

# Weekly Exercise 5

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## 1 Question 1

$$\sin A \cos B = (\sin (A - B) + \sin (A + B)) / 2$$

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$$\sin A \cos B = \frac{\sin(A-B)+\sin(A+B)}{2}$$

$$\sin A \cos B = \frac{\sin (A - B) + \sin (A + B)}{2}$$

$$\sin A \cos B = \frac{1}{2} (\sin (A - B) + \sin (A + B))$$

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$$e^{\phi} = \cos \phi + i \sin \phi$$

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$$f(x) = \frac{d}{dx} \left( \int_0^x f(u) du \right)$$

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$$\frac{d}{dx} \arctan(\sin(x^2)) = -2 \frac{\cos(x^2)x}{-2 + (\cos(x^2))^2}$$

$$\frac{d}{dx} \arctan(\sin(x^2)) = -2 \frac{\cos(x^2)x}{-2 + (\cos(x^2))^2}$$

$$\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

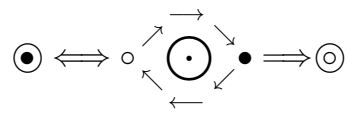
$$\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

## 2 Question 2

Let  $A$  be a finite set and  $n \in \mathbb{N}$ .

- $A^2 \stackrel{\text{def}}{=} A \times A$  is the *cartesian product* of  $A$  by itself.
- If  $A = \{a, b\}$  then  $A^2 = \{(a, a), (a, b), (b, a), (b, b)\}$ .
- $A^n \stackrel{\text{def}}{=} \overbrace{A \times A \times \cdots \times A}^{n \text{ times}}$  is  $A$  to the power of  $n$  or the set of all  $n$ -tuples that can be produced from  $A$ . Especially  $A^0 = \{()\}$ .
- The *powerset* of  $A$  is  $\mathcal{P}(A) = \{B \mid B \subseteq A\}$ , the set of all subsets of  $A$ . It is often denoted also as  $2^A$ .
- When the elements of  $A$  are interpreted as characters, it is a custom to leave the parenthesis and commas. It is also a custom to denote  $() = \varepsilon$ .
- $A^* \stackrel{\text{def}}{=} A^0 \cup A^1 \cup \cdots = \lim_{n \rightarrow \infty} \bigcup_{i=0}^n A^i$ . The elements of  $A^{*:n}$  are all *finite* (strings). But, if  $|A| \neq 0$  then  $|A^*| = \infty$ .
- $A^\omega = \underbrace{A \times A \times \cdots}_{\infty \text{ times}}$  is the set of all  $\infty$ -tuples of  $A$  (if  $A = \emptyset$  then  $A^\omega = \emptyset$ ).
- $A^\infty \stackrel{\text{def}}{=} A^* \cup A^\omega$ , the set of all finite and infinite tuples that can be produced from  $A$ .
- The *catenation* of strings  $\alpha, \beta$  can be defined so that  $\alpha\beta$  is the catenation of  $\alpha$  and  $\beta$  if and only if  $\{\alpha\beta\} = \{\alpha\} \times \{\beta\}$ .

### 3 Question 3



### 4 Question 4

$$\frac{1+\sqrt{5}}{2} = \frac{1}{1+\frac{1}{1+\frac{1}{1+\ddots}}}$$

$$\pi = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{1 + \ddots}}}}$$

$$\sqrt{x} = 1 + \frac{x-1}{1+\sqrt{x}} = 1 + \frac{x-1}{2 + \frac{x-1}{2 + \frac{x-1}{2 + \frac{x-1}{2 + \ddots}}}}$$

### 5 Question 5

$$\begin{aligned} D(a,\mathbf{r}) &= \left\{z \in \mathbb{C} \left| |z-a| < \mathbf{r} \right. \right\}, \\ \text{seg}(a,\mathbf{r}) &= \left\{z \in \mathbb{C} \left| \Im z = \Im a \wedge |z-a| < \mathbf{r} \right. \right\}, \\ c(e,\Theta,\mathbf{r}) &= \left\{ (x,y) \in \mathbb{C} \left| |x-e| < \tan \Theta \wedge \right. \right. \\ &\quad \left. \left. 0 < y < \mathbf{r} \right. \right\}, \text{and} \\ \mathcal{C}(E,\Theta,\mathbf{r}) &= \bigcup_{e \in E} c(e,\Theta,\mathbf{r}). \end{aligned}$$