Wednesday, November 23, 2022 10:53 AM

Linear Regression

Squareloss $\mathcal{L}(w_0, w_1) = \frac{1}{2}\sum_{n=1}^{N}(y_n - (w_0 + w_1 x_n))^2$

$$\boxed{\frac{\partial \mathcal{L}}{\partial w_0} = \sum_{n=1}^{N}(y_n - w_0 - w_1 x_n)(-1) = 0}, \quad \boxed{\frac{\partial \mathcal{L}}{\partial w_1} = \sum_{n=1}^{N}(y_n - w_0 - w_1 x_n)(-x_n) = 0}$$

$$\sum_{n=1}^{N} y_n = w_0 N + w_1 \sum_{n=1}^{N} x_n, \quad \sum_{n=1}^{N} x_n y_n = w_0 \sum_{n=1}^{N} x_n + w_1 \sum_{n=1}^{N} x_n^2$$

$$\frac{\sum_{n=1}^{N} y_n}{N} = w_0 + w_1 \frac{\sum_{n=1}^{N} x_n}{N}, \quad \frac{\sum_{n=1}^{N} x_n y_n}{N} = w_0 \frac{\sum_{n=1}^{N} x_n}{N} + w_1 \frac{\sum_{n=1}^{N} x^2}{N}$$

$$\Rightarrow E[x] = \sum_{n=1}^{N} x_n / N, \quad E[x^2] = \sum_{n=1}^{N} x_n^2 / N$$

$$E[y] = w_0 + w_1 E[x], \quad E[xy] = w_0 E[x] + w_1 E[x^2]$$

$$w_0 = E[y] - w_1 E[x], \quad E[xy] = E[y]E[x] - w_1 (E[x])^2 + w_1 E[x^2]$$

$$\frac{E[xy] - E[y]E[x]}{E[x^2] - E[x]^2} = w_1$$

$$\frac{E[(x - E[x])(y - E[y])]}{E[(x - E[x])^2]} = w_1$$

多次元

$$f(x) = w_0 + w_1 x_1 + \cdots + w_D x_D$$
$$= w^T x$$

$$w = (w_0, w_1, \ldots, w_D)^T, \quad x = (1, x_1, \ldots, x_D)^T$$

$$\mathcal{L}(w) = \frac{1}{2}\sum_{n=1}^{N}(y_n - w^T x_n)^2$$
$$= \frac{1}{2}\|Xw - y\|_2^2$$

$$\hat{w} = \mathrm{argmin}_w \mathcal{L}(w)$$

$$\nabla_w \mathcal{L}(w) = X^T X w - X^T y = 0$$

$$w = (X^T X)^{-1} X^T y$$

$$= \underbrace{X^{\dagger}}_{\text{擬似逆行列}} y \qquad X 正則 \quad X^{\dagger} = X^{-1}$$

多項式

$$f(x) = w_0 + w_1 x + w_2 x^2$$

$$f(x) = w^T x$$

$$w = (w_0, w_1, \ldots)^T, \quad x = (x^0, x^1, x^2)^T$$

線型基底関数モデル

$$f(x) = w_0 + w_1 \phi_1(x) + \quad \cdots \quad + w_{M-1} \phi_{M-1}(x)$$
$$= w^T \phi(x)$$