Assignment 8

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	System A	System B
Sample size	$n_1 = 120$	$n_2 = 100$
Sample mean	$\bar{x}_1 = 4{,}1$	$\bar{x}_2 = 3{,}4$
Population std dev.	$\sigma_1 = 2{,}2$	$\sigma_2 = 1{,}5$

$H_0: \mu_1 - \mu_2 = 0$, $\alpha = 0{,}05$

$H_A: \mu_1 - \mu_2 \neq 0$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(\sigma_{\bar{x}_1})^2}{n_1} + \frac{(\sigma_{\bar{x}_2})^2}{n_2}} = \sqrt{\frac{(2{,}2)^2}{120} + \frac{(1{,}5)^2}{100}} = 0{,}25$$

$$\bar{x}_1 - \bar{x}_2 = 4{,}1 - 3{,}4 = 0{,}7$$

$$Z = \frac{(\bar{x} - \bar{x}_2) - D_0}{\sqrt{\frac{(\sigma_{\bar{x}_1})^2}{n_1} + \frac{(\sigma_{\bar{x}_2})^2}{n_2}}} = \frac{0{,}7 - 0}{0{,}25} = 2{,}8$$

P-value $= P(z \geq 2{,}8) + P(z \leq -2{,}8) = 0{,}0026 + (1 - 0{,}9974) = 0{,}0052$

p-value $< 0{,}05$ -> we reject H_0

We reject that the two systems do not differ, and can conclude that system B is preferred.

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a)

	2015	2016
Sample size	$n_1 = 33$	$n_2 = 46$
Sample mean	$\bar{x}_1 = 653$	$\bar{x}_2 = 540$
Sample std dev.	$s_1 = 101$	$s_2 = 93,5$

Point estimate: $\bar{x}_1 - \bar{x}_2 = 653 - 540 = 113$

The point estimate indicates that monthly lease payments have declined from 2015 to 2016.

b) 99% confidence interval

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{101^2}{33} + \frac{93.5^2}{46}\right)^2}{\frac{1}{33-1}\left(\frac{101^2}{33}\right)^2 + \frac{1}{46-1}\left(\frac{93{,}5^2}{46}\right)^2} \approx 65{,}77 \approx 65$$

every source says round down

$\alpha = 1 - 0{,}99 = 0{,}01 \rightarrow \alpha/2 = 0.005 \rightarrow t_{\alpha/2} = 2{,}654$

Margin of error: $t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2{,}654\sqrt{\frac{101^2}{33} + \frac{93{,}5^2}{46}} = 59{,}29$

99% confidence interval: $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 113 \pm 59{,}29$

$= [53{,}71,\ 172{,}29]$

We are 99% confident that $53{,}71 \leq \mu_1 - \mu_2 \leq 172{,}29$

c) Since we are 99% confident that $53{,}71 \leq \mu_1 - \mu_2 \leq 172{,}29$, we can conclude that monthly lease payments have declined by about 53 to 172 dollars from 2015 to 2016.

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a)

Kitchen	Master Bedroom
25.2	18.0
17.4	22.9
22.8	26.4
21.9	24.8
19.7	26.9
23.0	17.8
19.7	24.6
16.9	21.0
21.8	
23.6	

$\rightarrow$ Calculations in Excel

	Kitchen	Master bedroom
Sample size	$n_1 = 10$	$n_2 = 8$
Sample mean	$\bar{x}_1 = 21{,}24$	$\bar{x}_2 = 22{,}8$
Sample std dev.	$S_1 = 2{,}77$	$S_2 = 3{,}55$

Point estimate: $\bar{X}_1 - \bar{X}_2 = 21{,}24 - 22{,}8 = -1{,}56$

b) 90% confidence interval:

$$df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2} = \frac{\left(\frac{2{,}77^2}{10} + \frac{3{,}55^2}{8}\right)^2}{\frac{1}{10-1}\left(\frac{2{,}77^2}{10}\right)^2 + \frac{1}{8-1}\left(\frac{3{,}65^2}{8}\right)^2} \approx 13{,}07 \approx 13$$

$\alpha = 1 - 0{,}90 = 0{,}10 \rightarrow \alpha/2 = 0.050 \rightarrow t_{\alpha/2} = 1{,}771$

Margin of error: $t_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = 1{,}771\sqrt{\frac{2{,}77^2}{10} + \frac{3{,}55^2}{8}} = 2{,}71$

90% confidence interval: $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = -1{,}56 \pm 2{,}71$

$= [-4{,}27 \ , \ 1{,}15]$

We are 90% confident that $-4{,}27 \leq \mu_1 - \mu_2 \leq 1{,}15$. It is therefore dificult to conclude that one remodeling project is cheaper than another.

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a) using python to estimate the proportions:

$\bar{p}_1 = 0{,}726$
$\bar{p}_2 = 0{,}545$

b) $1-\alpha = 0{,}95 \rightarrow \alpha = 0.05 \rightarrow \alpha/2 = 0.025 \rightarrow 1-\alpha/2 = 1-0.025 = 0.975$

$z_{\alpha/2} = 1{,}96$

95% confidence interval $= \bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

$$= 0{,}726 - 0{,}545 \pm 1{,}96\sqrt{\frac{0{,}726(1-0726)}{150} + \frac{0{,}545(1-0{,}545)}{200}}$$

$$= 0{,}18 \pm 0{,}098 = [0{,}083 , 0{,}279]$$

c) we can see from the 95% confidence interval that we are 95% confident that $\bar{p}_1 > \bar{p}_2$, and we can therefore conclude that $\bar{p}_1$ exceeds $\bar{p}_2$.

we can show the same result with hypothesis testing:

$H_0: \bar{p}_1 - \bar{p}_2 \leq 0$
$H_A: \bar{p}_1 - \bar{p}_2 > 0$

$$\bar{p} = \frac{n_1 \cdot \bar{p}_1 + n_2 \cdot \bar{p}_2}{n_1 + n_2} = \frac{150 \cdot 0{,}726 + 200 \cdot 0{,}545}{150 + 200} = 0{,}623$$

$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{0{,}623(1-0{,}623)\left(\frac{1}{150}+\frac{1}{200}\right)} = 0{,}0523$$

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{s_{\bar{p}_1 - \bar{p}_2}} = \frac{0{,}726 - 0{,}545}{0{,}0523} = 3{,}46$$

$z_{\alpha/2} = z_{0{,}025} = 1{,}96$, $z > z_{0.025} \rightarrow$ we reject H_0

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a) $\bar{p}_1 = 0{,}276$ $\bar{p}_2 = 0{,}487$ $n = 240$

$1-\alpha = 0{,}95 \rightarrow \alpha = 0.05 \rightarrow \alpha/2 = 0.025 \rightarrow 1-\alpha/2 = 1-0.025 = 0.975$

$z_{\alpha/2} = 1{,}96$

95% confidence interval $= \bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\dfrac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \dfrac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

$= 0{,}276 - 0{,}487 \pm 1{,}96\sqrt{\dfrac{0{,}276(1-0{,}276)}{240} + \dfrac{0{,}487(1-0{,}487)}{240}}$

$= -0{,}21 \pm 0{,}089 = [-0{,}300,\ -0{,}122]$

b) $\bar{p}_1 = 0{,}276$ $\bar{p}_2 = 0{,}397$ $n = 240$

Rejection of Ho will allow us to conclude that the most recent bullish sentiment is weaker than one month ago

If $\bar{p}_1 - \bar{p}_2 < 0 \rightarrow \bar{p}_1$ is weaker than $\bar{p}_2$

$H_0: \bar{p}_1 - \bar{p}_2 \geq 0$
$H_A: \bar{p}_1 - \bar{p}_2 < 0$

c) $\bar{p} = \dfrac{n_1 \cdot \bar{p}_1 + n_2 \cdot \bar{p}_2}{n_1 + n_2} = \dfrac{240 \cdot 0{,}276 + 240 \cdot 0{,}397}{240 + 240} = 0{,}337$

$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0{,}337(1-0{,}337)\left(\frac{1}{240} + \frac{1}{240}\right)} = 0{,}0432$

$z = \dfrac{(\bar{p}_1 - \bar{p}_2)}{s_{\bar{p}_1 - \bar{p}_2}} = \dfrac{0{,}276 - 0{,}397}{0{,}0432} = -2{,}80$

$z_{\alpha/2} = z_{0{,}005} = 2{,}58$, $z < -z_{\alpha/2} \rightarrow$ we reject Ho