Assignment 3

59)

a) $P(X=0) = 0{,}096$ $P(X=1) = 0{,}570$ $P(X=2) = 0{,}238$
$P(X=3) = 0{,}077$ $P(X=4) = 0{,}019$

X	P(x)
0	.096
1	.570
2	.238
3	.077
4	.019
	1.000

b) $E(X) = 0 \cdot 0{,}096 + 1 \cdot 0{,}570 + 2 \cdot 0{,}238 + 3 \cdot 0{,}077 + 4 \cdot 0{,}019 = 1{,}353$

The expected value is 1,353, which means that the expected wind condition during a boating accident is something between light and moderate

c) $Var(x) = (0 - 1{,}353)^2 \cdot 0{,}096 + (1 - 1{,}353)^2 \cdot 0{,}570 + (2 - 1{,}353)^2 \cdot 0{,}238 + (3 - 1{,}353)^2 \cdot 0{,}077 + (4 - 1{,}353)^2 \cdot 0{,}019 = 0{,}688$

$\sigma = \sqrt{0{,}688} = 0{,}83$

The standard deviation is 0,83

d) We can see that a boating accident is expected to happen under light to moderate wind conditions, and the standard deviation indicates that most accidents happen during none to moderate conditions. A possible explanation is that most people don't use their boats during storms.
Also, if most people use their boat during light wind conditions, most accidents will happen during this condition because not all accidents are related to wind.

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a)

	Reading material (Y)			
Snacks (X)	0	1	2	Total
0	0,000	0,100	0,030	0,130
1	0,400	0,150	0,050	0,600
2	0,200	0,050	0,020	0,270
total	0,600	0,300	0,100	1,000

$P(X=2, Y=1) = 0{,}050$

$P(X=1) = 0{,}600$

$P(X=0, Y=0) = 0$ because the sales terminal cannot collect data when a customer doesn't buy anything.

b)

X	P(X)
0	0,130
1	0,600
2	0,270
	1,000

marginal probability distribution for snacks

$E(X) = 0 \cdot 0{,}130 + 1 \cdot 0{,}600 + 2 \cdot 0{,}270 = 1{,}14$

$Var(X) = (0-1{,}14)^2 \cdot 0{,}130 + (1-1{,}14)^2 \cdot 0{,}600 + (2-1{,}14)^2 \cdot 0{,}270 = 0{,}38$

$\sigma(X) = \sqrt{0{,}38} = 0{,}617$

c)

$E(Y) = 0 \cdot 0{,}600 + 1 \cdot 0{,}300 + 2 \cdot 0{,}100 = 0{,}5$

$Var(Y) = (0-0{,}5)^2 \cdot 0{,}600 + (1-0{,}5)^2 \cdot 0{,}300 + (2-0{,}5)^2 \cdot 0{,}100 = 0{,}45$

$\sigma(Y) = \sqrt{0{,}45} = 0{,}67$

d) $t = x + y$

t	$P(t)$	
0	0.000	
1	0,500	(0,400 + 0,100)
2	0,380	(0,200 + 0,150 + 0,030)
3	0,100	(0,050 + 0,050)
4	0,020	
	1,000	

$E(t) = 0 \cdot 0{,}000 + 1 \cdot 0{,}500 + 2 \cdot 0{,}380 + 3 \cdot 0{,}100 + 4 \cdot 0{,}020 = 1{,}64$

$Var(t) = (0 - 1{,}64)^2 \cdot 0{,}000 + (1 - 1{,}64)^2 \cdot 0{,}500 + (2 - 1{,}64)^2 \cdot 0{,}380 + (3 - 1{,}64)^2 \cdot 0{,}100$
$+ (4 - 1{,}64)^2 \cdot 0{,}020 = 0{,}55$

$\sigma(t) = \sqrt{0{,}55} = 0{,}74$

e) Covariance:

$$\sigma_{xy} = \frac{Var(t) - Var(x) - Var(y)}{2} = \frac{0{,}55 - 0{,}38 - 0{,}45}{2} = -0{,}14$$

We can see that x and y have a negative relationship. We calculate the correlation between x and y to get a better sense of the strength of the relationship.

Correlation:

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{0{,}74}{0{,}617 \cdot 0{,}67} = 1{,}79$$

We can see that the negative relationship is strong.

64)

a) $n=20$, $x=3$, $p=0{,}53$

$$P(X=3)=\binom{20}{3}\cdot 0{,}53^{3}\cdot(1-0{,}53)^{(20-3)}=\boxed{0{,}00045}$$

b) $n=20$, $X\leq 5$, $p=0{,}28$

$$P(X\leq 5)=P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$
$$=\binom{20}{0}\cdot 0{,}28^{0}\cdot(1-0{,}28)^{20}+\binom{20}{1}\cdot 0{,}28^{1}\cdot(1-0{,}28)^{19}$$
$$+\binom{20}{2}\cdot 0{,}28^{2}\quad(1-0{,}28)^{18}+\binom{20}{3}\cdot 0{,}28^{3}\cdot(1-0{,}28)^{17}$$
$$+\binom{20}{4}\cdot 0{,}28^{4}\cdot(1-0{,}28)^{16}+\binom{20}{5}\cdot 0{,}28^{5}\cdot(1-0{,}28)^{15}$$
$$=\boxed{0{,}495}$$

c) $n=2000$, $p=0{,}49$

$$E(X)=n\cdot p=2000\cdot 0{,}49=\boxed{980}$$

d) $n=2000$, $p=0{,}36$

$$E(x)=n\cdot p=2000\cdot 0{,}36=\boxed{720}$$

$$Var(x)=n\cdot p\cdot(1-p)=2000\cdot 0{,}36\cdot(1-0{,}36)=\boxed{460{,}80}$$

$$\sigma=\sqrt{var(x)}=\sqrt{460{,}80}=\boxed{21{,}47}$$

66)

a) $p = 0{,}01 \qquad n = 5 \qquad x = 0$

$$P(X=0) = \binom{5}{0} \cdot 0{,}01^0 \cdot 0{,}99^5 = 0{,}95$$

b) $p = 0{,}01 \qquad n = 5 \qquad x = 1$

$$P(X=1) = \binom{5}{1} \cdot 0{,}01^1 \cdot 0{,}99^4 = 0{,}048$$

c) $P(X \geq 1) = 1 - P(X=0) = 0{,}05$

d) We can see that the probability that one component is found to be defective is nearly 5%. This exceeds the 1% limit, and I would not feel comfortable accepting the shipment.

68)

$p = 0{,}235 \qquad n = 200$

X = the number of respondents who list price as the most important factor in selecting a home builder.

a) $E(X) = n \cdot p = 200 \cdot 0{,}235 = 47$

b) $\sigma = \sqrt{\sigma^2} = \sqrt{n \cdot p\,(p-1)} = \sqrt{47 \cdot (1-0{,}235)} = 6$

c) Y = the number of respondents who do not list price as the most important factor in selecting a home builder.

$E(Y) = 200 \cdot 0{,}765 = 153$

$\sigma = \sqrt{\sigma^2} = \sqrt{153 \cdot 0{,}235} = 6$