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Lab 1+2

The spring elastic constant evaluation

1. Purpose

The objective of the experiment is to determine the spring constant of a spiral spring using Hooke's law and the period of oscillatory motion in response to a weight.
Apparatus: A spiral spring, a set of weights, a weight hanger, a stop watch, and a lab scale.

2. Theory

A. Static method

We use a spiral spring with elastic constant *k* and undeformed length l_o and bodies with different mass.

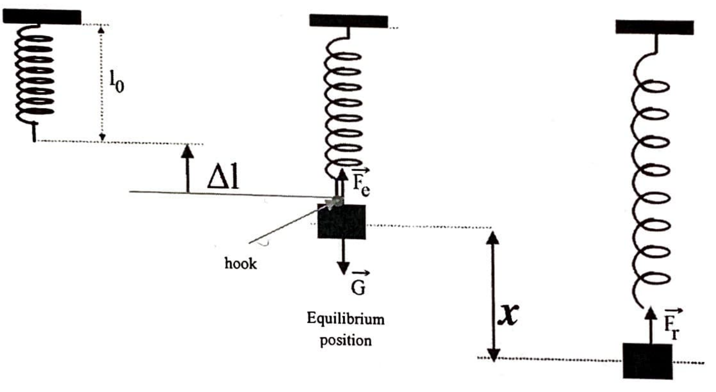

Fig. 1.

When a spring is stretched, according to Hooke's law, a restoring force F proportional to its elongation, x (or $\Delta l = l - l_o$) appears. Every spring obeys the Hooke's law if the deformation is not too great.

$$\vec{F_e} = -k\overrightarrow{\Delta l} \qquad (1)$$

For the equilibrium

$$\Rightarrow mg = k\Delta l \qquad (2)$$

$$k = \frac{mg}{\Delta l} \qquad (3)$$

B. Dynamic method

When we move the body connected to a spring from its equilibrium position it starts to oscillate around the equilibrium position under the action of restoring (elastic) force. With x the distance from equilibrium position the Newton's second law is:

$$ma = -kx \qquad (4)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad (5)$$

We note $\omega = \sqrt{\frac{k}{m}}$ and we call it the natural angular frequency. The second Newton law for the spring (5) is a differential equation having the solution

$$x(t) = A\sin(\omega t + \varphi) \quad (7)$$

The period for the harmonic oscillation is connected to the natural angular frequency throught:

$$\omega = \frac{2\pi}{T} \quad (8)$$

$$k = m\omega^2 = 4\pi^2 \frac{m}{T^2} \quad (9)$$

3. What to do

Static method

1) We measure the undeformed spring length l_o.
2) We hang the weight hanger with mass m_1 on the spring and we measure the deformed spring length l_1. We calculate the spring elongation Δl_1.
3) Successively we hang weights (m_i) and we calculate corresponding spring elongations Δl_i.
4) Use the Table 1. for experimental data and calculate elastic constant using relation (3).
5) Plot the graph of force (the deformative force) produced by different masses ($F = m \cdot g$) as a function of the displacement from equilibrium Δl: **$F(\Delta l)$**. The data should be linear. Hence, the slope of the line will be equal to the spring constant k according to the relation 2.
6) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\qquad$ $\overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ $\qquad$ $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

Table 1.

$l_0 = 44$ cm

Nr. crt	m [kg]	l [cm]	Δl [m]	F [N]	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
1	0,05	50,5	0,065	0,49	7,538		0,366		
2	0,066	53,5	0,095	0,646	6,8		-0,372		
3	0,083	55	0,11	0,813	7,39		0,218		
4	0,091	56,5	0,125	0,891	7,128		-0,044		
5	0,099	57,5	0,135	0,97	7,185	7,172	0,013	0,061	7,172 ± 0,061
6	0,105	58,5	0,145	1,029	7,096		-0,076		
7	0,134	62,5	0,185	1,313	7,097		-0,075		
8	0,178	68,5	0,245	1,744	7,118		-0,054		
9	0,207	72	0,28	2,028	7,242		0,07		
10	0,24	77	0,33	2,352	7,127		-0,045		

Dynamic method

1) Hang the weight hanger with several weights (mass m_1) on the spring and set the equilibrium position of the system.
2) Pull the system out of its equilibrium position to make oscillations with 1-2cm amplitude.
3) Record the time for n=20 oscilations and find the period: $T = t/n$.
4) Repeat 1), 2) and 3) for different masses.
5) Complete the Table 2 using relation (9) for elastic constant.
6) Plot the graph of $T^2(s^2)$ as function of m(kg). The data should be linear. Find elastic constant from the slope (i.e. $T^2 = \frac{4\pi^2}{k} m \Leftrightarrow y = \text{slope} \cdot x$).
7) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\qquad$ $\overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ $\qquad$ $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

Table 2.

Nr. crt.	m [kg]	t [s]	n	T [s]	T^2 (s^2)	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
1	0,06	6,96	12	0,58	0,336	7,047		-0,202		
2	0,09	8,306	12	0,692	0,478	7,43		0,181		
3	0,12	9,763	12	0,813	0,66	7,175		-0,074		
4	0,15	10,875	12	0,906	0,82	7,218	7,249	-0,031	0,048	7,249±0,048
5	0,18	11,905	12	0,992	0,984	7,218		-0,031		
6	0,21	12,743	12	1,061	1,125	7,366		0,117		
7	0,24	13,576	12	1,131	1,279	7,405		0,156		
8	0,27	14,666	12	1,222	1,493	7,136		-0,113		

Compare the results obtained by the 2 methods, respectively by arithmetic and graphic mediation !!

Table 1 :

1. $m = 0{,}05\ Kg$

$l = 50{,}5\ cm$

$l_0 = 44\ cm$

$$\begin{array}{r} 0{,}05 \\ 9{,}8 \\ \hline 040 \\ 045 \\ \hline 0{,}49 \end{array}$$

$\Delta l = 50{,}5 - 44 = 6{,}5\ cm \Rightarrow \Delta l = 0{,}065\ m$

$F = m \cdot g \quad \Rightarrow F = 0{,}05 \cdot 9{,}8 = 0{,}49\ N$

$g = 9{,}8\ m/s^2$

$$K = \frac{m \cdot g}{\Delta l} = \frac{0{,}49}{0{,}065} = 7{,}538\ N/m$$

2. $m = 0{,}066\ Kg$

$l = 53{,}5\ cm$

$l_0 = 44\ cm$

$$\begin{array}{r} 0{,}066 \\ 9{,}8 \\ \hline 0528 \\ 0594 \\ \hline 0{,}6468 \end{array}$$

$\Delta l = 53{,}5 - 44 = 9{,}5\ cm \Rightarrow \Delta l = 0{,}095\ m$

$F = 0{,}066 \cdot 9{,}8 = 0{,}646\ N$

$$K = \frac{0{,}646}{0{,}095} = 6{,}8\ N/m$$

3. $m = 0{,}083\ Kg$

$l = 55\ cm$

$l_0 = 44\ cm$

$$\begin{array}{r} 0{,}083 \\ 9{,}8 \\ \hline 0664 \\ 0747 \\ \hline 0{,}8134 \end{array}$$

$\Delta l = 55 - 44 = 11\ cm \Rightarrow \Delta l = 0{,}11\ m$

$F = 0{,}083 \cdot 9{,}8 = 0{,}8134\ N$

$$K = \frac{0{,}813}{0{,}11} = 7{,}39\ N/m$$

4. $m = 0{,}091\ Kg$

$l = 56{,}5\ cm$

$l_0 = 44\ cm$

$$\begin{array}{r} 0{,}091 \\ 9{,}8 \\ \hline 0728 \\ 0819 \\ \hline 0{,}8918 \end{array}$$

$\Delta l = 56{,}5 - 44 = 12{,}5\ cm \Rightarrow \Delta l = 0{,}125\ m$

$F = 0{,}091 \cdot 9{,}8 = 0{,}891\ N$ $\quad K = \frac{0{,}891}{0{,}125} = 7{,}128\ N/m$

5. $m = 0{,}099\ Kg$

$l = 57{,}5\ cm$

$l_0 = 44\ cm$

$\Delta l = 57{,}5 - 44 = 13{,}5\ cm \Rightarrow \Delta l = 0{,}135\ m$

$F = 0{,}099 \cdot 9{,}8 = 0{,}97\ N$

$k = \frac{0{,}97}{0{,}135} = 7{,}185\ N/m$

$$\begin{array}{r} 0{,}099 \cdot \\ 9{,}8 \\ \hline 0792 \\ 0891 \\ \hline 0{,}9702 \end{array}$$

6. $m = 0{,}105\ Kg$

$l = 58{,}5\ cm$

$l_0 = 44\ cm$

$\Delta l = 58{,}5 - 44 = 14{,}5\ cm \Rightarrow \Delta l = 0{,}145\ m$

$F = 0{,}105 \cdot 9{,}8 = 1{,}029\ N$

$K = \frac{1{,}029}{0{,}145} = 7{,}096\ N/m$

$$\begin{array}{r} 0{,}105 \cdot \\ 9{,}8 \\ \hline 0840 \\ 0945 \\ \hline 1{,}0290 \end{array}$$

7. $m = \cancel{0{,}135}\ 0{,}134\ Kg$

$l = 62{,}5\ cm$

$l_0 = 44\ cm$

$\Delta l = 62{,}5 - 44 = 18{,}5\ cm \Rightarrow \Delta l = 0{,}185\ m$

$F = 0{,}134 \cdot 9{,}8 = 1{,}313\ N$

$K = \frac{1{,}313}{0{,}185} = 7{,}097\ N/m$

$$\begin{array}{r} 0{,}134 \\ 9{,}8 \\ \hline 1072 \\ 1206 \\ \hline 1{,}3132 \end{array}$$

8. $m = 0{,}178\ Kg$

$l = 68{,}5\ cm$

$l_0 = 44\ cm$

$\Delta l = 68{,}5 - 44 = 24{,}5\ cm \Rightarrow \Delta l = 0{,}245\ m$

$F = 0{,}178 \cdot 9{,}8 = 1{,}744\ N.$

$K = \frac{1{,}744}{0{,}245} = 7{,}118\ N/m$

$$\begin{array}{r} 0{,}178 \cdot \\ 9{,}8 \\ \hline 1424 \\ 1602 \\ \hline 1{,}7444 \end{array}$$

9.

$m = 0{,}207\ kg$

$l = 72\ cm$

$l_0 = 44\ cm$

$\Delta l = 72 - 44 = 28\ cm \Rightarrow \Delta l = 0{,}28\ m$

$F = 0{,}207 \cdot 9{,}8 = 2{,}028\ N$

$k = \frac{2{,}028}{0{,}28} = 7{,}242\ N/m$

$$\begin{array}{r} 0{,}207 \cdot \\ 9{,}8 \\ \hline 1656 \\ 1863 \\ \hline 2{,}0286 \end{array}$$

10.

$m = 0{,}24\ kg$

$l = 77\ cm$

$l_0 = 44\ cm$

$\Delta l = 77 - 44 = 33\ cm \Rightarrow \Delta l = 0{,}33\ m$

$F = 0{,}24 \cdot 9{,}8 = 2{,}352\ N$

$k = 7{,}127\ N/m$

$$\begin{array}{r} 0{,}24 \cdot \\ 9{,}8 \\ \hline 192 \\ 216 \\ \hline 2{,}352 \end{array}$$

$$\bar{k} = \frac{7{,}538 + 6{,}8 + 7{,}39 + 7{,}128 + 7{,}185 + 7{,}096 + 7{,}097 + 7{,}118 + 7{,}242 + 7{,}12}{10}$$

$$\begin{array}{r} 7{,}538 + \\ 6{,}800 \\ \hline 14{,}338 + \\ 7{,}390 \\ \hline 21{,}728 + \\ 7{,}128 \\ \hline 28{,}856 + \\ 7{,}185 \\ \hline 36{,}041 + \\ 7{,}096 \\ \hline 43{,}137 + \\ 7{,}097 \\ \hline 50{,}234 + \\ 7{,}118 \\ \hline 57{,}352 + \\ 7{,}242 \\ \hline 64{,}594 + \\ 7{,}127 \\ \hline 71{,}721 \end{array}$$

$\bar{k} = \frac{71{,}721}{10} = \frac{71{,}721 \cdot 1000}{10}$

~~$\bar{k} = 71{,}721$~~

$\bar{k} = \frac{71721}{10} = \frac{7172{,}1}{1000}$

$\bar{k} = 7{,}1721\ N/m$

$$\begin{array}{r} 71{,}721 \cdot \\ 1000 \\ \hline 71721 \end{array}$$

$$\begin{array}{l} 71721 \mid 10 \\ 70 \quad\ \mid 7172{,}1 \\ \hline 17 \\ 10 \\ \hline 72 \\ 70 \\ \hline 21 \\ 20 \\ \hline 10 \\ 10 \\ \hline == \end{array}$$

$\Delta K_1 = 7{,}538 - 7{,}172 = 0{,}366\ N/m$

$\Delta K_2 = 6{,}8 - 7{,}172 = -0{,}372\ N/m$

$\Delta K_3 = 7{,}39 - 7{,}172 = 0{,}218\ N/m$

$\Delta K_4 = 7{,}128 - 7{,}172 = -0{,}044\ N/m$

$\Delta K_5 = 7{,}185 - 7{,}172 = 0{,}013\ N/m$

$\Delta K_6 = 7{,}096 - 7{,}172 = -0{,}076\ N/m$

$\Delta K_7 = 7{,}095 - 7{,}172 = -0{,}075\ N/m$

$\Delta K_8 = 7{,}118 - 7{,}172 = -0{,}054\ N/m$

$\Delta K_9 = 7{,}242 - 7{,}172 = 0{,}07\ N/m$

$\Delta K_{10} = 7{,}127 - 7{,}172 = -0{,}045\ N/m$

$$\sigma\bar{K} = \sqrt{\frac{\sum_1^{10}(0{,}366 - 0{,}372 + 0{,}218 - 0{,}044 + 0{,}013 - 0{,}076 - 0{,}075 - 0{,}054 + 0{,}07 - 0{,}0}{10(10-1)}}$$

$\sigma\bar{K} = 0{,}061\ N/m$

$K_{true} = 7{,}172 \pm 0{,}061.$