Q1.3: $\max_{k_i} \sum_i s_i k_i^v$

s.t. $K = \sum_i k_i$

$\mathcal{L}: \sum_i s_i k_i^v - \mu(\sum_i k_i)$

$[k_i]: s_i k_i^v + s_2 k_2^v + \cdots + s_{i-1} k_{i-1}^v + v s_i k_i^{v-1} - \mu(K - \sum_{j=1}^{i-1} k_j) + \mu = 0$

$$k_i^e = \sqrt[v-1]{\frac{\mu(K - \sum_{j=1}^{i-1} k_j) - \mu - \sum_{j=1}^{i-1} s_j k_j^v}{v s_i}}$$

$[M]: \sum_i s_i k_i^v - \sum_i k_i = 0$

$\Rightarrow \sum_i s_i k_i^v = K$

So the optimal allocation $Y^e = K = 10110$

Q1.4: Compare the optimal k^e.

The optimal allocations are $k_i^e = k_i$

Q1.5: Output gains from reallocation: $\frac{Y^e}{Y} = 1.20$.

Q1.6.3: The optimal allocation $Y^e = K = 10519.66$

Q1.6.4: Compute the optimal k^e ~~Compare~~ The optimal allocations are $k_i^e = k_i$

Q1.6.5: Output gains from reallocation: $\frac{Y^e}{Y} = 1.06$

Q2.3 The optimal allocation: $Y^e = K = 10358.96$

Q2.4 Compute the optimal k^e

Q2.5 Output gains from reallocation: $1.13 = \frac{Y^e}{Y}$

Q4.2: $Y_e = \max_i \sum s_i(a_i, k_i) k_i^v$

$K = \sum_i k_i$

$\mathcal{L}: \sum_i [\alpha a_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) k_i^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \cdot k_i^v - \mu(K - \sum k_i)$

~~$[k_i]: k_i^{-\frac{1}{\sigma}} (1-\alpha) [(1-\alpha) k_i^{\frac{\sigma-1}{\sigma}} + \alpha a_i^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} + \mu = 0$~~

~~$[\mu]: K = \sum k_i$~~

~~$\Rightarrow Y^e = \sum \frac{\mu^\sigma}{(1-\alpha)^\sigma} \cdot k_i^{v+1}$~~

$[k_i]: k_i^{-\frac{1}{\sigma}} (1-\alpha) [(1-\alpha) k_i^{\frac{\sigma-1}{\sigma}} + \alpha a_i^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} \cdot k_i + [\alpha a_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) k_i^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} + \mu = 0$ (1)

$[\mu]: K = \sum k_i$

$[K]: \mu = 0$

(1) $\rightarrow k_i^{1-\frac{1}{\sigma}} \cdot (1-\alpha) [A]^{\frac{1}{\sigma-1}} + [A]^{\frac{\sigma}{\sigma-1}} = 0 \qquad A = (1-\alpha) k_i^{\frac{\sigma-1}{\sigma}} + \alpha a_i^{\frac{\sigma-1}{\sigma}}$

$Y^e = \sum_i (\alpha - 1) k_i^{1-\frac{1}{\sigma}} [A]^{\frac{1}{\sigma-1}} \cdot k_i^v = 0.5 \sum_i k_i^v \cdot s_i = 0.5 Y = 4961.19$

Q4.3: ~~So~~ $Y^e = 0.5Y = \sum_i 0.5 k_i^v s_i$

The best allocations satisfy $\sum_i 0.5 k_i^v s_i = 0.5Y$, $k_i^e < k_i$

Q4.4: Output gains from reallocation: $\frac{Y^e}{Y} = \frac{1}{2}$

Q45.2 $\sigma = 0.5$

$$Y^e = \sum_i (\alpha-1) k_i^{1-\frac{1}{\sigma}} [A]^{\frac{1}{\sigma-1}} \cdot k_i^v$$

$$= \sum_i (\alpha-1) k_i^{1-\frac{1}{\sigma}+v} S_i^{\frac{1}{\sigma}}$$

$$= 0.5 \sum k_i^{0.5} S_i^2$$

$$= \cancel{4961.19} \quad 12198.63$$

Q46.3: $Y^e = \sum_i 0.5 k_i^{0.5} S^2$

The best allocations satisfy $Y^e = \sum_i 0.5 k_i^{0.5} S^2$, $k_i^e > k_i$

Q45.4: Output gains from reallocation: $\frac{Y^e}{Y} = 1.23$

Q45.2 $\sigma = 2$

$$Y^e = \sum_i (\alpha-1) k_i^{1-\frac{1}{\sigma}+v} S_i^{\frac{1}{\sigma}}$$

$$= 0.5 \sum_i k_i S_i^{0.5}$$

$$= 5530.76$$

Q45.3. The best allocations satisfy $Y^e = 0.5 \sum_i k_i S_i^{0.5}$ $k_i^e < k_i$

Q45.4 Output gains from reallocation: $\frac{Y^e}{Y} = 0.56$

Q46 Compare with Q1: $k_i^e = k_i$, because in Q4, k_i as a component of a_i is considered, so in Q4 $k_i^e \neq k_i$.

P3.