Constructing Set Constraints for ReScript

Joonhyup Lee

1 Calculations

$$
\begin{aligned}
i\hbar\partial_t\rho &= \hbar\omega[a^\dagger a, \rho] + \hbar\chi^{(2)}[\frac{\alpha^* e^{2i\omega t}a^2 - \alpha e^{-2i\omega t}(a^\dagger)^2}{2i}, \rho] \\
&= \hbar\omega[a^\dagger a, \rho] + \frac{\hbar\chi^{(2)}}{2i}([\alpha^* e^{2i\omega t}a^2, \rho] - [\alpha e^{-2i\omega t}(a^\dagger)^2, \rho])
\end{aligned}
$$

Let $\rho = \exp(-i\omega t a^\dagger a)\tilde{\rho}\exp(i\omega t a^\dagger a)$, then

$$
\partial_t\tilde{\rho} = -\frac{\chi^{(2)}}{2}(\alpha^*[a^2, \tilde{\rho}] - \alpha[(a^\dagger)^2, \tilde{\rho}])
$$

We want to look at the time evolution of the Wigner quasiprobability distribution. The Von Neumann equation above can be translated into a partial differential equation for the Wigner function by the following rules:

$$
\begin{aligned}
a\rho &\leftrightarrow (z + \frac{1}{2}\partial_{z^*})W & z &\leftrightarrow q + ip \\
a^\dagger\rho &\leftrightarrow (z^* - \frac{1}{2}\partial_z)W & z^* &\leftrightarrow q - ip \\
\rho a &\leftrightarrow (z - \frac{1}{2}\partial_{z^*})W & \partial_z &\leftrightarrow \frac{1}{2}(\partial_q - i\partial_p) \\
\rho a^\dagger &\leftrightarrow (z^* - \frac{1}{2}\partial_z)W & \partial_{z^*} &\leftrightarrow \frac{1}{2}(\partial_q + i\partial_p)
\end{aligned}
$$

Then the Von Neumann equation turns into

$$
\partial_t W = -\frac{\chi^{(2)}}{2}(2\alpha z^*\partial_z W + 2\alpha^* z\partial_{z^*}W)
$$

References

[1] Kwangkeun Yi and Sukyoung Ryu. "Towards a Cost-Effective Estimation of Uncaught Exceptions in SML Programs". In: *Proceedings of the 4th International Symposium on Static Analysis*. SAS '97. Berlin, Heidelberg: Springer-Verlag, 1997, pp. 98–113. ISBN: 3540634681.