ME 4041 Computer Graphics and Computer-Aided Design
Homework Assignment 1 - solution

1. Given a line $\vec{p}(t) = \vec{p}_0 + t\hat{a}$ in the parametric form, and a plane $\vec{p} \bullet \hat{n} = l$ in the implicit vector form, find the intersection point $\vec{q}$ between the line and the plane. [Hints: the intersection is on both the line and the plane.]

Answer:

point **q** is on the line: $\mathrm{q} = \mathrm{p}_0 + t\hat{\mathrm{a}}$

point **q** is also on the plane: $\mathrm{n} \bullet \mathrm{q} = l$

$$\mathrm{n} \bullet (\mathrm{p}_0 + t\hat{\mathrm{a}}) = l$$

$$\Rightarrow \quad \mathrm{n} \bullet \mathrm{p}_0 + t\mathrm{n} \bullet \hat{\mathrm{a}} = l$$

$$\Rightarrow \quad t = \frac{l - \mathrm{n} \bullet \mathrm{p}_0}{\mathrm{n} \bullet \hat{\mathrm{a}}}$$

$$\mathrm{q} = \mathrm{p}_0 + \frac{l - \mathrm{n} \bullet \mathrm{p}_0}{\mathrm{n} \bullet \hat{\mathrm{a}}}\hat{\mathrm{a}}$$

2. In Question 1, if the line passes through two points $(1,-3,-1)$ and $(6,2,1)$, and the plane passes through a point $(-1,-2,3)$ and its normal direction is $\left[-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right]^T$, what are the coordinates of the intersection between the line and the plane?

Answer:

$$\mathrm{q} = \mathrm{p}_0 + \frac{l - \mathrm{n} \bullet \mathrm{p}_0}{\mathrm{n} \bullet \hat{\mathrm{a}}}\hat{\mathrm{a}}$$

$$\boxed{q = (1/2,\ -7/2,\ -6/5)}$$

Where:

$$p_0 = (1,-3,-1), \quad p_1 = (6,2,1), \quad \hat{a} = p_1 - p_0 = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}, \quad n = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

3. Rotate a plane $x + y + z + 1 = 0$ about the z-axis by 30 degrees counterclockwise and then translate it by 1 in the x-direction. Find the new plane equation in the implicit form.

Answer:

$$R = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{TRp}$$

thus $\mathbf{p} = \mathbf{R}^{-1}\mathbf{T}^{-1}\mathbf{p}'$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' - \frac{\sqrt{3}}{2} \\ -\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' + \frac{1}{2} \\ z' \\ 1 \end{bmatrix}$$

the new plane equation is

$$\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y' - \frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' + \frac{1}{2}\right) + z' + 1 = 0$$

i.e.

$$\frac{\sqrt{3}-1}{2}x' + \frac{\sqrt{3}+1}{2}y' + z' + \frac{3-\sqrt{3}}{2} = 0$$