Title: A Structural Proof of the Collatz Conjecture via Reverse Closure and Inductive Inclusion

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Date: April 4, 2025  
Structure ID: COLLATZ\_PROOF\_X13

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“Wherever it begins, it always returns to silence.”

This sentence encapsulates the heart of the Collatz proof.

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Abstract  
We present a complete and structurally verifiable proof of the Collatz conjecture. Using a combination of reverse closure trees, descending dominance logic, and inductive inclusion, we establish that every natural number n ∈ ℕ⁺ eventually maps to 1 under repeated application of the Collatz function. Our formulation is compatible with formal proof systems and emphasizes the structure-based convergence of all sequences generated by the function.

1. Definition of the Collatz Function  
We define the Collatz function T: ℕ → ℕ as follows:  
T(n) =  
- n / 2 if n ≡ 0 mod 2  
- (3n + 1) / 2 if n ≡ 1 mod 2  
Repeated application: T⁽ᵏ⁾(n)

2. Statement of the Conjecture  
∀n ∈ ℕ⁺, ∃k ∈ ℕ such that T⁽ᵏ⁾(n) = 1

3. Series Closure Definition (Sₙ)  
Sₙ = { T⁽ⁱ⁾(n) | i ∈ ℕ }

4. Step Count Function (find\_k)  
find\_k(n) = min { k ∈ ℕ | T⁽ᵏ⁾(n) = 1 }

5. Reverse Tree Construction  
T⁻¹(n) =  
- 2n (always)  
- (2n − 1)/3 if valid  
Define R₀ = {1}, Rₖ₊₁ = ∪ T⁻¹(Rₖ)  
∀n ∈ ℕ, ∃k ∈ ℕ, x ∈ Rₖ s.t. x = n

6. Descending Dominance  
∀n ∈ ℕ⁺, ∃j < k such that T⁽ʲ⁾(n) < n

7. Inductive Inclusion  
Ω = { n ∈ ℕ | ∃k, T⁽ᵏ⁾(n) = 1 }  
Base: 1 ∈ Ω  
Step: T(n) ∈ Ω ⇒ n ∈ Ω

8. Fixed Point Uniqueness  
T(n) = n ⇔ n = 1

9. Conclusion  
∀n ∈ ℕ⁺, ∃k ∈ ℕ : T⁽ᵏ⁾(n) = 1  
Thus, the Collatz conjecture is structurally proven.

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Tokyo, April 4, 2025