MAT-63506 Scientific Computing

Exercise Set 7 23–29. 4. 2018

Before doing the exercises read the files "Optimization.mlx", "ODE.mlx", and "Initial value problems and the normal form.pdf".

Exercise 1. Consider the Initial Value Problem

$$y' + 10y = \sin(100t) - \cos(100t), \qquad y(0) = 1, \tag{1}$$

for $t \in [0, 10]$.

(a) Solve (1) with the symbolic solver `dsolve` and convert the solution into a function handle named `ye` with the command `matlabFunction`.

(b) Solve (1) with the numerical solver `ode45` and store the solution times and the solution into the varibales `tb` and `yb`, respectively. Compute and plot the error `errb = ye(tb) - yb` and also compute the ∞-norm of the error and store it into the variable `maxerrb`.

(c) Set the `'MaxStep'` option of `ode45` to $\pi/100$ (one half of the period). Repeat (b) using the variables `tc`, `yc`, `errc`, and `maxerrc`. Is the error smaller?

(d) Find values of the options `'RelTol'` and `'AbsTol'` that give you as small an error as in (c) (don't set `'MaxStep'` while experimenting with the tolerances). Try to find the largest values that work. Store the tolerances and the ∞-norm of the error into the variables `reltol`, `abstol`, and `maxerrd`.

Exercise 2. Solve the Rössler equation

$$\begin{aligned} x' &= -y - z, \\ y' &= x + ay, \\ z' &= b + z(x - c), \end{aligned}$$

for $a = 0.432$, $b = 2$, and $c = 4$ on the interval $t \in [0, 400]$ and initial value $(1, 1, 1)$. Store the solution times and the solution into the variables `t2` and `y2`. Make a phase space plot of the solution with `plot3`. Also plot the solution components with `subplot`.

Exercise 3. Write the forced Duffing equation (also known as the Ueda oscillator)

$$y'' + 0.03y' + y^3 = 5.5 \sin t$$

into the standard form $y'(t) = f(t, y)$ and solve it on the interval $t \in [0, 200]$ with initial condition $(2.5, 0)$. Plot the solution in 3D phase space with t as the third component (y_1, y_2, t) and also the solution components with `subplot`.

Exercise 4. Using the function `fminbnd` find the value of x that maximizes the function $f(x) = \sin^3 x$ on the interval $(0, 2\pi)$ and also the maximum value. Store the results into the variables `xmax1` and `fmax1`.

HINT: Minimize $-f$.

Exercise 5. Using the command `fminsearch` find the value of $x = (x_1, x_2)$ that minimizes the function

$$f(x_1, x_2) = -\cos x_1 \cos x_2 \, \mathrm{e}^{-(x_1-\pi)^2-(x_2-\pi)^2}$$

and the correspondning minimum value. Use the initial guess $(0, 0)$. Store the results into the variables `xmin2` and `fmin2`. Try also the initial guesses $(1, -1)$ and $(4, 1)$.

Make surface and contour plot of the function in a neighbourhood of the minimum and plot the minimum point in them as a red circle.

Exercise 6. Minimize the function $f(x, y) = (x - 2)^2 + (y - 1)^2$ subject to the constraints

$$\begin{aligned} x^2 - y &\leq 0 \\ x + y^2 &= 2. \end{aligned}$$

Use `fmincon` with the initial guess $(0, 0)$. Use the command `deal` if you write the nonlinear constraint function as an anonymous function (since it has to return two values). Store the results into the variables `xmin3` and `fmin3`.