Assignment 5

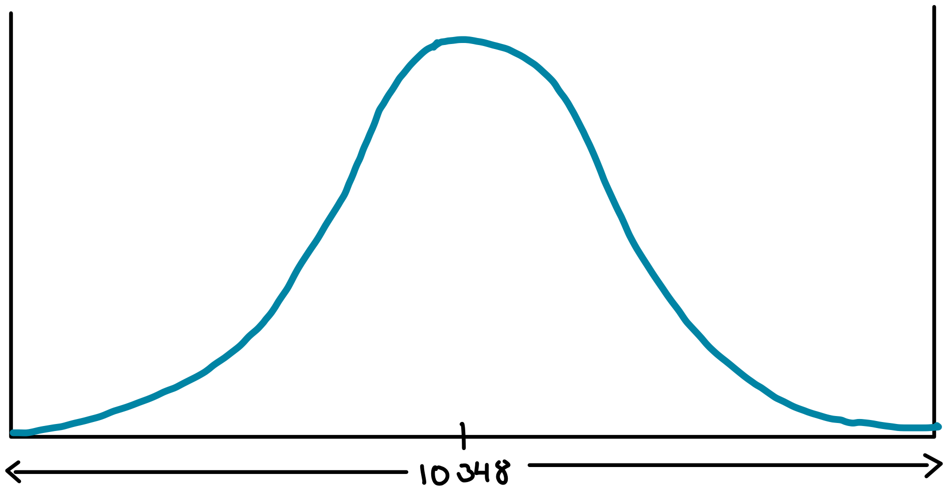
51

$\mu: 10\,348$, $\sigma: 2500$, $n = 100$

a) Since $n = 100$ -> the sampling distribution is approximately normal distribution with $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

$\mu_{\bar{x}} = \mu = 10348$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2500/10 = 250$



b) $P(10148 \leq \bar{x} \leq 10548) = P(\bar{x} \leq 10548) - P(\bar{x} \leq 10148)$

$P(\bar{x} \leq 10548)$ -> $z = \frac{200}{250} = 0{,}8$, $P(Z \leq 0{,}8) = 0{,}7881$

$P(\bar{x} \leq 10148)$ -> $z = \frac{-200}{250} = -0{,}8$, $P(Z \leq -0{,}8) = 0{,}2119$

$P(10148 \leq \bar{x} \leq 10548) = 0{,}7881 - 0{,}2119 = 0{,}5762$

c) $P(\bar{x} > 12000) = 1 - P(\bar{x} \leq 12000)$

$z = \frac{12000 - 10348}{250} = 6.61$, $P(Z \leq 6.61) \rightarrow 1$

$P(\bar{x} > 12000) \rightarrow 0$. It is highly unlikely that the sample mean is greater than 12000. I would question the sampling procedures

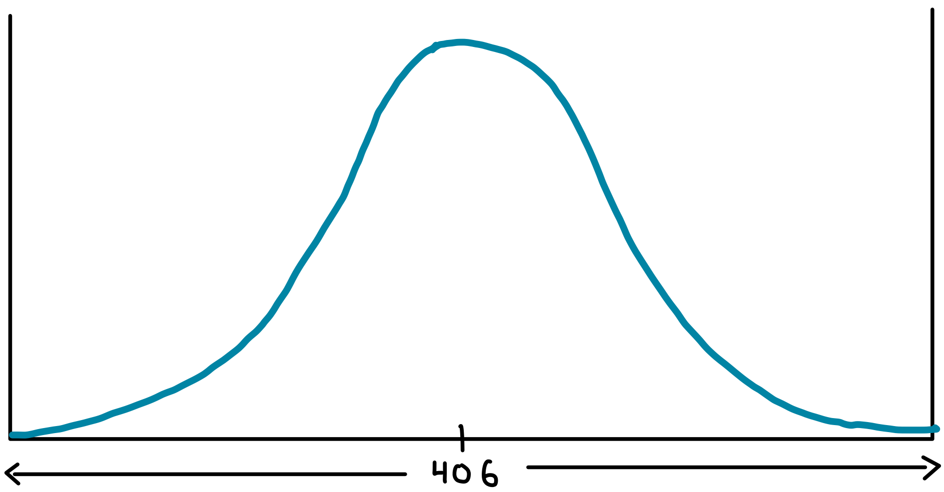
52

$\mu = 406$, $\sigma = 80$, $N = 3400$, $n = 64$

a) $n = 64 \rightarrow$ the sampling distribution is approximately normal distribution with $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

$\mu_{\bar{x}} = \mu = 406$

$\sigma_{\bar{x}} = \sigma / n = 80/\sqrt{64} = 10$



b) $P(391 \leq \bar{x} \leq 421) = P(\bar{x} \leq 421) - P(\bar{x} \leq 391)$

$P(\bar{x} \leq 421) = \quad z = \frac{15}{10} = 1{,}5$, $P(z \leq 1{,}5) = 0{,}9332$
$P(\bar{x} \leq 391) = \quad z = \frac{-15}{10} = -1{,}5$, $P(z \leq -1{,}5) = 0{,}0668$

$P(391 \leq \bar{x} \leq 421) = 0{,}9332 - 0{,}0668 = 0{,}8664$

c) $P(\bar{x} \geqslant 380)$

$z = \frac{380 - 406}{10} = -2{,}6$, $P(z \leq -2{,}6) = 0{,}047$

The probability of finding a sample mean of 380 or less is 4,7%.
This sample is an unusually low performing group of stores.

54

$\mu = 27\,175, \quad \sigma = 7400, \quad n = 60$

a) $\sigma_{\bar{x}} = 7400/\sqrt{60} = 955,34$

b) $P(\bar{x} > 27\,175) = 1 - P(\bar{x} \leq 27\,175)$

$z = 0 \rightarrow P(Z \leq 0) = 0,5$

$P(\bar{x} > 27\,175) = 0,5$

c) $P(26\,175 \leq \bar{x} \leq 28\,175) = P(\bar{x} \leq 28\,175) - P(\bar{x} \leq 26\,175)$

$z_1 = \frac{1000}{955,34} = 1,05 \quad , \quad P(Z \leq 1,05) = 0,8531$
$z_2 = \frac{-1000}{955,34} = -1,05 \quad , \quad P(Z \leq -1,05) = 0,1469$

$P(26\,175 \leq \bar{x} \leq 28\,175) = 0,8531 - 0,1469 = 0,7062$

d) $n = 100 \rightarrow \sigma_{\bar{x}} = 7400/\sqrt{100} = 740$

$z_1 = \frac{1000}{740} = 1,35 \quad , \quad P(Z \leq 1,35) = 0,9115$
$z_2 = \frac{-1000}{740} = -1,35 \quad , \quad P(Z \leq -1,35) = 0,0885$

$P(26\,175 \leq \bar{x} \leq 28\,175) = 0,9115 - 0,0885 = 0,8230$

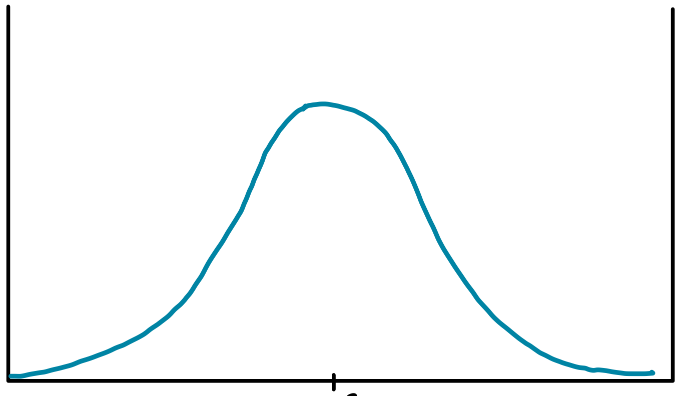
58

$p = 0{,}15$, $n = 240$, $np = 36$

a) The sampling distribution of $\bar{p}$ is approximately normal distribution with $\mu_{\bar{p}}$ and $\sigma_{\bar{p}}$

$$\mu_{\bar{p}} = p = 0{,}15$$

$$\sigma_{\bar{p}} = \sqrt{\frac{0.15(1-0.15)}{240}} = 0{,}0230$$



b) $P(0{,}11 \le \bar{p} \le 0{,}19) = P(\bar{p} \le 0{,}19) - P(\bar{p} \le 0{,}11)$

$z_1 = \frac{0{,}04}{0{,}023} = 1{,}74$, $P(z_1 \le 1{,}74) = 0{,}9582$

$z_2 = \frac{-0{,}04}{0{,}023} = -1{,}74$, $P(z_2 \le -1{,}74) = 0{,}0409$

$P(0{,}11 \le \bar{p} \le 0{,}19) = 0{,}9582 - 0{,}0409 = 0{,}9173$

c) $P(0{,}13 \le \bar{p} \le 0{,}17) = P(\bar{p} \le 0{,}17) - P(\bar{p} \le 0{,}13)$

$z_1 = \frac{0{,}02}{0{,}023} = 0{,}87$, $P(z_1 \le 0{,}87) = 0{,}8078$

$z_2 = \frac{-0{,}02}{0{,}023} = -0{,}87$, $P(z_2 \le -0{,}87) = 0{,}1922$

$P(0{,}13 \le \bar{p} \le 0{,}17) = 0{,}8078 - 0{,}1922 = 0{,}6156$

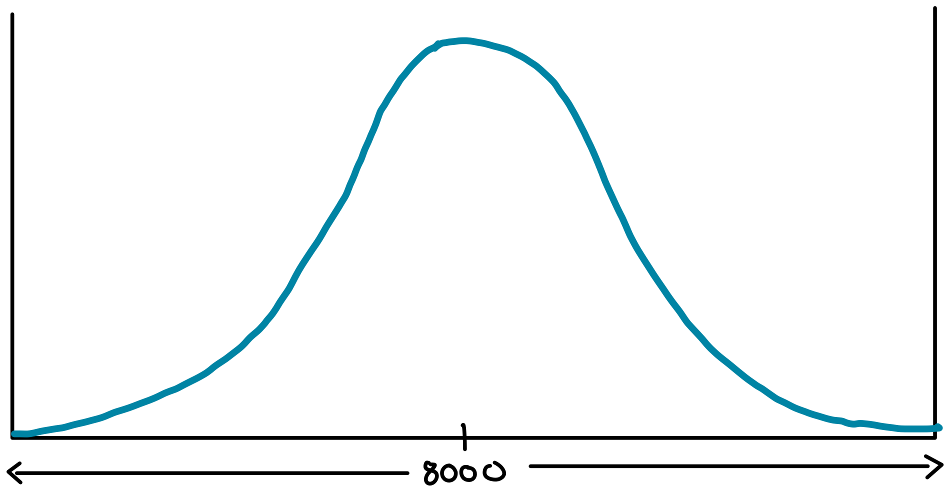
63

$\mu = 8000$, $\sigma = 480$, $n = 35\,000$

a) $n = 35000 \rightarrow$ the sampling distribution is approximately normal distribution with $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

$$\mu_{\bar{x}} = \mu = 8000$$

$$\sigma_{\bar{x}} = 480/\sqrt{35000} = 2{,}5657$$



b) $P(7996 \leq \bar{x} \leq 8004) = P(\bar{x} \leq 8004) - P(\bar{x} \leq 7996)$

$z_1 = \frac{4}{2{,}5657} = 1{,}56$, $P(Z \leq 1{,}56) = 0{,}9406$

$z_2 = \frac{-4}{2{,}5657} = -1{,}56$, $P(Z \leq -1{,}56) = 0{,}0594$

$P(7996 \leq \bar{x} \leq 8004) = 0{,}9406 - 0{,}0594 = 0{,}8812$

c) $P(7999 \leq \bar{x} \leq 8001) = P(\bar{x} \leq 8001) - P(\bar{x} \leq 7999)$

$z_1 = \frac{1}{2{,}5657} = 0{,}39$, $P(Z \leq 0{,}39) = 0{,}6517$

$z_2 = \frac{-1}{2{,}5657} = -0{,}39$, $P(Z \leq -0{,}39) = 0{,}3483$

$P(7999 \leq \bar{x} \leq 8001) = 0{,}6517 - 0{,}3483 = 0{,}3034$

d) The probability that the mean life of a sample differs from the population mean life by more than four hours is $1 - 0{,}8812 = 0{,}1188$, which is unlikely, but possible. This means that half of the 11,88 % has a lower mean life, while the other half has a longer mean life.

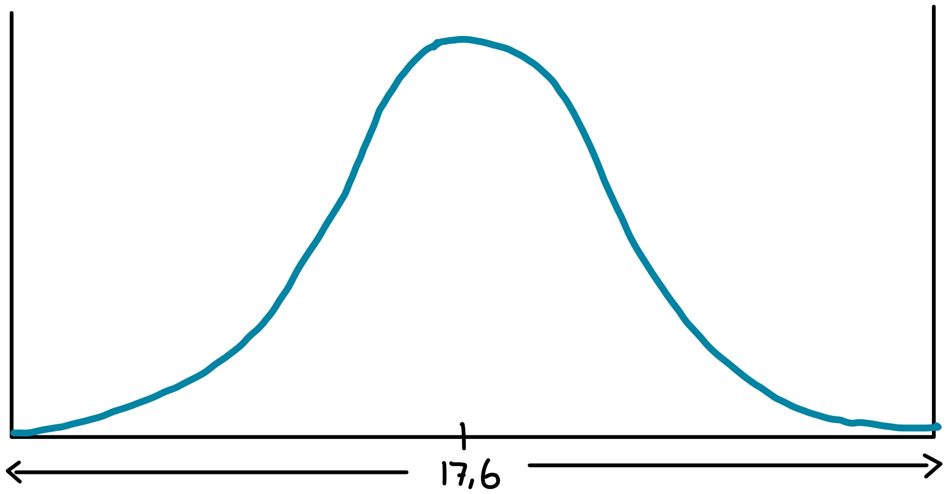
64

$\mu = 17{,}6$, $\sigma = 5{,}1$, $n = 85\,020$

a) $n = 85020 \rightarrow$ the sampling distribution is approximately normal distribution with $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

$\mu_{\bar{x}} = \mu = 17{,}6$

$\sigma_{\bar{x}} = 5{,}1 / \sqrt{85020} = 0{,}0175$



b) 3 minutes : 0,05 hours

$P(85019{,}95 \leq \bar{x} \leq 85020{,}05) = P(\bar{x} \leq 85020{,}50) - P(\bar{x} \leq 85019{,}95)$

$z_1 = \frac{0{,}05}{0{,}0175} = 2{,}86$, $P(z \leq 2{,}86) = 0{,}9979$

$z_2 = \frac{-0{,}05}{0{,}0175} = -2{,}86$, $P(z \leq -2{,}86) = 0{,}0021$

$P(85019{,}95 \leq \bar{x} \leq 85020{,}05) = 0{,}9979 - 0{,}0021 = 0{,}9958$

c) It is highly unlikely that the mean time spent on the Internet by the sample of 85020 Floridians differs from the U.S. population mean by more than three minutes. In that case, all of Florida would have to spend considerable less time on the Internet than the rest of America (or more)