Handout

Introduction

An American physicist by the name Arthur Compton observed a phenomenon in the 1920s, which came to be known as the Compton effect or Compton scattering. He stated that the wavelength of X-rays scattered by a paraffin layer was greater than the wavelength of the original rays hitting it. Later the same effect was observed with gamma rays, and it can be generalized that the effect happens every time electromagnetic radiation is scattered by a charged particle, usually a proton or an electron. Compton scattering is a paradigm of inelastic scattering. The effect proved the dual nature of light, because the classical theory of electromagnetism could not explain the change in wavelength caused by scattering. During a time when the interpretation of the photoelectric effect and the particle-like nature of light (photons) were still being argued, Compton's measurements gave clear and independent evidence for light's particle-like nature. Compton was given the Nobel Prize in Physics in 1927.

Compton effect and its theory

The change of wavelength in Compton scattering depends on the scattering angle following the formula

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta), \qquad (1)$$

where λ is the wavelength of the original radiation and λ' the wavelength of scattered light and λ_C is a constant that is characteristic to the particle that is doing the scattering, known as the *Compton wavelength*. Picture 1 illustrates the measuring of the energy distribution of the scattered radiation.

According to the quantum theory, the energy of a quantum of electromagnetic radiation, also known as a photon, is calculated using the equation

$$E = h\nu, \qquad (2)$$

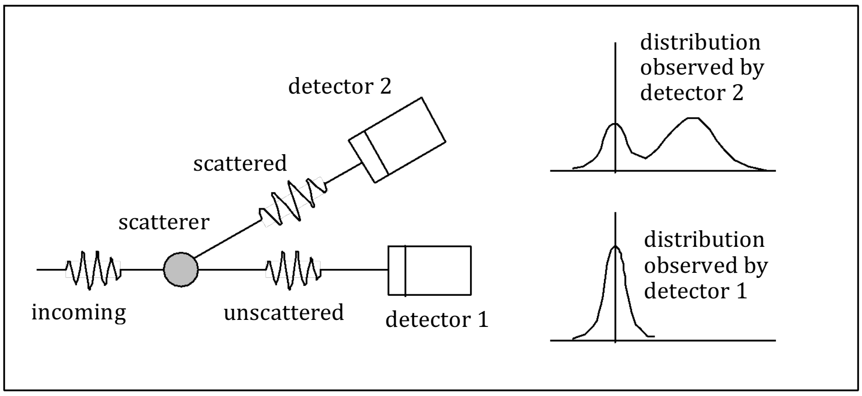
where h is the Planck constant and ν is the frequency of the photon. The classical relationship between frequency and wavelength is expressed by

$$c = \lambda\nu, \qquad (3)$$

where c is the speed of light. According to the theory of relativity, the total energy of a free particle is

$$E = c\sqrt{{m_e}^2c^2 + p_e^2}, \qquad (4)$$

where m_e is the resting mass of the particle and p_e its momentum.



***Picture 1** The Compton effect. The wavelength distribution of scattered and unscattered radiation.*

A resting particle has a significant amount of energy, known as rest energy

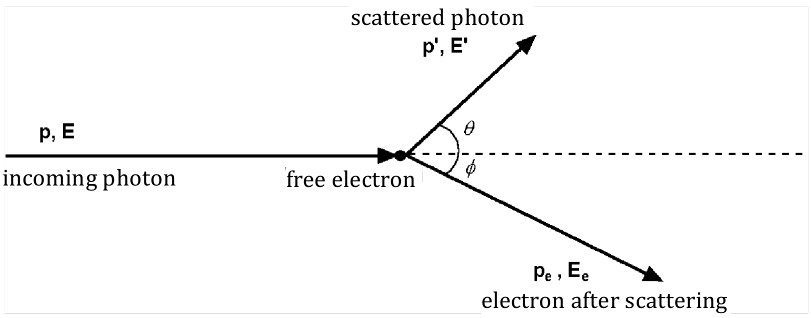
$$E_0 = m_e c^2. \tag{5}$$

A photon can be assumed to be a particle with zero rest mass. Then it is possible to state the photon's energy using its momentum

$$E = pc. \tag{6}$$

A photon's energy can be therefore expressed using two equations (2) and (6).

If a photon's energy is notably greater than an electron's binding energy in an atom, the electron can be thought of as a free particle. The translational momentum of an electron like this is zero.



***Picture 2** The elastic collision of a photon and a free electron follows the laws of conservation of momentum and energy. In the picture's case, a photon is scattered by a free electron, but generally any charged particle can do the scattering.*

Momentum is conserved in the collision between a photon and a free electron, so

$$p = p_e + p', \tag{7}$$

where p_e is the electron's relativistic Lorentz invariant momentum (vector) gained from the collision impulse. The directions of the momentum vectors follow picture 2. The change of wavelength in Compton scattering (equation (1)) can now be derived by using laws of conservation for relativistic total energy and momentum. When the scattering is done by an electron, a theoretical value for the Compton wavelength is obtained by

$$\lambda_C = \frac{h}{m_e c} = 2{,}426 \cdot 10^{-12}\, m, \tag{8}$$

where $m_e = 9{,}1091 \cdot 10^{-31}\,kg$ is the electron's rest mass. The Compton wavelength is the average value of the scattered radiation's change of wavelength. An equation for the energy of a scattered phonot can be derived

$$E' = \frac{E}{1+(1-\cos\theta)\cdot\frac{E}{m_e c^2}} \qquad (9)$$

and it fluctuates between

$$(E'_{max}, E'_{min}) = \left(E, \frac{E}{1+2E/m_e c^2}\right). \qquad (10)$$

Correspondingly the energy received by the electron during the scattering fluctuates between

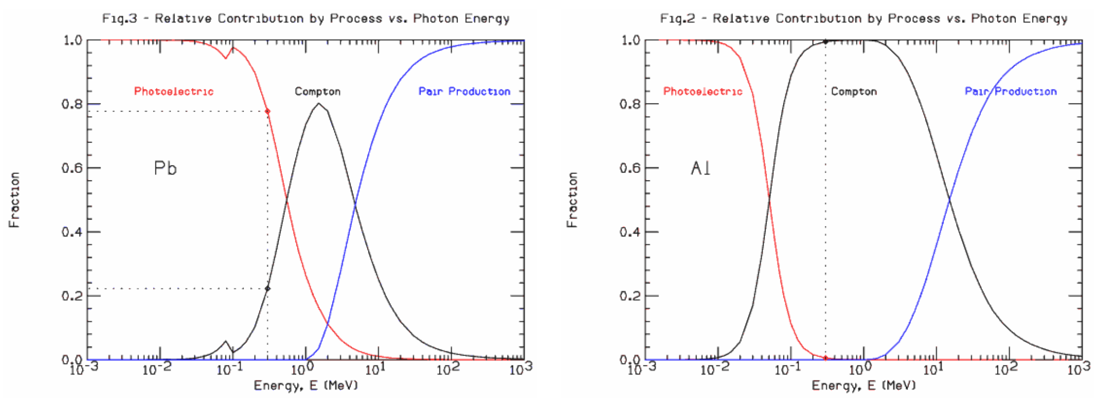
$$(E'_{emin}, E'_{emax}) = \left(0, \frac{E}{1+m_e c^2/2E}\right). \qquad (11)$$

Theoretical observation of the Compton effect should be based on Dirac's relativistic theory of the electron. The interaction between an electron and the field represented by a photon is treated as a three-part process. First a photon and a free, practically at rest, electron are assumed. Then the electron absorbs the photon and finally, the electron emits a new photon while transitioning to a new energy level. This leads to the expressions for scattering cross sections, which correspond to experimental results.

The interaction between electromagnetic radiation and matter

Three mechanisms for electromagnetic radiation and matter interaction can be separated, where the photon loses energy or disappears completely: the photoelectric effect, Compton effect and pair production.

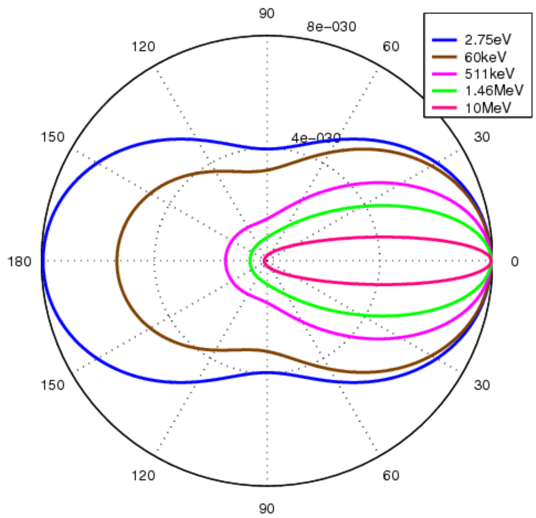
The probability for an interaction mechanism depends on the photons energy and the material. Picture 3 shows the relative probabilities as a function of the photon's energy, when the target materials are lead and aluminum. The graph shows what is the relative probability of a mechanism, when a photon of set energy interacts with the material. For example using a photon with an energy of 120 keV and lead as the material, the photoelectric effect is observed with a probability of approximately 78% and the Compton effect with an approximate probability of 22%, while with aluminum and a photon with 120 keV of energy, the probability for the Compton effect is almost 100%.



Picture 3 *The relative probability of an electromagnetic radiation's and material's inelastic interaction mechanism, when the target material is a) lead and b) aluminum.*

The scattering angle of a scattered photon in relation to the direction of incidence in the Compton effect is dependent on the photon's energy. This is demonstrated in picture 4.

Picture 4 *The scattering angle of a scattered photon in the Compton effect in relation to the angle of incidence with different photon energies according to the Klein-Nishina theory*



Literary sources

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Compton, Arthur H. (May 1923). "*A Quantum Theory of the Scattering of X-Rays by Light Elements*". Physical Review 21 (5): 483–502. doi:10.1103/PhysRev.21.483.
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Pictures:

Picture 3.
a) http://durpdg.dur.ac.uk/vvc/egs/lab/figures/fig3.gif
b) http://durpdg.dur.ac.uk/vvc/egs/lab/figures/fig2.gif

Picture 4.
Klein-Nishina formula, [internet][referenced:28.5. 2015], available:
http://en.wikipedia.org/wiki/Klein%E2%80%93Nishina_formula
http://demonstrations.wolfram.com/KleinNishinaFormulaForComptonEffect/