Exercise Session 1 IaCV

Vector product:

$$x = l \times l' = \begin{vmatrix} l_1 & l_2 & l_3 \\ l_1' & l_2' & l_3' \end{vmatrix} = (l_2 l_3' - l_2' l_3,\ l_1' l_3 - l_1 l_3,$$
$$l_1 l_2' - l_1' l_2)$$

1.) a.)

$$l^T x = 0,\ l'^T x = 0$$

$$(l_1 l_2 l_3) x = ?$$

b.) $l = x \times x',\ l^T x = 0 = (x^T \times x) x = 0$

2.) a.)

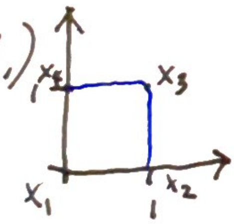
$$y = Ax \Rightarrow x = A^{-1} y$$

$$l^T x = 0 \Rightarrow l^T (A^{-1} y) = \hat{l}^T y,\ \hat{l} = A^{-T} l$$

$$(\hat{l})^T = (A^T l)^T = l^T A^{-1}$$

b.) $x^T C x = 0 \Rightarrow y^T \underbrace{A^{-T} C A^{-1}}_{C' \text{ - pos def}} y = y^T C' y$

3.)



i.)

$$A = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

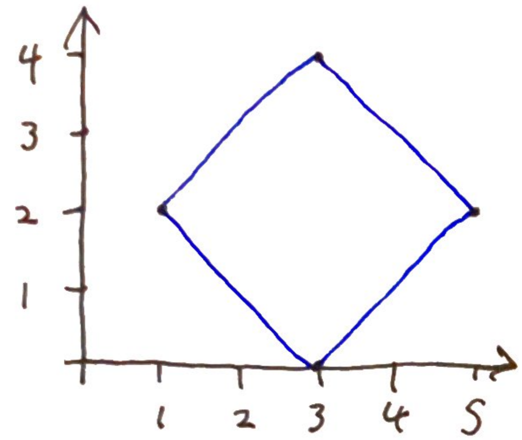
$$x_1' = A x_1 = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix} (0\ \ 0\ \ 1)^T = (1\ \ 2\ \ 1)^T \rightarrow (1, 2)$$

$$x_2' = A x_2 = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \rightarrow (3, 0)$$

3.) i.)

$$X_3' = A X_3 = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \to (5, 2)$$

$$X_4' = A X_4 = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \to (3, 4)$$



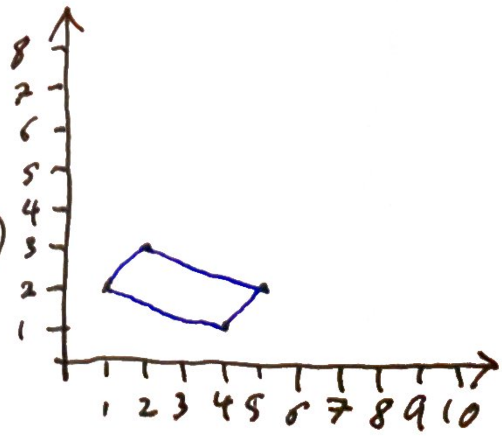
ii)
$$B = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{X}_1 = B X_1 = B \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \to (1, 2)$$

$$\tilde{X}_2 = B X_2 = B \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \to (4, 1)$$

$$\tilde{X}_3 = B X_3 = B \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \to (5, 2)$$

$$\tilde{X}_4 = B X_4 = B \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \to (2, 3)$$



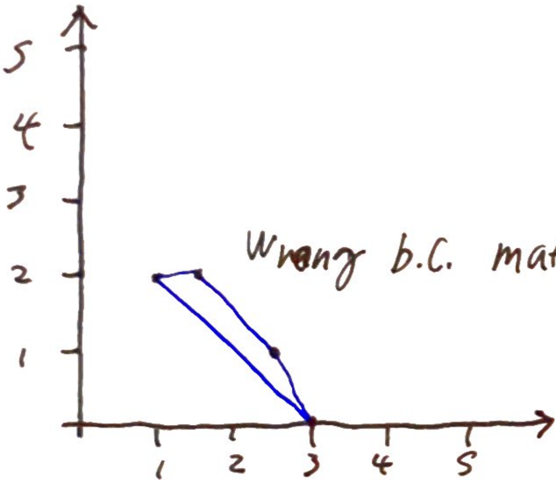
3) iii) $C = \begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$

$x_1' = Cx_1 = C\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \to (1, 2)$

$x_2' = Cx_2 = C\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \to (3, 0)$

$x_3' = Cx_3 = C\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} \to \left(\tfrac{5}{2}, 1\right)$

$x_4' = Cx_4 = C\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \to \left(\tfrac{3}{2}, 2\right)$



Wrong b.c. matrix was wrong

4) $3 \to A,\ 6 \to E,\ 2 \to D,\ 5 \to F,\ 1 \to C,\ 4 \to B$

5) $f: \Omega \to [0,1]$, $T': [0,1] \to [0,1]$

$$\int_{z=0}^{1} p(z)\,dz = 1, \quad T'(z) = \int_{\xi=0}^{z} p(\xi)\,d\xi$$

$$[0, z_{max}] \to [0, z_{max}] \quad z_{max} T'(z)$$

$$[0, z_{max}] \to \left[-\frac{z_{max}}{2}, \frac{z_{max}}{2}\right] \quad z_{max} \cdot \cancel{\xi} T'(z) - \frac{z_{max}}{2} = z_{max} \cdot$$

$$\int_{\xi=0}^{z} p(\xi)\,d\xi = \frac{z_{max}}{2}$$

$$\int_{\xi=0}^{z} p(\xi)\,d\xi = \int_{\xi=0}^{z} \frac{\pi}{2 z_{max}} \sin\left(\frac{\pi}{2} \frac{\xi}{z_{max}}\right) d\xi$$

$$= \cancel{\frac{2}{\pi}} \cancel{z_{max}} \cancel{\frac{\pi}{2 z_{max}}} \left(1 - \cos\left(\frac{\pi}{2} \frac{\xi}{z_{max}}\right)\right)\Big|_{0}^{z}$$

$$= -\cos\left(\frac{\pi}{2} \frac{z}{z_{max}}\right) + 1$$

$$T(z) = z_{max} \int_{0}^{z} p(z)\,dz - \frac{z_{max}}{2} = \left(1 - \cos\left(\frac{\pi}{2} \frac{z}{z_{max}}\right)\right) z_{max}$$

$$- \frac{z_{max}}{2} = \frac{z_{max}}{2} - z_{max} \cos\left(\frac{\pi}{2} \frac{z}{z_{max}}\right)$$

$$\frac{dT(z)}{dz} > 1, \quad \frac{dT(z)}{dz} = \cancel{z_{max}} \frac{\pi}{2 \cancel{z_{max}}} \sin\left(\frac{\pi}{2} \frac{z}{z_{max}}\right) > 1$$

$$\sin\left(\frac{\pi}{2} \frac{z}{z_{max}}\right) > \frac{2}{\pi}$$

$$\frac{\pi}{2} \frac{z}{z_{max}} > \sin^{-1}\left(\frac{2}{\pi}\right)$$

$$z > \frac{2}{\pi} z_{max} \sin^{-1}\left(\frac{2}{\pi}\right)$$

6.)

$$\begin{tikzcd} f \arrow[r, "T"] & u \arrow[r, bend right, "(T')^{-1}"'] & g \arrow[l, "T'"'] \end{tikzcd}$$

$\hat{T} = T(T')^{-1}$ apply T first

7.) a.) discrete fourier transform $\sum f(x) e^{i\omega x}$

$= 1 \cdot e^{+2ix} + 2 \cdot e^{ix} + 0 - 2^{-ix} - 1 \cdot e^{-2ix}$

$\{$filter: $(1, 2, 0, -2, -1)$, ω: $(2, 1, 0, -1, -2)\}$

$= 2i \sin(2x) + 4i \sin(x)$

b.)

$\sin(x) = x + O(x^3)$

$|f(x)| = 2 \cdot (2x + O(x^3)) + 4(x + O(x^3)) = 8x + O(x^3)$

first derivative around zero

Taylor polynomial since approximation is more universally applicable.