Homework 5

FE621 Computational Methods in Finance
due 23:55ET, Thursday May 5, 2017

Specifications. For all the problems in this assignment you need to design and use a computer program, output and present the results in nicely formatted tables and gures. The computer program may be written in any programming language you want. Please submit an archive containing a written report (pdf), where you detail your results and copy your code into an Appendix. The archive should also contain the code with comments. Any part of the problems that asks for implementation should contain a reference to the relevant code submitted.

Problem 1. Comparing different Monte Carlo schemes. Consider the Black-Scholes setup (geometric Brownian motion) with $r = 6\%,\ \delta = 0.03, \sigma = 20\%,\ S_0 = 100$, and assume we want to price an European option with strike $K = 100$ and maturity $T = 1$.

(a) Implement a simple Monte Carlo scheme using m simulation trials for European Call and Put options. This should be a function of n (number of time steps) and m. In all practical applications you should use at least 300 time steps and at least 1 million simulated paths. Furthermore, implement a calculation of the standard error of the estimate of the option price and a way to time the simulation routine.

(b) Implement a Monte Carlo scheme for European call and put options using the antithetic variates method (see section 4.3 of the textbook), the delta–based control variate (section 4.5 of the textbook) with $\beta_1 = -1$, and the combined antithetic variates with delta-based control variate method. Report the values obtained in four columns: Monte Carlo (MC), MC with Antithetic Variates, MC with Delta-based Control Variate, and MC with both Antithetic Variates and Delta-based Control Variate. Report the estimated option values, the corresponding standard deviations, as well as the time it takes to obtain each result. Write a paragraph comparing the results you obtained. Discuss the methods implemented.

Problem 2. Simulating the Heston model. Consider the Heston stochastic volatility model with parameters: $S_0 = 100,\ V_0 = 0.010201,\ \kappa = 6.21,\ \theta = 0.019,\ \sigma = 0.61,\ \rho = -0.7,\ r = 3.19\%$.

(a) Apply the Euler discretization schemes as presented in Table 1 of the paper [1]. Please implement all the five schemes listed there and use Monte Carlo simulations to price a call option with strike $K = 100$ and maturity $T = 1$. The exact call option price (benchmark) is in this case $C_0 = 6.8061$. Provide a table listing the estimated call option price, the bias, the root mean square error (RMSE), and the computation time in seconds. Report the results for each of the five schemes in one table.

(b) (Bonus) Recall that in Homework 1 you had to price a call option for the Heston model, using a quadrature integration method. Price the same call option as in part (a), and list the bias, the root mean square error (RMSE), and the computation time in seconds. Comment on your findings.

(c) (Bonus) Implement the FFT approach to price a standard European call option in the Heston model, please see Homework 4, problem 3. Compare, as above, your obtained value with the benchmark. Choose an appropriate $\alpha > 0$ in your implementation.

Problem 3. Generating correlated BM and pricing basket options.

(a) Given a correlation matrix A,

$$A = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.5 & 1.0 & -0.4 \\ 0.2 & -0.4 & 1.0 \end{bmatrix},$$

perform a Cholesky decomposition of the matrix A, which is a decomposition of the form $A = LL^T$, when A is symmetric and positive definite matrix. The function you construct should return the lower triangular matrix, L.

Consider three assets, starting with $S(0) = [100, 101, 98]$. The assets are assumed to follow a standard geometric Brownian motion of the form

$$dS_i(t) = \mu_i S_i(t)dt + \sigma_i S_i(t)dW_i(t).$$

We assume $\mu = [0.03, 0.06, 0.02]$, the volatility $\sigma = [0.05, 0.2, 0.15]$, and the BM's have the correlation matrix A (e.g., $d < W_1, W_2 >= a_{12}dt = 0.5dt$).

(b) We take maturity $T = 100$ days and the number of simulated paths is $m = 1000$, Consider one day sampling frequency, i.e. $\Delta t = 1/365$. Generate a $3-$dimensional matrix where each row represents a time step, each column represent a separate simulation for a specific asset and the 3-rd dimension represents different assets in the basket. Plot one realization (sample path) for this $3-$dimensional process.

(c) Basket options are options on a basket of assets. A commonly traded basket option is a vanilla call/put option on a linear combination of assets. To clarify, suppose $S_i(t), i = 1, \ldots, N$ are the prices of N stocks at time t and let $a_i, i = 1, \ldots, N$ are real constants. Set

$$U(t) = \sum_{i=1}^{N} a_i S_i(t)$$

A vanilla basket option is simply a vanilla option on $U(T)$. Specifically, on the exercise date T, the payoff of the option is $\max\{\alpha(U(T) - K), 0\}$, where K is the exercise price and $\alpha = 1$ for a call and $\alpha = -1$ for a put. Price an European call option and an European put option with $K = 100$ on the 3 asset basket given in part (b), using a Monte Carlo simulation. Consider a simple average basket $a_1 = a_2 = a_3 = 1/3$.

(d) Please price an exotic option on the basket in part (b), described using the following conditions where we use $B = 104$, and $K = 100$:

(i) If the asset 2 (S_2) hits the barrier $B < S_2(t)$ for some t then the payoff of the option is equal to an European Call option written on the asset 2;

(ii) If $\max_{t \in [0,T]} S_2(t) > \max_{t \in [0,T]} S_3(t)$, then the payoff of the option is $(S_2^2(T) - K)_+$;

(iii) Take $A_i(0,T) := \sum_{t=1}^{T} S_i(t)$, the average of the daily values for stock i. If $A_2(0,T) > A_3(0,T)$, then the payoff is $(A_2(0,T) - K)_+$;

(iv) otherwise, the option is a vanilla call option on the basket, similar to part (c) of this problem.

References

[1] Lord, Roger, Remmert Koekkoek, and Dick Van Dijk. *A comparison of biased simulation schemes for stochastic volatility models.* Quantitative Finance 10.2 (2010): 177-194.