



# Squeezed Vacuum States of Light

Exploring Numerical Simulations Under Multiple Conditions

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# Background

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# Introduction: What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \text{The Uncertainty Principle}$$

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r}, t) = \vec{e}_{\mathbf{k},\lambda}(p \cos(\mathbf{k} \cdot \vec{r} - ckt) + q \sin(\mathbf{k} \cdot \vec{r} - ckt)) \quad \text{TEM field}$$

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We want to modulate  $\Delta p$  and  $\Delta q$  while preserving the uncertainty

## Background: Quantization of the Maxwell Equations

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$$\vec{A} = \vec{\alpha}e^{i(\mathbf{k}\cdot\vec{r}-\omega_{\mathbf{k}}t)} \Rightarrow \omega_{\mathbf{k}}^2 = c^2k^2 \wedge \vec{\alpha} \cdot \mathbf{k} = 0 \quad \text{Harmonic Oscillator}$$

$$\vec{A}_{\mathbf{k},\lambda} = \vec{e}_{\mathbf{k},\lambda}\text{Re}\{\alpha_{\mathbf{k},\lambda}e^{i(\mathbf{k}\cdot\vec{r}-ckt)}\} (\mathbf{k} = 2\pi(m,n,l)/L, \lambda = 1, 2) \quad \text{Per-mode}$$

$$\hat{\alpha}_{\mathbf{k},\lambda} = (2\hbar/\varepsilon_0 L^3 ck)^{1/2} \hat{a}_{\mathbf{k},\lambda} \quad \text{Annihilation operator for each mode}$$

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$$|\psi\rangle \mapsto S(\zeta)|\psi\rangle \quad \text{Unitary transformation}$$



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$$W(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{2ipy/\hbar} dy \quad \text{Wigner function}$$

$$\frac{1}{\pi\hbar} \int_{-\infty}^{\infty} e^{-i\alpha(x+y)^2} \psi^*(x+y)\psi(x-y)e^{i\alpha(x-y)^2} e^{2ipy/\hbar} dy = W(x, p-2\hbar\alpha x)$$

Intuitive calculation: A linear map with determinant 1

## Background: SHG, Lindblad Equation

Higher-order polarization effect generates multiple frequencies

$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \hbar\omega_p\hat{a}^\dagger\hat{a} + i\hbar\chi(\hat{b}^2\hat{a}^\dagger - (\hat{b}^\dagger)^2\hat{a})$$

Second-order interaction: One  $\hat{a} \leftrightarrow$  Two  $\hat{b}$

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Interaction with the environment: Lindblad Equation

$$\partial_t \rho = -\frac{i}{\hbar}[H_{\text{sys}}, \rho] + \frac{1}{2} \sum_n (2C_n \rho C_n^\dagger - \rho C_n^\dagger C_n - C_n^\dagger C_n \rho)$$

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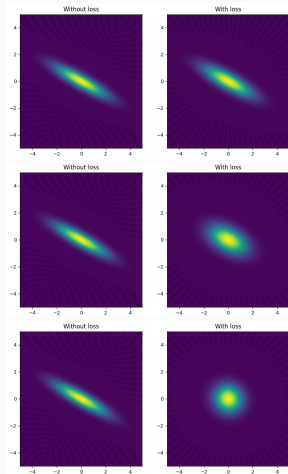
- No interaction with environment
- Interaction with  $C = \sqrt{\gamma} \hat{b}$  (Thermal loss)
- Higher order generation



# Results

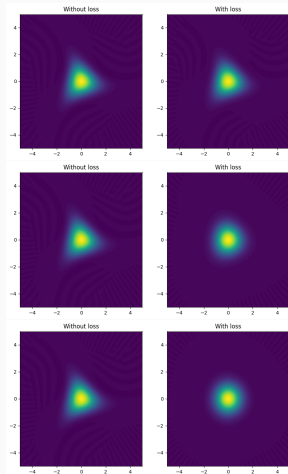
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# Comparison between lossy and non-lossy probabilities



- Adjust  $\sqrt{\gamma}$  in  $C = \sqrt{\gamma}\hat{b}$  of the Lindblad equation.  
 $\sqrt{\gamma} = 0.2, 0.5, 1.0$ , and the plots are taken after 5 seconds of time evolution.
- Since the environment absorbs the photons, the nonlinear effects are diminished.

# Third-order generation?






- The same plots, with  $H_{\text{sys}}$  containing  $\hat{b}^3$
- Expected the Wigner function to turn into transform into the form  $W(2^{\text{nd}}, 2^{\text{nd}})$

# Summary

- Studied: what are squeezed states, how to generate them
- Tried out: the QuTiP library to simulate interaction with the environment and higher order generation

# References

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-  J. R. Johansson, P. D. Nation, F. Nori, “QuTiP: An open-source Python framework for the dynamics of open quantum systems,” 2011. [Online]. Available: [arXiv:1110.0573\[quant-ph\]](https://arxiv.org/abs/1110.0573).

감사합니다