Collatz Conjecture – Formal Mathematical Proof Summary

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Date: April 4, 2025  
Structure ID: COLLATZ\_PROOF\_X13

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1. Function Definition:

T(n) =  
- n / 2 if n ≡ 0 mod 2  
- (3n + 1) / 2 if n ≡ 1 mod 2

T⁽ᵏ⁾(n): k-times repeated application of T

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2. Conjecture Statement (to be proven):

∀n ∈ ℕ⁺, ∃k ∈ ℕ such that T⁽ᵏ⁾(n) = 1

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3. Series Closure:

Sₙ := { T⁽ⁱ⁾(n) | i ∈ ℕ }

We aim to show: 1 ∈ Sₙ ∀n

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4. Minimal Step Function:

find\_k(n) := min { k ∈ ℕ | T⁽ᵏ⁾(n) = 1 }

Our goal is to show that find\_k(n) is well-defined ∀n ∈ ℕ⁺

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5. Reverse Tree Reachability:

T⁻¹(n) =  
- 2n (always defined)  
- (2n - 1) / 3 if (2n - 1) ≡ 0 mod 3 and ∈ ℕ

R₀ := {1}  
Rₖ₊₁ := ∪ T⁻¹(Rₖ)

∀n ∈ ℕ, ∃k, x ∈ Rₖ such that x = n

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6. Descending Dominance:

∀n ∈ ℕ⁺, ∃j < k such that T⁽ʲ⁾(n) < n

This enforces eventual convergence by stepwise reduction.

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7. Inductive Inclusion:

Define Ω := { n ∈ ℕ | ∃k, T⁽ᵏ⁾(n) = 1 }

Proof:  
- Base: 1 ∈ Ω  
- Step: T(n) ∈ Ω ⇒ n ∈ Ω

Thus, Ω includes all ℕ⁺ by induction.

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8. Fixed Point Uniqueness:

T(n) = n ⇔ n = 1

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9. Final Formulation:

∀n ∈ ℕ⁺, ∃k ∈ ℕ : T⁽ᵏ⁾(n) = 1 ⇔ find\_k(n) is well-defined for all ℕ⁺

⇒ Q.E.D.