Assignment 7

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$\sigma = 0{,}8$. $n = 30$. $\alpha = 0{,}05$. $\mu_0 = 16$

a) Overfilling or underfilling requires the operator to shut down the line for readjustment

We want to challenge if $\mu = 16$ -> $H_0: \mu = 16$
$H_A: \mu \neq 16$

b) $\bar{X} = 16.32$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{16.32 - 16}{0.8 / \sqrt{30}} = 2.19$$

P-value = $P(z \leq -2{,}19) + P(z \geq 2{,}19) = 0{,}0110 + (1 - 0{,}9857) = 0{,}0253$

$0{,}0253 < 0{,}05$ -> we reject H_0

I would recommend to change the filling method.

c) $\bar{X} = 15.82$

$$z = \frac{x - \mu_0}{\sigma / \sqrt{n}} = \frac{15.82 - 16}{0.8 / \sqrt{30}} = -1{,}23$$

P-value = $P(z \leq -1{,}23) + P(z \geq 1{,}23) = 0{,}1093 + (1 - 0{,}8907) = 0{,}2186$

$0{,}2186 > 0{,}05$ -> we do not reject H_0

With a sample mean of 15.82, I cannot recommend to change the filling method

d) $\alpha = 0{,}05$, $z_{\alpha/2} = z_{0.025} = -1{,}96$ v $1{,}96$

Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

a) $z = 2{,}19$, $z > 1{,}96$ -> we reject H_0
b) $z = -1{,}23$, $z > -1{,}96$ -> we do not reject H_0

We reach the same conclusion with the critical value approcah

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$\mu_0 = 4$, $n = 60$, $\bar{x} = 4,5$

a) we want to challenge if $\mu > 4$ -> $H_0: \mu \leq 4$, $H_A: \mu > 4$

b) $\sigma = 1,5$

$$z = \frac{4,5 - 4}{1,5/\sqrt{60}} = 2,58$$

p-value = $P(z \geq 2,58) = 1 - P(z \leq 2,58) = 1 - 0,9951 = 0,0049$

c) $\alpha = 0,01$ -> Reject H_0 if p-value $\leq \alpha$ -> we reject H_0

we conclude that the children from low-income families are exposed more than 4 hours of daily background television. (with a significance level of 0,01)

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$\mu_0 = 493$, $n = 50$

a) we want to test the claim that the mean weekly pay for workers who have received a high school diploma is significantly greater than the mean weekly pay for workers who have not received a high school diploma.

$H_0: \mu \leq 493$

$H_A: \mu > 493$

b) we calculate the sample mean and the sample standard deviation in Python and get: $\bar{x} = 778$ and $s = 1045,13$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{778 - 493}{1045,13/\sqrt{50}} = 1,93 \quad , \quad df = 49$$

p-value = $P(t \geq 1,93)$ = 0,025 to 0,05

c) $\alpha = 0,05$, $0,025 < \text{p-value} < 0,05$ -> we reject H_0

we conclude that workers with a high school diploma has a higher mean weekly pay than workers without a high school diploma. (with a significance level of 0,05)

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$\mu_0 = 23$, $n = 8783$, $\alpha = 0.01$

we want to test the claim that people spend less time channel surfing during December.

$H_0: \mu \geq 23$
$H_A: \mu < 23$

we calculate the sample mean and the sample standard deviation in Python and get: $\bar{x} = 22,81$ and $s = 6,44$

$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{22,81 - 23}{6,44 / \sqrt{8783}} = -2,76$, $df = 8782$

Critical value: $-t_\alpha = -t_{0,01} = -2,327$

Reject H_0 if $t \leq -t_\alpha$ → we reject H_0

we conclude that people don't spend less time channel surfing in december. We have used $\alpha = 0,01$, but the result would be the same if we had used $\alpha = 0,05$ or $\alpha = 0,1$ instead. In order to not reject H_0, we would need a significance level of 0,001.

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$p_0 = 0,02$, $n = 10\,531$, $\alpha = 0,5$

we want to test if the proportion of travelers waiting more than 20 minutes in TSA security lines at the major U.S. Airport is different than the proportion of travelers waiting more than 20 minutes in TSA security lines across the United states.

H_0: $p = 0,02$
H_A: $p \neq 0,02$

we calculate the sample proportion in python and get $\bar{p} = 0,0186$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0,0186 - 0,02}{\sqrt{0,0196/10531}} = -1,03$$

p-value : $P(z \leq -1,03) + P(z \geq 1,03) = 2 \cdot 0,1524 = 0,3048$

$\alpha = 0,05$, p-value : $0,3048 > \alpha = 0,05$ -> we do not reject H_0

we conclude that we cannot say that the proportions are different