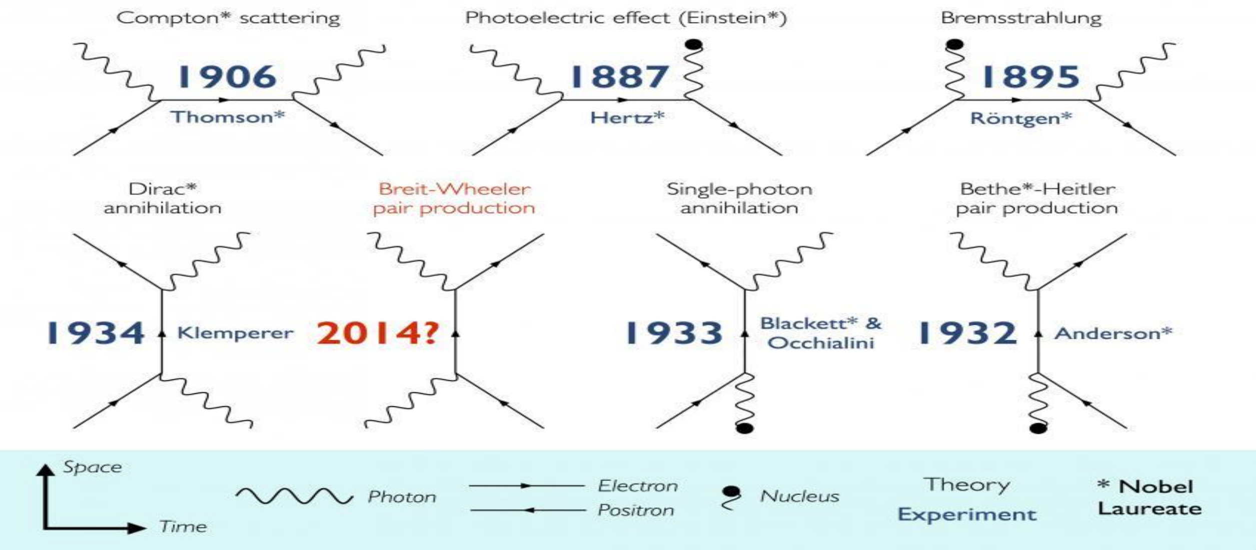


Figure 3: *Some of the well-known matter wave interaction [3].*

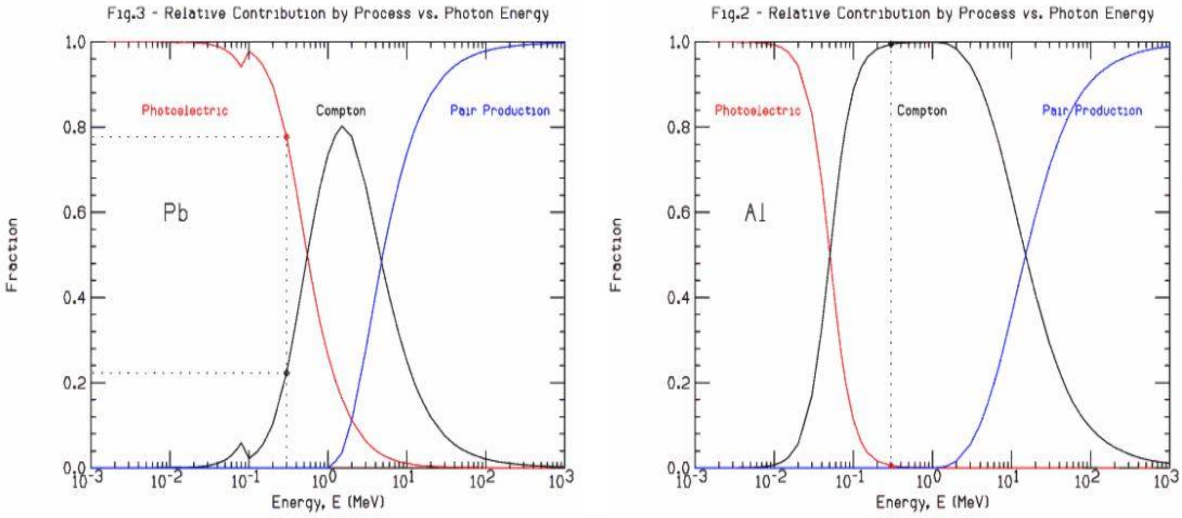


Figure 4: *Matter-wave interaction as a function of photon energy [4].*

3. Distribution functions

Distribution function helps us understand how the data are spread-out. The widely used distribution functions with experimental data can be categorized in two types.

3.1 Normal Distribution or Gaussian distribution

Data obeying Gaussian distribution tend to be around a central value with no bias on left or right. A set of data can be said is normally distributed if

1. Mean, median and mode of the data are same.
2. The distribution is symmetry about the center, and
3. 50% of data are less than mean and 50% greater than mean.

The probability distribution function of normal distribution is,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right), \sigma > 0 \tag{26}$$

and the cumulative distribution function is,

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dt, \sigma > 0 \tag{27}$$

where, mean = μ, variance = σ^2, and standard deviation = σ.

The probability density function is the probability that the variate has the value x. The cumulative distribution function is the probability that the variable takes a value less than or equal to x.

3.2 Cauchy Distribution or Lorentzian distribution

Cauchy distribution resembles normal distribution graphically. However, it has a taller peak than a normal distribution. And unlike the normal distribution, its thick tail decays much slowly. The expected value of Cauchy distribution is essentially zero. However, the median and mode do exist and they are equal. They define the line of symmetry.

Uses of Cauchy distribution

1. Robustness studies
2. Models the ratio of two normal random variables.
3. In quantum mechanics, it models the distribution of energy of an unstable state.

The probability distribution function of standard Cauchy distribution is,

$$f(x) = \frac{1}{s\pi(1+(\frac{x-t}{s})^2)} \tag{28}$$

where t is the location parameter and s is the scale parameter. The case where t = 0 and s =1 is called the standard Cauchy distribution. Then the equation becomes,

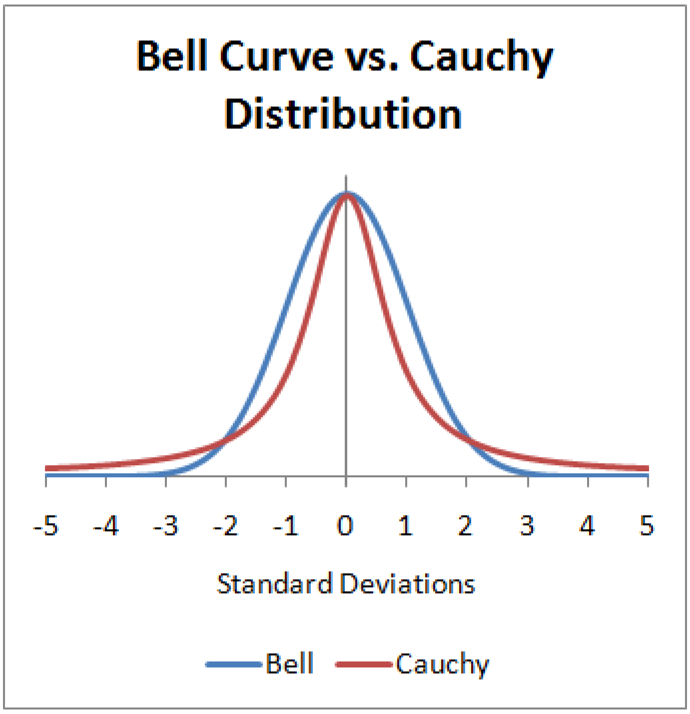
$$f(x) = \frac{1}{\pi(1+x^2)} \tag{29}$$

The cumulative distribution function of Lorentz distribution function is,

$$F(x) = 0.5 + \frac{\arctan(x)}{\pi} \tag{30}$$

3.3 Normal Distribution vs Cauchy distribution

In the figures below, We compare the normal probability with mean 0 and standard deviation of 1. Whereas, the Cauchy probability distribution in the same figure is the standard Cauchy distribution as stated in section 3.2. On the right, the cumulative distribution is shown with the same parameter.



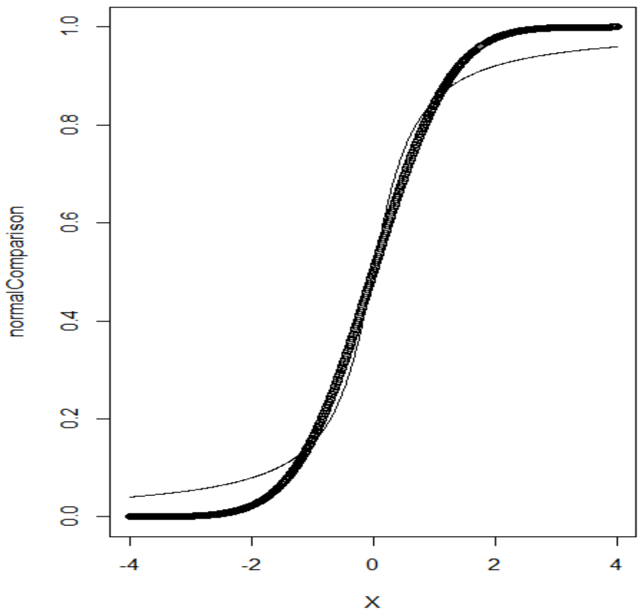


Figure 5: *normal distribution versus Cauchy distribution (left) and cumulative distribution of normal and Cauchy distribution (thick black line – normal, thin line Cauchy) [5].*

4. Procedure and Experimental Details

4.1 Experimental Apparatus

The apparatus is shown in the figure below. The apparatus contains 1) a measurement chamber consisting of ^{241}Am gamma source with radioactivity of 1670 Mbq. The radioactive gamma source is fully contained inside the chamber, which also collimates the gamma rays emanating from it. Additionally, the chamber has an angle slider on the bottom that allows us to change the angle of scattering. A sample holder lies on the top middle of the chamber that lets us feed-through the scatterer or ^{241}Am sample for calibration. Finally, the chamber has an XR-100CR X-ray detector on its edge consisting of semiconductor crystal to detect the scattered or incident radiation. 2) An aluminum scatterer, used for scattering the incoming gamma ray. 3) A Pocket MCA 8000A multichannel analyzer system consisting of Spectrum techniques model AmpTek ADMCA with measurement software, and 4) AmpTek PX2T/CR power supply and amplifier for the X-ray detector.

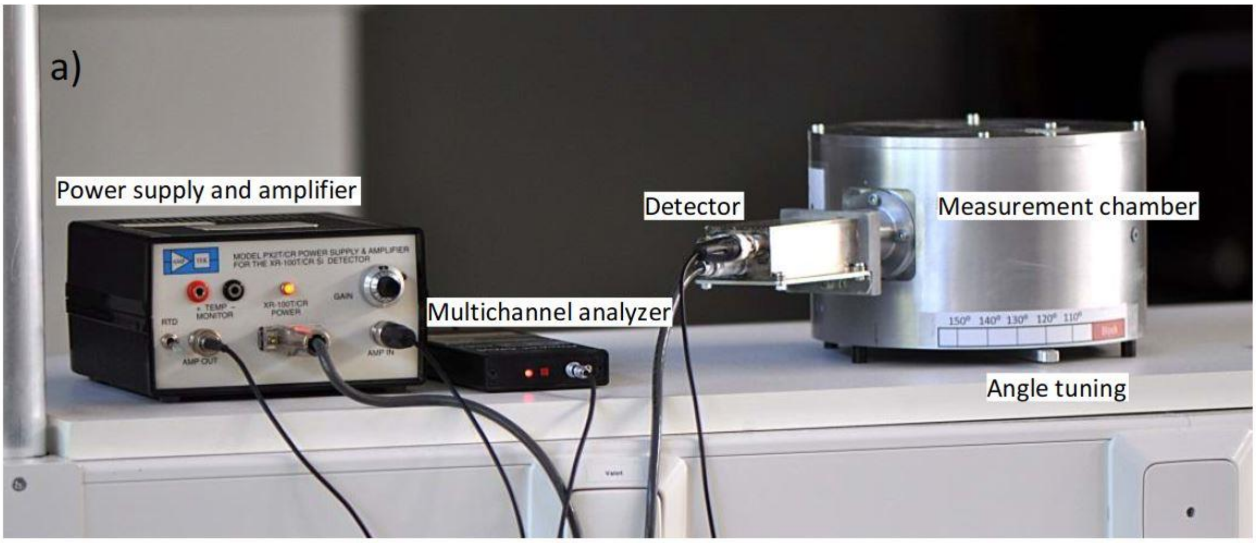


Figure 6: *Experimental apparatus [4].*

4.2 Experimental setup operating principle

In order to study the Compton Effect, a gamma ray spectroscopy method is needed to measure the gamma ray's energy before and after an interaction. The x-ray detector consisting semiconductor crystal is used to measure the gamma ray spectra. The detector system produces a voltage pulse that is proportional to the energy deposited in the crystal by the absorbed gamma ray. The detected gamma ray may be an incident one or scattered one. The size of the voltage pulse, and hence the energy deposited in the detector, is measured with a multichannel analyzer. The deposited energy depends on the type of matter-wave interaction. An MCA measures the distribution of voltage pulse heights, or spectrum of voltage pulses, for multiple gamma rays interacting in the crystal depending on the type of interaction that occurs.

4.3 Effect of collection time on the measured spectrum

MCA measures two types of times, real time (Measurement time) and live time (Spectrum collection time). Measurement time is the time for the whole duration of measurement. Spectrum collection time, in the other hand, is the actual duration of time for which MCA is sensitive to incoming counts. The MCA doesn't need a lot of time to analyze the signal, and during this analysis time, it is insensitive to any other incoming signals and would miss processing them. Therefore, it pauses the timer during the processing time. At low signal rates, there is not much difference in either of these times. However, at high signal rates, the difference is quite significant. It is best to use live time to accumulate data for equal times for comparison.

4.4 Energy calibration

For all intention and purposes, we want our devices, MCA, detector to be linear devices; however, that is not the cases. The devices may have a small quadratic or higher power dependency. To avoid this non-linearity, we need to calibrate the MCA. The calibrations can be done using one-point, two-point or 3-point calibration technique. In our experiment, we will use two-point calibration technique. In this technique, we will use Two gamma ray energies value of our sample ^{241}Am for calibration. This forces the relationship between energy and channels to be linear. We can do the calibration with following steps:

1. We will place the sample ^{241}Am through the sample holder.
2. The radiation coming from the sample is blocked inside the chamber from reaching the detector.
3. When the spectrum is collected, the energy calibration is done. The calibration requires two calibration point in this case (channel value and known reference value for the energy.)

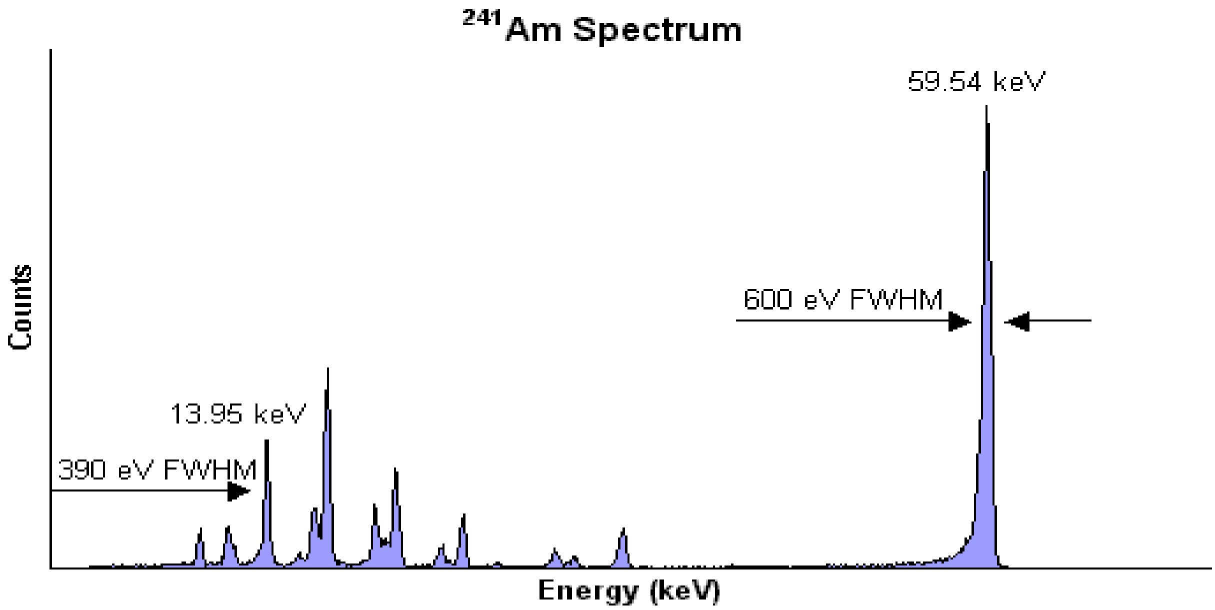


Figure 7: *Gamma spectrum of ^{241}Am and reference values for some spikes in range 0-70kev [6].*

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