# Squeezed Vacuum States of Light

## Exploring Numerical Simulations Under Multiple Conditions

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# Background

# Introduction: What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$ The Uncertainty Principle

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r},t) = \vec{e}_{\mathbf{k},\lambda}(p\cos(\mathbf{k}\cdot\vec{r} - ckt) + q\sin(\mathbf{k}\cdot\vec{r} - ckt))$$ TEM field

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Rewrite $p\cos\theta(t) + q\sin\theta(t) = A\sin(\theta(t) + \phi)$

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We want to modulate $\Delta A$ and $\Delta\phi$ while preserving the uncertainty

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$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \underbrace{\vec{J}}_{=0} - \nabla(\partial_t\phi + \underbrace{\nabla \cdot \vec{A}}_{=0}), \qquad \nabla^2\phi = -\underbrace{\rho}_{=0} - \partial_t(\underbrace{\nabla \cdot \vec{A}}_{=0})$$

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$$\vec{A} = \vec{\alpha}e^{i(\mathbf{k}\cdot\vec{r}-\omega_{\mathbf{k}}t)} \Rightarrow \omega_{\mathbf{k}}^2 = c^2k^2 \wedge \vec{\alpha}\cdot\mathbf{k} = 0 \qquad \text{Harmonic Oscillator}$$

$$\vec{A}_{\mathbf{k},\lambda} = \vec{e}_{\mathbf{k},\lambda}\mathrm{Re}\{\alpha_{\mathbf{k},\lambda}e^{i(\mathbf{k}\cdot\vec{r}-ckt)}\}(\mathbf{k} = 2\pi(m,n,l)/L, \lambda = 1,2) \qquad \text{Per-mode}$$

$$\hat{\alpha}_{\mathbf{k},\lambda} = (2\hbar/\varepsilon_0 L^3 ck)^{1/2}\hat{a}_{\mathbf{k},\lambda} \qquad \text{Annihilation operator for each mode}$$

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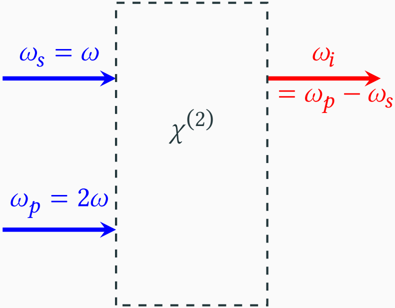
$$W(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}dy$$ Wigner function

$$\frac{1}{\pi\hbar} \int_{-\infty}^{\infty} e^{-i\alpha(x+y)^2}\psi^*(x+y)\psi(x-y)e^{i\alpha(x-y)^2}e^{2ipy/\hbar}dy = W(x, p-2\hbar\alpha x)$$

Intuitive calculation: A linear map with determinant 1

# Background: Parametric amplification




$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \hbar\omega_P\hat{a}^\dagger\hat{a} + \underbrace{i\hbar\chi(\hat{b}^2\hat{a}^\dagger - (\hat{b}^\dagger)^2\hat{a})}_{\text{second-order interaction}}$$

One pump $\hat{a} \leftrightarrow$ Two $\hat{b}$

Approximate: $\hat{a}$ very strong

$$\Rightarrow \hbar\omega\hat{b}^\dagger\hat{b} + \mathrm{Im}\{ce^{i\omega_P t}\hat{b}^2\}$$

Higher-order polarization effect generates multiple frequencies

# Background: Lindblad master equation for simulation

Interaction with the environment: Lindblad Equation

$$\partial_t \rho = -\frac{i}{\hbar}[H_{sys}, \rho] + \frac{1}{2}\sum_n (2C_n \rho C_n^\dagger - \rho C_n^\dagger C_n - C_n^\dagger C_n \rho)$$

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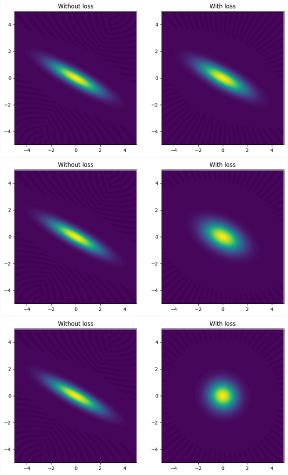
## What to solve

- No interaction with environment
- Interaction with $C = \sqrt{\gamma}\hat{b}$ (Thermal loss)
- Higher order generation

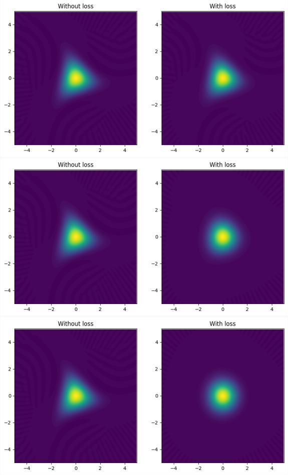
# Results

# Comparison between lossy and non-lossy probabilities




- Adjust $\sqrt{\gamma}$ in $C = \sqrt{\gamma}\hat{b}$ of the Lindblad equation. $\sqrt{\gamma} = 0.2, 0.5, 1.0,$ and the plots are taken after 5 seconds of time evolution.
- Since the environment absorbs the photons, the nonlinear effects are diminished.

# Third-order generation?




- The same plots, with $H_{sys}$ containing $\hat{b}^3$
- Expected the Wigner function to turn into transform into the form $W(2^{nd}, 2^{nd})$

## Summary

- Studied: what are squeezed states, how to generate them
- Tried out: the QuTiP library to simulate interaction with the environment and higher order generation
- All results + this slide: available on GitHub

## References


- R. Loudon, *The Quantum Theory of Light*. Oxford, UK: OUP Oxford, 2000.
- H. Seifoory, S. Doutre, and M. M. Dignam, “The properties of squeezed optical states created in lossy cavities,” 2016. [Online]. Available: arXiv:1608.05005[quant-ph].
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