

Weekly Exercise 6

Shubo Yan

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Exercise 1

Solving an equation of 3rd degree

By *Vieta's formula* it is possible to solve an equation of 3rd degree.

Let the original equation be

$$ax^3 + bx^2 + cx = 0, \quad \text{where } a \neq 0 \quad (1)$$

Doing $x \leftarrow t - \frac{b}{3a}$ (*Tschirnhaus transformation*) we get for *the equation to be solved*:

$$t^3 + pt + q = 0 \quad (2)$$

By using *Vieta's formula*

$$t = w - \frac{p}{3w}, \quad \text{where } w \neq 0, \quad (3)$$

we get the equation

$$w^3 + q - \frac{p^3}{27w^3} = 0, \quad (4)$$

By multiplying both sides by w^3 we get *an equation of 6th degree* but actually it is 2nd degree of w^3 :

$$w^6 + qw^3 - \frac{p^3}{27} = 0, \quad (5)$$

Let us solve this for w^3 . Let w_1, w_2 , and w_3 w^3 be the *the cubic roots*.

All real numbers, except 0, have exactly one *real* cubic root, and two complex conjugates, and all non-zero complex numbers have three distinct complex cubic roots.

For example:

$$\sqrt[3]{0} = 0 \quad (6a)$$

$$\sqrt[3]{8} = \begin{cases} 2 \\ -1 - i\sqrt[3]{3} \\ -1 + i\sqrt[3]{3} \end{cases} \quad (6b)$$

$$\sqrt[3]{-27i} = \begin{cases} 3i \\ \frac{3\sqrt{3}}{2} - \frac{3}{2}i \\ -\frac{3\sqrt{3}}{2} - \frac{3}{2}i \end{cases} \quad (6c)$$

Now the roots of equation (2) are

$$t_1 = w - \frac{p_1}{3w_1}, \quad (7a)$$

$$t_2 = w - \frac{p_2}{3w_2}, \text{ and} \quad (7b)$$

$$t_3 = w - \frac{p_3}{3w_3}. \quad (7c)$$

Exercise 2

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad (8)$$

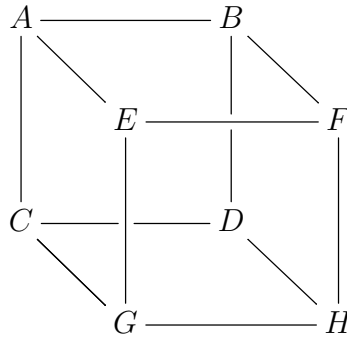
The adjungated matrix is computed as follows:

$$\text{adj}(A) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{21} & a_{21} \\ a_{32} & a_{33} \end{vmatrix} \\ - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{32} & a_{33} \end{vmatrix} \\ + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix} \quad (9)$$

Thus

$$\text{adj} \begin{pmatrix} -3 & 2 & -5 \\ -1 & 0 & -2 \\ 3 & -4 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 18 & -4 \\ -5 & 12 & -1 \\ 4 & -6 & 2 \end{pmatrix} \quad (10)$$

Exercise 3



Exercise 4

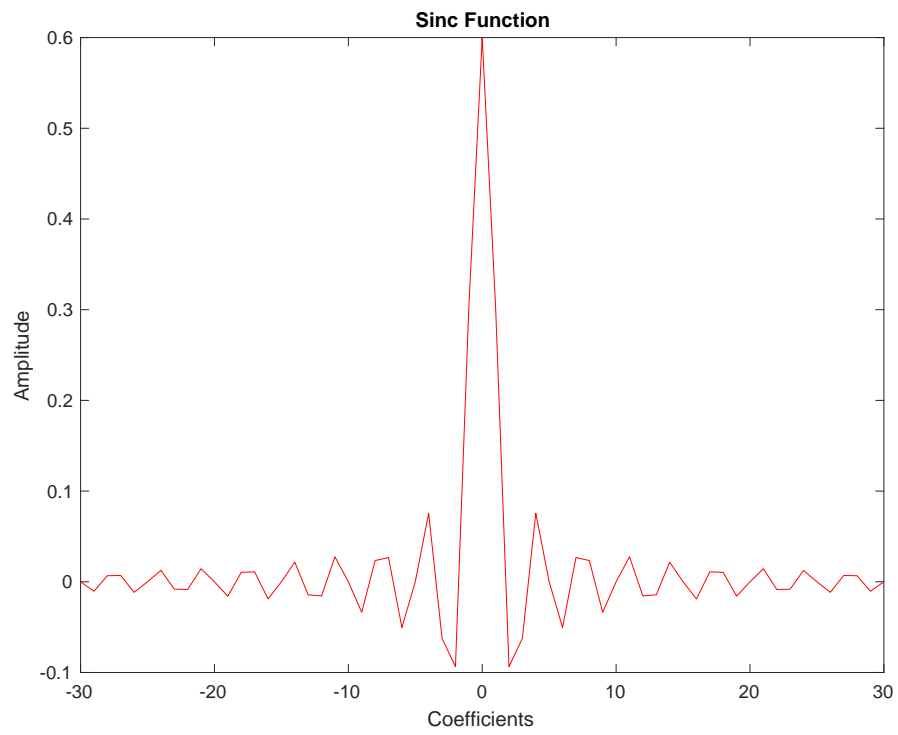


Figure 1: “interesting” curve

Exercise 5

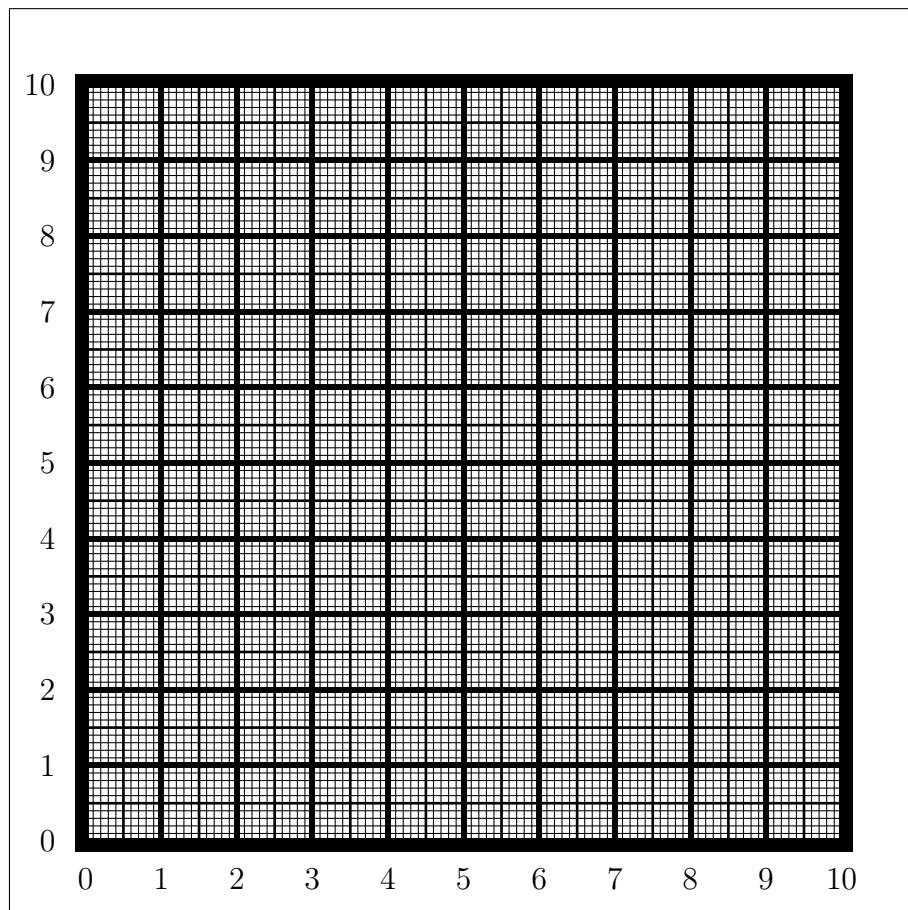


Figure 2: mm grid