Weekly Exercise 5

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1 Question 1

$\sin A \cos B = \left(\sin\left(A-B\right)+\sin\left(A+B\right)\right)/2$

$\boxed{\sin A \cos B = \left(\sin\left(A-B\right)+\sin\left(A+B\right)\right)/2}$

$\sin A \cos B = \frac{\sin(A-B)+\sin(A+B)}{2}$

$$\boxed{\sin A \cos B = \frac{\sin\left(A-B\right)+\sin\left(A+B\right)}{2}}$$

$\sin A \cos B = \frac{1}{2}\left(\sin\left(A-B\right)+\sin\left(A+B\right)\right)$

$$\boxed{\sin A \cos B = \frac{1}{2}\left(\sin\left(A-B\right)+\sin\left(A+B\right)\right)}$$

$\sin A \cos B = \frac{1}{2}\big(\sin\left(A-B\right)+\sin\left(A+B\right)\big)$

$$\boxed{\sin A \cos B = \frac{1}{2}\big(\sin\left(A-B\right)+\sin\left(A+B\right)\big)}$$

$e^{\phi} = \cos\phi + i\sin\phi$ $\boxed{e^{\phi} = \cos\phi + i\sin\phi}$

$f(x) = \frac{d}{dx}\left(\int_0^x f(u)du\right)$ $$\boxed{f(x) = \frac{d}{dx}\left(\int_0^x f(u)du\right)}$$

$\frac{d}{dx}\arctan\big(\sin\left(x^2\right)\big) = -2\frac{\cos\big(x^2\big)x}{-2+\big(\cos(x^2)\big)^2}$

$$\boxed{\frac{d}{dx}\arctan\big(\sin\left(x^2\right)\big) = -2\frac{\cos\left(x^2\right)x}{-2+\big(\cos\left(x^2\right)\big)^2}}$$

$\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$ $$\boxed{\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}}$$

2 Question 2

Let A be a finite set and $n \in \mathbb{N}$.

- $A^2 \stackrel{\text{def}}{=} A \times A$ is the *cartesian product* of A by itself.
- If $A = \{a, b\}$ then $A^2 = \{(a, a), (a, b), (b, a), (b, b)\}$.
- $A^n \stackrel{\text{def}}{=} \overbrace{A \times A \times \cdots \times A}^{n \text{ times}}$ is A to the power of n or the set of all n-tuples that can be produced from A. Especially $A^0 = \{()\}$.
- The *powerset* of A is $\mathcal{P}(A) = \left\{B \;\middle|\; B \subseteq A\right\}$, the set of all subsets of A. It is often denoted also as 2^A.
- When the elements of A are interpreted as characters, it is a custom to leave the parenthesis and commas. It is also a custom to denote $() = \varepsilon$.
- $A^* \stackrel{\text{def}}{=} A^0 \cup A^1 \cup \cdots = \lim_{n\to\infty} \bigcup_{i=0}^{n} A^i$. The elements of A^*:n are all *finite* (strings). But, if $|A| \neq 0$ then $|A^*| = \infty$.
- $A^\omega = \underbrace{A \times A \times \cdots}_{\infty \text{ times}}$ is the set of all ∞-tuples of A (if $A = \emptyset$ then $A^\omega = \emptyset$).
- $A^\infty \stackrel{\text{def}}{=} A^* \cup A^\omega$, the set of all finite and infinite tuples that can be produced from A.
- The *catenation* of strings α, β can be defined so that $\alpha\beta$ is the catenation of α and β if and only if $\{\alpha\beta\} = \{\alpha\} \times \{\beta\}$.

3 Question 3

$$
\circledbullet \Longleftrightarrow \circ
\begin{array}{ccc} \nearrow & \longrightarrow & \searrow \\ & \bigodot & \\ \nwarrow & \longleftarrow & \swarrow \end{array}
\bullet \Longrightarrow \circledcirc
$$

4 Question 4

$$
\frac{1+\sqrt{5}}{2} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}
$$

$$
\pi = 3 + \cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}}
$$

$$
\sqrt{x} = 1 + \frac{x-1}{1+\sqrt{x}} = 1 + \cfrac{x-1}{2+\cfrac{x-1}{2+\cfrac{x-1}{2+\cfrac{x-1}{2+\ddots}}}}
$$

5 Question 5

$$
\begin{aligned}
D(a,\mathbf{r}) &= \left\{ z \in \mathbb{C} \;\middle|\; |z-a| < \mathbf{r} \right\}, \\
\mathsf{seg}(a,\mathbf{r}) &= \left\{ z \in \mathbb{C} \;\middle|\; \Im z = \Im a \wedge |z-a| < \mathbf{r} \right\}, \\
c(e,\Theta,\mathbf{r}) &= \big\{ (x,y) \in \mathbb{C} \;\big|\; |x-e| < \tan\Theta \wedge \\
&\qquad\qquad\qquad 0 < y < \mathbf{r} \big\}, \text{and} \\
\mathcal{C}(E,\Theta,\mathbf{r}) &= \bigcup_{e \in E} c(e,\Theta,\mathbf{r}).
\end{aligned}
$$