Squeezed Vacuum States of Light

Exploring Numerical Simulations Under Multiple Conditions

이준협

2023년 3월 23일

전기공학설계프로젝트 중간발표

Background

Introduction: What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \text{The Uncertainty Principle}$$

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r},t) = \vec{e}_{\mathbf{k},\lambda}(p\cos(\mathbf{k}\cdot\vec{r} - ckt) + q\sin(\mathbf{k}\cdot\vec{r} - ckt)) \quad \text{TEM field}$$

$$\Delta p \Delta q \geq \frac{\hbar c k}{2\varepsilon_0 L^3} \qquad \text{Uncertainty Principle for } \vec{E}$$

Introduction: What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$ The Uncertainty Principle

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r},t) = \vec{e}_{\mathbf{k},\lambda}(p\cos(\mathbf{k}\cdot\vec{r} - ckt) + q\sin(\mathbf{k}\cdot\vec{r} - ckt))$$ TEM field

$$\Delta p \Delta q \geq \frac{\hbar c k}{2\varepsilon_0 L^3}$$ Uncertainty Principle for $\vec{E}$

We want to modulate Δp and Δq while preserving the uncertainty

Background: Quantization of the Maxwell Equations

$\vec{E} = -\nabla\phi - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A}$ The potentials

Background: Quantization of the Maxwell Equations

$$\vec{E} = -\nabla\phi - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A} \qquad \text{The potentials}$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \underbrace{\vec{J}}_{=0} - \nabla(\partial_t\phi + \underbrace{\nabla\cdot\vec{A}}_{=0}), \qquad \nabla^2\phi = -\underbrace{\rho}_{=0} - \partial_t(\underbrace{\nabla\cdot\vec{A}}_{=0})$$

Background: Quantization of the Maxwell Equations

$$\vec{E} = -\nabla\phi - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A} \qquad \text{The potentials}$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \underbrace{\vec{J}}_{=0} - \nabla(\partial_t\phi + \underbrace{\nabla\cdot\vec{A}}_{=0}), \qquad \nabla^2\phi = -\underbrace{\rho}_{=0} - \partial_t(\underbrace{\nabla\cdot\vec{A}}_{=0})$$

$$\vec{A} = \vec{\alpha}e^{i(\mathbf{k}\cdot\vec{r}-\omega_{\mathbf{k}}t)} \Rightarrow \omega_{\mathbf{k}}^2 = c^2k^2 \wedge \vec{\alpha}\cdot\mathbf{k} = 0 \qquad \text{Harmonic Oscillator}$$

$$\vec{A}_{\mathbf{k},\lambda} = \vec{e}_{\mathbf{k},\lambda}\mathrm{Re}\{\alpha_{\mathbf{k},\lambda}e^{i(\mathbf{k}\cdot\vec{r}-ckt)}\}(\mathbf{k} = 2\pi(m,n,l)/L, \lambda = 1,2) \qquad \text{Per-mode}$$

$$\hat{\alpha}_{\mathbf{k},\lambda} = (2\hbar/\varepsilon_0 L^3 ck)^{1/2}\hat{a}_{\mathbf{k},\lambda} \qquad \text{Annihilation operator for each mode}$$

Background: Squeezing, Wigner Function

$$|\psi\rangle \mapsto S(\zeta)|\psi\rangle \qquad \text{Unitary transformation}$$

Background: Squeezing, Wigner Function

$|\psi\rangle \mapsto S(\zeta)|\psi\rangle$ Unitary transformation

$S(\zeta) = \exp(i \cdot \mathrm{Im}\{\zeta^* \hat{a}^2\})$ Squeeze by $e^{|\zeta|}$ rotate by $\arg \zeta/2$

Background: Squeezing, Wigner Function

$|\psi\rangle \mapsto S(\zeta)|\psi\rangle$ Unitary transformation

$S(\zeta) = \exp(i \cdot \mathrm{Im}\{\zeta^* \hat{a}^2\})$ Squeeze by $e^{|\zeta|}$ rotate by $\arg\zeta/2$

What do you squeeze? The “pdf”

Background: Squeezing, Wigner Function

$|\psi\rangle \mapsto S(\zeta)|\psi\rangle$ Unitary transformation

$S(\zeta) = \exp(i \cdot \mathrm{Im}\{\zeta^* \hat{a}^2\})$ Squeeze by $e^{|\zeta|}$ rotate by $\arg \zeta/2$

What do you squeeze? The "pdf"

$$W(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} \psi^*(x + y)\psi(x - y)e^{2ipy/\hbar}dy$$ Wigner function

$$\frac{1}{\pi\hbar} \int_{-\infty}^{\infty} e^{-i\alpha(x+y)^2}\psi^*(x+y)\psi(x-y)e^{i\alpha(x-y)^2}e^{2ipy/\hbar}dy = W(x, p-2\hbar\alpha x)$$

Intuitive calculation: A linear map with determinant 1

Background: SHG, Lindblad Equation

Higher-order polarization effect generates multiple frequencies

$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \hbar\omega_P\hat{a}^\dagger\hat{a} + i\hbar\chi(\hat{b}^2\hat{a}^\dagger - (\hat{b}^\dagger)^2\hat{a})$$

Second-order interaction: One $\hat{a} \leftrightarrow$ Two $\hat{b}$

Approximate: $\hat{a}$ very strong $\Rightarrow \hbar\omega\hat{b}^\dagger\hat{b} + \mathrm{Im}\{ce^{i\omega_P t}\hat{b}^2\}$

Background: SHG, Lindblad Equation

Higher-order polarization effect generates multiple frequencies

$$\hat{H} = \hbar\omega\hat{b}^{\dagger}\hat{b} + \hbar\omega_P\hat{a}^{\dagger}\hat{a} + i\hbar\chi(\hat{b}^2\hat{a}^{\dagger} - (\hat{b}^{\dagger})^2\hat{a})$$

Second-order interaction: One $\hat{a} \leftrightarrow$ Two $\hat{b}$

Approximate: $\hat{a}$ very strong $\Rightarrow \hbar\omega\hat{b}^{\dagger}\hat{b} + \mathrm{Im}\{ce^{i\omega_P t}\hat{b}^2\}$

Interaction with the environment: Lindblad Equation

$$\partial_t\rho = -\frac{i}{\hbar}[H_{sys}, \rho] + \frac{1}{2}\sum_n(2C_n\rho C_n^{\dagger} - \rho C_n^{\dagger}C_n - C_n^{\dagger}C_n\rho)$$

Methodology

Methodology

How to solve

- Projection onto finite dimemsional Hilbert space spanned by $\{|i\rangle\}_{0\leq i\leq n}$
- Use the open-source Python library QuTiP to solve the Lindblad equation

Methodology

How to solve

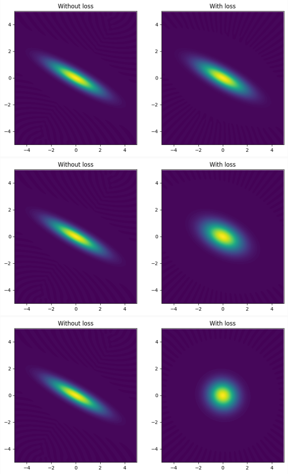
- Projection onto finite dimemsional Hilbert space spanned by $\{|i\rangle\}_{0\leq i\leq n}$
- Use the open-source Python library QuTiP to solve the Lindblad equation

What to solve

- No interaction with environment
- Interaction with $C = \sqrt{\gamma}\hat{b}$ (Thermal loss)
- Higher order generation

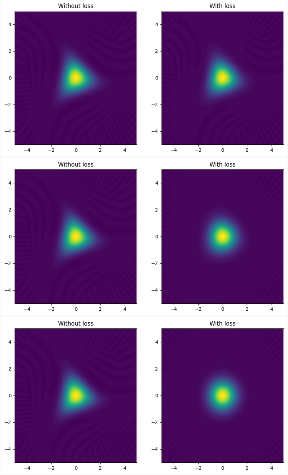
Results

Comparison between lossy and non-lossy probabilities



- Adjust $\sqrt{\gamma}$ in $C = \sqrt{\gamma}\hat{b}$ of the Lindblad equation. $\sqrt{\gamma} = 0.2, 0.5, 1.0,$ and the plots are taken after 5 seconds of time evolution.
- Since the environment absorbs the photons, the nonlinear effects are diminished.

Third-order generation?



- The same plots, with H_{sys} containing $\hat{b}^3$
- Expected the Wigner function to turn into transform into the form $W(2^{nd}, 2^{nd})$

Summary

- Studied: what are squeezed states, how to generate them
- Tried out: the QuTiP library to simulate interaction with the environment and higher order generation

References

- R. Loudon, *The Quantum Theory of Light*. Oxford, UK: OUP Oxford, 2000.
- H. Seifoory, S. Doutre, and M. M. Dignam, “The properties of squeezed optical states created in lossy cavities,” 2016. [Online]. Available: arXiv:1608.05005[quant-ph].
- J. R. Johanssona, P. D. Nationa, F. Nori, “QuTiP: An open-source Python framework for the dynamics of open quantum systems,” 2011. [Online]. Available: arXiv:1110.0573[quant-ph].

감사합니다