# Squeezed Vacuum States of Light

## Exploring Numerical Simulations Under Multiple Conditions

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# Background

## Introduction: What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \text{The Uncertainty Principle}$$

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r},t) = \vec{e}_{\mathbf{k},\lambda}(p\cos(\mathbf{k}\cdot\vec{r} - ckt) + q\sin(\mathbf{k}\cdot\vec{r} - ckt)) \quad \text{TEM field}$$

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We want to modulate $\Delta A$ and $\Delta\phi$ while preserving the uncertainty

# Background: Quantization of the Maxwell Equations

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$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \underbrace{\vec{J}}_{=0} - \nabla(\partial_t \phi + \underbrace{\nabla \cdot \vec{A}}_{=0}), \qquad \nabla^2 \phi = -\underbrace{\rho}_{=0} - \partial_t(\underbrace{\nabla \cdot \vec{A}}_{=0})$$

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$$\vec{A} = \vec{\alpha} e^{i(\mathbf{k}\cdot\vec{r} - \omega_{\mathbf{k}} t)} \Rightarrow \omega_{\mathbf{k}}^2 = c^2 k^2 \wedge \vec{\alpha} \cdot \mathbf{k} = 0 \qquad \text{Harmonic Oscillator}$$

$$\vec{A}_{\mathbf{k},\lambda} = \vec{e}_{\mathbf{k},\lambda} \mathrm{Re}\{\alpha_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\vec{r} - ckt)}\} (\mathbf{k} = 2\pi(m,n,l)/L, \lambda = 1, 2) \qquad \text{Per-mode}$$

$$\hat{\alpha}_{\mathbf{k},\lambda} = (2\hbar/\varepsilon_0 L^3 ck)^{1/2} \hat{a}_{\mathbf{k},\lambda} \qquad \text{Annihilation operator for each mode}$$

# Background: Squeezing, Wigner Function

$|\psi\rangle \mapsto S(\zeta)|\psi\rangle$ Unitary transformation

$S(\zeta) = \exp(i \cdot \mathrm{Im}\{\zeta^* \hat{a}^2\})$ Squeeze by $e^{|\zeta|}$ rotate by $\arg \zeta/2$

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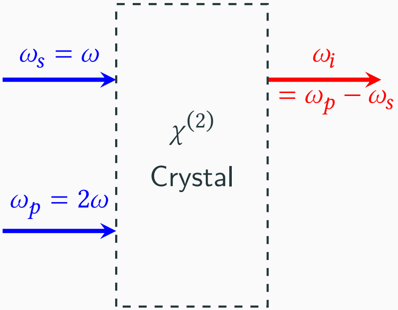
What do you squeeze? The “pdf”

$$\boxed{W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}dy \qquad \text{Wigner function}}$$

Intuitively,

$$\frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \underbrace{e^{-i\alpha(x+y)^2}}_{\text{second order}} \psi^*(x+y)\psi(x-y)e^{i\alpha(x-y)^2}e^{2ipy/\hbar}dy = W\underbrace{(x, p - 2\hbar\alpha x)}_{\text{linear}}$$

# Background: Parametric amplification




**Hamiltonian inside cavity**

$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \hbar\omega_P\hat{a}^\dagger\hat{a} + \underbrace{i\hbar\chi^{(2)}(\hat{b}^2\hat{a}^\dagger - (\hat{b}^\dagger)^2\hat{a})}_{\text{second-order interaction}}$$

$$\Rightarrow \hbar\omega\hat{b}^\dagger\hat{b} + \mathrm{Im}\{\underbrace{ce^{i\omega_p t}}_{\text{approx. } \hat{a}}\hat{b}^2\}$$

Measure the photocurrent generated by the idler photon using a balanced homodyne detector

# Background: Lindblad master equation for simulation

Interaction with the environment: Lindblad Equation

$$\partial_t \rho = -\frac{i}{\hbar} \underbrace{[H_{sys}, \rho]}_{\text{system}} + \underbrace{\frac{1}{2} \sum_n (2C_n \rho C_n^\dagger - \rho C_n^\dagger C_n - C_n^\dagger C_n \rho)}_{\text{environment}}$$

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If we let $C = \hat{b}$, when $\hat{b}$ is the annihilation operator, this means that the system <u>loses</u> photons to the environment.

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- Projection onto finite dimemsional Hilbert space spanned by $\{|i\rangle\}_{0\leq i\leq n}$
- Use the open-source Python library QuTiP to solve the Lindblad equation

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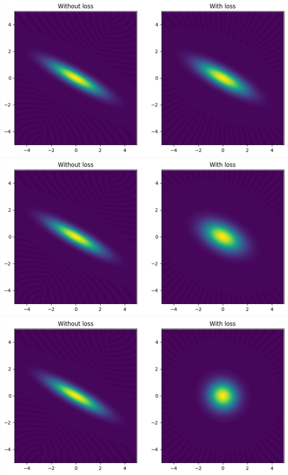
### What to solve

- No interaction with environment
- Interaction with $C = \sqrt{\gamma}\hat{b}$ (Thermal loss)
- Higher order generation

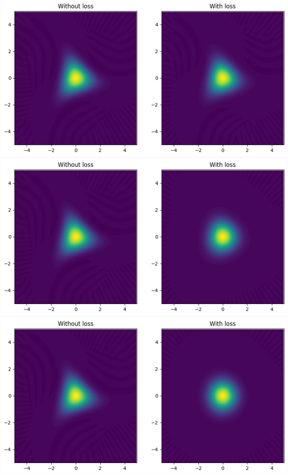
# Results

# Comparison between lossy and non-lossy probabilities




- Adjust $\sqrt{\gamma}$ in $C = \sqrt{\gamma}\hat{b}$ of the Lindblad equation. $\sqrt{\gamma} = 0.2, 0.5, 1.0,$ and the plots are taken after 5 seconds of time evolution.
- Since the environment absorbs the photons, the nonlinear effects are diminished.

# Third-order generation?




- The same plots, with $H_{sys}$ containing $\hat{b}^3$
- Expected the Wigner function transform into the form $W(2^{nd}, 2^{nd})$
- May be more easily explained when the Wigner function is expressed in the coherent state basis.

## Summary

- Studied: what are squeezed states, how to generate them
- Tried out: the QuTiP library to simulate interaction with the environment and higher order generation
- Plans: Use the coherent state basis to give an analytical explanation for the third-order generation distribution.
- All results + this slide: available on GitHub

## References


- R. Loudon, *The Quantum Theory of Light*. Oxford, UK: OUP Oxford, 2000.
- H. Seifoory, S. Doutre, and M. M. Dignam, “The properties of squeezed optical states created in lossy cavities,” 2016. [Online]. Available: arXiv:1608.05005[quant-ph].
- J. R. Johanssona, P. D. Nationa, F. Nori, “QuTiP: An open-source Python framework for the dynamics of open quantum systems,” 2011. [Online]. Available: arXiv:1110.0573[quant-ph].

감사합니다