Homework 3

FE621 Computational Methods in Finance
due 23:55ET, Thursday March 23, 2017

Specifications. For all the problems in this assignment you need to design and use a computer program, output and present the results in nicely formatted tables and figures. The computer program may be written in any programming language you want. Please submit an archive containing a written report (pdf), where you detail your results and copy your code into an Appendix. The archive should also contain the code with comments. Any part of the problems that asks for implementation should contain a reference to the relevant code submitted.

Problem 1 (50 points). Finite difference methods.

(a) Implement the Explicit Finite Difference method to price both European Call and Put options. *Hint.* See Chapter 3 in [1].

(b) Implement the Implicit Finite Difference method to price European Call and Put options.

(c) For both the Explicit and Implicit Finite Difference schemes estimate the numbers Δt, Δx as well as the total number N_j of points on the space grid x to obtain a desired error of $\varepsilon = 0.001$. *Hint.* You need to this part in a theoretical way. Please use the convergence order as the actual error of the estimate.

(d) Consider $S_0 = 100, K = 100$, $T = 1$ year, $\sigma = 25\%, r = 6\%, \delta = 0.03$. Calculate and report the price for European Call and Put using both explicit and implicit FD methods and the number of steps that you calculated in the previous point (part c).

(e) Repeat part (c) of this problem but this time get the empirical number of iterations. Specifically, obtain the Black Scholes price for the data in (d), then do an iterative procedure to figure out the Δx, Δt, N, and N_j to obtain an accuracy of $\varepsilon = 0.001$.

(f) Using the parameters from part (d), plot on the same graph the implicit finite difference probabilities p_u, p_m, p_d as a function of σ, $\sigma \in \{0.05, 0.1, 0.15, ..., 0.6\}$. Write detailed comments on your observations.

(g) Implement the Crank-Nicolson Finite Difference method and price both European Call and Put options. Use the same parameters as in part (d) and the same number of steps in the grid. Put the results of the 3 methods (EFD, IFD, CNFD) side by side in a table and write your observations.

(h) Calculate the hedge sensitivities for the European call option using the Explicit Finite Difference method. You need to calculate Delta, Gamma, Theta, and Vega.

Problem 2 (20 points). Finite difference methods applied to market data. We will use here the algorithms implemented in Problem 1 to price European Call and Put options using market data, and compare these methods.

(a) Download Option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for an equity, for 3 different maturities (1 month, 2 months, and 3 months) and 10 strike prices. Use the same method from Homework 1 to calculate the implied volatility. Set the current short-term interest rate equal to 0.75%.

(b) Use the Explicit, Implicit, and Crank-Nicolson Finite Difference schemes implemented in Problem 1 to price European Call and Put options. Use the calculated implied volatility to obtain a space dimension Δx that insures stability and convergence with an error magnitude of no greater than 0.001.

(c) For the European style options above (puts and calls) calculate the corresponding Delta, Gamma, Theta, and Vega using the Explicit finite difference method.

(d) Create a table with the following columns: time to maturity T, strike price K, type of the option (Call or Put), ask price A, bid price B, market price $C_M = (A+B)/2$, implied volatility from the BSM model σ_{imp}, option price calculated with EFD, IFD, and CNFD. Plot on the same graph A, B, C_M, and the 3 option prices obtained with finite difference schemes, as a function of K and T. What can you observe?

Problem 3 (30 points). Explicit Finite Difference scheme for the Heston PDE. Consider the Heston PDE for the value $U(S, v, t)$ of an option, when the spot price is S, the volatility is cv and the time to expiry is t

$$\frac{\partial U}{\partial t} = \mathcal{L}U(t),$$

where the operator $\mathcal{L}$ is given in equation (10.3), see Chapter 10 in [2]. For this problem please implement the equation (10.20) in an Explicit Finite difference scheme. Use the scheme to price an European Put option. Please pay attention to the boundary conditions. Use the same parameters as in the paper [3], and choose $N_T = 1000,\ N_S = N_V = 40$. Compare the price you obtained here with the one obtained via the analytical formulas in [3].

Problem 4. Pricing Exotic Options with Finite Difference Methods. All parts are BONUS.

(a) (20 points)The Asian Options are quite popular among derivative traders and risk managers, please see Chapter 15 in [4]. It can be shown that the pricing problem for the Asian option can be formulated in terms of a single–state–variable PDE. Using synthetic parameters, i.e. chosen by you, implement the finite difference scheme detailed in Section 15.2.3 of [4].

(b) (20 points) We consider here the lookback option, please see Chapter 17 in [4] for assumptions, theoretical specifications and results. Price the lookback option using the finite difference method, see Section 17.2.2. Please note that, once the PDE (17.27) is solved, the prices are obtained through the integral representation (17.4), see also equation (17.29).

(c) (20 points) Price an American Up and Out put option using a trinomial tree, see Section 5.4 in [1]. Use synthetic parameters to test your algorithm.

References

[1] Clewlow, Les and Strickland, Chris. *Implementing Derivative Models (Wiley Series in Financial Engineering)*, John Wiley and Sons 1996.

[2] Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C*, 2013, John Wiley and Sons.

[3] Mikhailov, Sergei and Nögel, Ulrich. *Heston's stochastic volatility model: Implementation, calibration and some extensions* 2004, John Wiley and Sons.

[4] Fusai, Gianluca and Roncoroni, Andrea. *Implementing models in quantitative finance: methods and cases.* 2007 Springer Science & Business Media.