MAT-63506 Scientific Computing

Exercise Set 4 26.3.–1.4. 2018

Before doing the exercises read the files "Graphics3D.mlx" and "Animation.mlx".

Exercise 1. Make a surface (`surf`) and contour (`contour`) plots for $x, y \in [-3.5, 3.5]$ of the function $z = f(x, y)$ defined by

$$f(x, y) = \sin(2x^2 + y^2 + 2) + \cos(x^2 + 3y^2 + 3).$$

Set the shading to interpolated and try different colormaps, light directions, light positions (`camlight`), surface materials and view angles until you find a combination that produces a nice looking picture.

Exercise 2. The Möbius strip is a surface with only one side. It is given parametrically by

$$\begin{aligned} x(u, v) &= \big(1 + 0.5v\cos(u/2)\big)\cos(u), \\ y(u, v) &= \big(1 + 0.5v\cos(u/2)\big)\sin(u), \\ z(u, v) &= 0.5v\sin(u/2), \end{aligned}$$

where $(u, v) \in [0, 2\pi] \times [-1, 1]$. Plot it and experiment in the same way as in Exercise 1.

Exercise 3. Plot the implicitly given surface $f(x, y, z) = \mathrm{e}^z\cos(x) - \cos(y) = 0$ with `fimplicit3` for $x, y \in [-5, 5]$. Experiment in the same way as in Exercise 1.

Exercise 4. Let $f : \mathbb{C} \to \mathbb{C}$ be the complex function given by

$$f(z) = \frac{z - 2}{z^2 + z + 2}.$$

Make a surface plot of $\ln|f(z)|$ for $z = x + iy$ and $x, y \in [-2, 2]$. Color the surface with the argument of $f(z)$ (`angle`).

Exercise 5. A two-dimensional random walk is a sequence $((x_n, y_n))_{n=1}^{\infty} \subset \mathbb{R}^2$ with $(x_0, y_0) = (0, 0)$ and for $n \geq 0$

$$(x_{n+1}, y_{n+1}) = (x_n, y_n) + \alpha(\cos\theta_n, \sin\theta_n),$$

where θ_n is a random variable uniformly distributed in the interval $[0, 2\pi]$ and α is the step size.

(a) Make an animation of the random walk using the animation examples in the file "Animation.mlx" as a model. Use the step size $\alpha = 0.1$ and a red circle of size 5 as the marker. The animation should stop when the user clicks in the figure window.

(b) Add to (a) printing the coordinates of the current point. Do this by adding two static text boxes at the lower left of the figure window with the `uicontrol`-command like this:

```
hx = uicontrol('Style', 'text', 'Units', 'char',...
 'Position', [1 1 11 1]);
```

Now you can write the x-coordinate to the text box using the handle `hx` to set the `String` property of the box. Similarly for the y-coordinate.

(c) The random walk might wander outside the current axes. Fix this by changing the `'XLim'` and `'YLim'` properties of the axes as follows. If the x-coordinate of the point is less than `XLim(1)`, decrease `XLim(1)` by a suitable amount, and if the x-coordinate is greater than `XLim(2)`, increase `XLim(2)`. Similarly for the y-coordinate. You get a handle to the current axes with the command `gca`.

Exercise 6. Using the example in "FileIO.mlx" as a guide, make an animation of the contours of the function $f(x, y) = x + y + 0.5 * m(x^2 + y^2 - 2)^2$ in the square $(x, y) \in [-2, 2] \times [-2, 2]$ for `m = 0.5:0.05:10` using the `contour`-command. Use contour levels from 0 to 20 in increments of 0.5. Save the animation in the file `contour.mp4` with the `writeVideo` command.