Analysis of Squeezed Vacuum States of Light by Means of Wigner Functions

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What is Squeezed Light?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \text{The Uncertainty Principle}$$

$$\vec{E}_{\mathbf{k},\lambda}(\vec{r},t) = \vec{e}_{\mathbf{k},\lambda}(p\cos(\mathbf{k}\cdot\vec{r} - ckt) + q\sin(\mathbf{k}\cdot\vec{r} - ckt)) \qquad \text{TEM field}$$

$$\Delta p \Delta q \geq \frac{\hbar ck}{2\varepsilon_0 L^3} \qquad \text{Uncertainty Principle for } \vec{E}$$

Rewrite $p\cos\theta(t) + q\sin\theta(t) = A\sin(\theta(t) + \phi)$

$$\Delta A \Delta \phi \geq \frac{\hbar ck}{2\varepsilon_0 L^3} \qquad \text{Uncertainty Principle for } \phi \text{ and } A$$

We want to modulate ΔA and $\Delta \phi$ while preserving the uncertainty

Background: Quantization of the Maxwell Equations

$$\vec{E} = -\nabla\phi - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A} \qquad \text{The potentials}$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \underbrace{\vec{J}}_{=0} - \nabla(\partial_t\phi + \underbrace{\nabla\cdot\vec{A}}_{=0}), \qquad \nabla^2\phi = -\underbrace{\rho}_{=0} - \partial_t(\underbrace{\nabla\cdot\vec{A}}_{=0})$$

$$\vec{A} = \vec{\alpha}e^{i(\mathbf{k}\cdot\vec{r} - \omega_{\mathbf{k}}t)} \Rightarrow \omega_{\mathbf{k}}^2 = c^2k^2 \wedge \vec{\alpha}\cdot\mathbf{k} = 0 \qquad \text{Harmonic Oscillator}$$

$$\vec{A}_{\mathbf{k},\lambda} = \vec{e}_{\mathbf{k},\lambda}\mathrm{Re}\{\alpha_{\mathbf{k},\lambda}e^{i(\mathbf{k}\cdot\vec{r} - ckt)}\}(\mathbf{k} = 2\pi(m,n,l)/L, \lambda = 1,2) \qquad \text{Per-mode}$$

$$\hat{\alpha}_{\mathbf{k},\lambda} = (2\hbar/\varepsilon_0 L^3 ck)^{1/2}\hat{a}_{\mathbf{k},\lambda} \qquad \text{Annihilation operator for each mode}$$

Background: Squeezing, Wigner Function

$$|\psi\rangle \mapsto S(\zeta)|\psi\rangle \qquad \text{Unitary transformation}$$

$$S(\zeta) = \exp(i \cdot \mathrm{Im}\{\zeta^*\hat{a}^2\}) \qquad \text{Squeeze by } e^{|\zeta|} \text{ rotate by } \arg\zeta/2$$

What do you squeeze? The "pdf"

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}dy \qquad \text{Wigner function}$$

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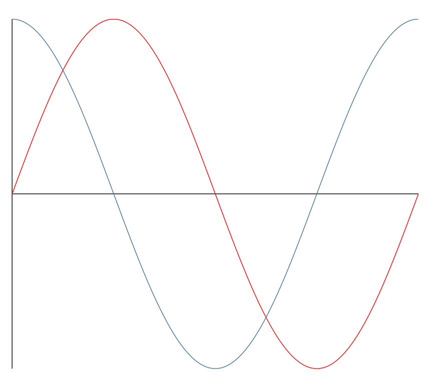

Figure 1. Another figure caption.

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First column	Second column	Third column	Fourth
Foo	13.37	384,394	α
Bar	2.17	1,392	β
Baz	3.14	83,742	δ
Qux	7.59	974	γ

Table 1. A table caption.

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References

[1] Claude E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, 1948.