Question 2

(a) $p(x=\text{apple}) = 0.2 \times \frac{3}{10} + 0.6 \times \frac{3}{10} + \frac{1}{2} \times 0.2 = 0.34$

(b) $p(x=\text{orange}) = 0.2 \times \frac{4}{10} + 0.6 \times \frac{3}{10} + 0.2 \times \frac{1}{2} = 0.36$

$p = \frac{0.6 \times \frac{3}{10}}{0.36} = \frac{1}{2}$

Question 3

(a) (1) $j \in \{1, \cdots, N\}$: $E[L] = \sum_{k=1}^{N} L_{kj} \, p(C_k | x)$

$$= l_s \cdot \sum_{k \neq j} p(C_k | x) + 0 \cdot p(C_j | x)$$

$$= l_s \cdot (1 - p(C_j | x))$$

To make $E[L]$ smallest, we wish $p(C_j|x)$ is biggest

So we want $p(C_j|x) \geqslant p(C_k|x)$ for any k, $x \to C_j$

(2) $j = N+1$: $E[L] = \sum_{k=1}^{N} L_{kj} \, p(C_k | x)$

$$= l_r \cdot \sum_{k=1}^{N} p(C_k | x)$$

$$= l_r$$

we get an fixed value l_r for $j = N+1$

To make $E[L]$ smallest, we wish $l_s \cdot (1 - p(C_j|x)) \leq l_r$

so the fixed value l_r is the upper bound

$$1 - p(C_j|x) \leq \frac{l_r}{l_s}$$

$$p(C_j|x) \geqslant 1 - \frac{l_r}{l_s}$$

This is the claim

(b) if $l_r = 0$

we have $classify(x) :\to \begin{cases} C_j, & \text{iff } \forall k: p(C_j|x) \geqslant p(C_k|x) \\ & \wedge\, p(C_j|x) = 1 \\ C_{rej}, & \text{otherwise} \end{cases}$

so unless $p(C_j|x) = 1$, we always choose C_{rej}

(c) if $l_r > l_s$, then $p(C_j|x)$ always $\geqslant 1 - \frac{l_r}{l_s}$

then we always get $classify(x) :\to C_j$

never C_{rej}

Question 4

$$p(x|\theta) = \theta^2 x \exp(-\theta x)\, g(x)$$

$$x_1, \dots, x_N > 0 \qquad g(x) = 1$$

$$p(x|\theta) = \theta^2 x \exp(-\theta x)$$

$$L(\theta) = \prod_{n=1}^{N} p(x_n|\theta)$$

$$E(\theta) = -\sum_{n=1}^{N} \ln p(x_n|\theta) = -\sum_{n=1}^{N} \ln\left(\theta^2 x_n \exp(-\theta x_n)\right)$$

$$\frac{\partial E(\theta)}{\partial \theta} = -\sum_{n=1}^{N} \frac{2\theta x_n \exp(-\theta x_n) + \theta^2 x_n \exp(-\theta x_n)(-x_n)}{\theta^2 x_n \exp(-\theta x_n)}$$

$$= -\sum_{n=1}^{N} \frac{2 - x_n \theta}{\theta} \overset{!}{=} 0$$

$$\Longleftrightarrow \quad 2N = \left(\sum_{n=1}^{N} x_n\right) \cdot \tilde{\theta}$$

$$\Longleftrightarrow \quad \tilde{\theta} = \frac{2N}{\sum_{n=1}^{N} x_n}$$