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Using Simulated Annealing to Solve Complex Optimization Problems

Minimizing Return on Investment Time of a Cooling Tower by Varying its Design and Minimization of Time to Accelerate from an Initial Velocity to an End Velocity

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Resumo Optimization techniques find widespread application across diverse industrial sectors due to their ability to significantly enhance efficiency, performance, and economic returns. This paper presents the use of Simulated Annealing, a potent optimization algorithm, in three distinct case studies: a power plant return on investment (ROI) by varying its cooling tower height, the gear ratios optimization of an Alfa Romeo Giulia Quadrifoglio, and a benchmark that comprises a truss structure optimization via 2 different approaches.

Palavras-Chave Optimization · Simulated Annealing · ROI · Power Plant · Gear Ratio · Acceleration

1 Introduction

In the modern industrial context, optimization techniques stand as instrumental tools for maximizing operational efficiency, financial viability, and overall performance. These techniques, often bolstered by computational modeling and algorithmic ingenuity, allow for precise manipulation of parameters to achieve a given set of objectives within a multitude of constraints. The paper showcases the application of Simulated Annealing, a renowned probabilistic method, in solving different optimization problems, as demonstrated through the presented case studies.

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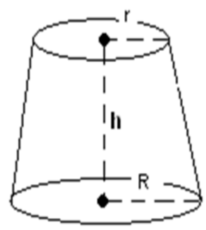
2 Defining Engineering Problems

2.1 Optimization of Cooling Tower Design for Minimized Return on Investment Time

In the case of the power plant design optimization, the primary goal was to minimize the Return on Investment (ROI) period. The ROI was computed as the ratio of total investment to total profit. Total investment included material cost (Cmaterial), related to the surface area (A) of the cooling tower, and construction cost (Cconstruction), proportional to the tower's height (h). Total profit was then obtained by subtracting the annual operational cost from the annual revenue.

The cooling tower's geometry, where the bottom and top diameters are functions of the height, affected the investment costs. The larger the surface area, the higher the material costs, and the taller the tower, the more substantial the construction costs, but also, better capacity to remove heat.



$$\text{Area} = \pi(R + r)\sqrt{(R - r)^2 + h^2}$$

$$\text{Volume} = \frac{\pi}{3}h(R^2 + r^2 + R * r)$$

Figura 1. Geometry of the Cooling Tower [6]

The total profit was influenced by the actual heat transfer (Qactual) of the cooling tower. This heat transfer depended on various factors, such as the difference in water and ambient air temperature at tower height, the tower's surface area, and a heat transfer coefficient (hc) impacted by wind speed. The annual revenue was directly linked to this heat transfer.

By varying the tower's height and calculating the corresponding ROI, it was now possible to locate an optimal solution: a tower height that minimized the ROI period while respecting all the other constraints of the problem.

2.2 Minimization of time to accelerate from an initial to an end velocity

The first step in defining the problem consists in the selection of a vehicle and the retrieval of its relevant parameters. Due to author's preference, the car chosen for the optimization problem is the Alfa Romeo Giulia Quadrifoglio, specifically the version equipped with a 6-speed manual transmission. The main parameters are shown in table 1 and are the starting point for the construction of the objective function.

Tabela 1. Vehicle specifications [1]

Parameter	
Weight	1650 kg
Transmission efficiency	0.9
Tyres (rear)	285/30 R19
Maximum power	500 HP
Top speed	307 km/h
I Gear ratio	0.247
VI Gear ratio	1.147

As seen in the table, the first and sixth gear ratios are fixed, and their values are taken from factory data sheets of the car. The four in between will be the decision variables of the optimization problem and will be defined by the vector $\mathbf{x} = [x_1,x_2,x_3,x_4] \in \mathbb{R}^4$. Note that the convention used to define gear ratios in this analysis is $\frac{\omega_{\mathrm{out}}}{\omega_{\mathrm{in}}}$.

The acceleration time of a vehicle can be calculated in many ways. The approach chosen for the purpose of this analysis, based on what was learnt in the author's home university [2], uses the concept of *excess power* P_{e}, defined as the difference between the power transmitted by the engine to the wheels, $P_{\mathrm{eng}}\eta_{\mathrm{t}}$, and the *resisting power* P_{n}, which is the amount of power required in order to travel at a constant speed (therefore winning the aerodynamic and rolling forces). P_{e} can thus be seen as a measure of the amount of power available, at a given velocity v, in order to accelerate with an acceleration a in full load conditions. The relation can be expressed as follows:

$$P_{\mathrm{e}} = P_{\mathrm{eng}}\eta_{\mathrm{t}} - P_{\mathrm{n}} = m_{\mathrm{a}}va, \tag{1}$$

where m_{a} is the apparent mass of the vehicle, which varies depending on the gear ratio. Given the dependency of m_{a} from the gear selected, P_{e} also varies with gear ratios and therefore needs to be defined for every gear. From equation 1 it is possible to derive the infinitesimal time $\mathrm{d}t$, which can then be integrated over a velocity interval to obtain the time to accelerate:

$$t = \int_0^t \mathrm{d}t = \int_{v_1}^{v_2} \frac{m_{\mathrm{a}}v}{P_{\mathrm{e}}}\mathrm{d}v \tag{2}$$

3 Optimization Problems Formulation

3.1 Optimization of Cooling Tower Design for Minimized Return on Investment Time

The formulation for this problem goes as follows:

$$\begin{aligned} &\text{Find } \mathbf{h} \text{ that :} \\ \text{Minimizes } ROI(\mathbf{h}) &= \frac{TotalInvestment(\mathbf{h})}{AnnualProfit(\mathbf{h})} && (3) \\ \text{Subject to } Q_{removed}(\mathbf{h}) &\geq Q_{min}, && (4) \\ h \in \mathbb{Z} \in [100{,}200] \end{aligned}$$

The unique geometry of the cooling tower is characterised by two diameters - the bottom diameter (D1) and the top diameter (D2) - both being linearly related to the tower's height. In addition, the surface area (A) of the tower, a derivation of these diameters and the tower height, determines the cost of materials (Cmaterial) as they share a direct proportional relationship.

The total investment in this context is a summation of two costs - the cost of materials and the cost of construction (Cconstruction). Interestingly, while the cost of materials bears proportionality to the tower's surface area, the cost of construction is directly proportional to the tower's height.

A key factor in the analysis is the ambient temperature at a specific height ($T_{\mathrm{air(h)}}$), computed as the sum of the base ambient temperature (T_{ambient}) and the product of the tower height squared and an air temperature gradient ($gradient_{\mathrm{air}}$). The heat transfer coefficient (h_{c}) is subsequently defined through a linear function involving wind speed (v).

Then it was needed to determine the actual heat transfer called ($Q_{\mathrm{actual(h)}}$), computed in megawatts, as a function of the heat transfer coefficient, the surface area of the tower, and the temperature difference between the water (T_{water}) and the ambient air at the height ($T_{\mathrm{air(h)}}$). A minimum heat transfer requirement as (Q_{min}) is also accounted for in the problem formulation. In terms of energy production, the model treats it as a constant (c) multiplied by the tower height. Accordingly, the annual revenue is derived from the energy production and the cost per megawatt-hour (E_{cost}).

Another key aspect is the total operational cost per year, which depends on the tower's height and an operation cost per meter per year ($C_{\mathrm{operation}}$), with an added penalty if the actual heat transfer fails to meet the minimum requirement.

Tabela 2. Parameters of the Cooling Tower

Parameter	**Value**
K_{material}	3000 m^2
$K_{\mathrm{construction}}$	100000
T_{water}	75 C
T_{ambient}	20 C
$\mathrm{gradient}_{\mathrm{air}}$	-0.006 C/m
a	15 W/($\mathrm{m}^2 K$)
b	120 W/($\mathrm{m}^2 K$)
E_{cost}	0.12 * 24 * 365
v	10 m/s
h_{c}	150
Q_{min}	500 MW

3.2 Minimization of Time to Accelerate from an Initial Velocity to an End Velocity

Find $\mathbf{x}$ that:

$$\text{minimizes } f(\mathbf{x}) = \sum_{i=1}^{6} \int_{v_{i-1}}^{v_i} \frac{m_{\mathrm{a}}^{(i)} v}{P_{\mathrm{e}}^{(i)}(\mathbf{x})} \mathrm{d}v + (i-1)\Delta t_{\mathrm{gc}}$$

$$\begin{aligned} \text{subj. to } g_1(\mathbf{x}) &= x_i - x_{i+1} < 0, & i &= 1, \ldots, 6 \\ g_2(\mathbf{x}) &= -x_i < 0, & i &= 1, \ldots, 6 \end{aligned}$$

The constraints ensure that the gear ratios are positive and in ascending order. Δt_{gc} accounts for the time required to physically change gear in the manual Alfa Romeo.

4 Sensitivity Analysis

4.1 Optimization of Cooling Tower Design for Minimized Return on Investment Time

The sensitivity analysis for the optimization problem involved varying each parameter by 25% relative to the cooling tower's height, to evaluate their individual impacts on the return on investment (ROI). Out of all the parameters, only five displayed significant alterations - the material constant (K_{material}), the construction constant ($K_{\mathrm{construction}}$), wind velocity (v), energy price (E_{cost}), and operation cost ($C_{\mathrm{operation}}$). Therefore only these five influential parameters were included in the graphical representations.

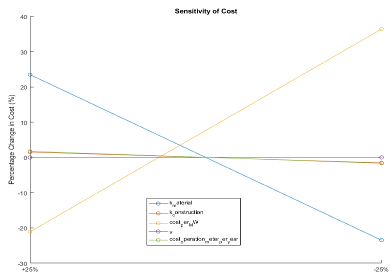


Figura 2. Geometry of the Cooling Tower

4.2 Minimization of Time to Accelerate from an Initial Velocity to an End Velocity

Drawing inspiration from a previous student's approach within the curricular unit [3], two different configurations of the gearbox were tested: a shorter configuration, achieved by reducing the intermediate gear ratios by 10%, and a longer configuration where the gear ratios were instead increased by 10%.

Figure 3 illustrates the comparison of the objective function's behaviours in the two scenarios, taking into account a target velocity of 300 km/h, and includes the reference result obtained using the factory gearbox. The perturbation of the gear ratios yielded predictable outcomes: the shorter gearbox demonstrates improved performance at lower speeds, while compromising acceleration at higher speeds. Conversely, the longer gearbox compromises acceleration overall, yet it potentially allows for a higher top speed to be achieved (not shown in the graph). In general, the variation in the gear ratios resulted in outcomes that were close to the reference value, with only a slight increase of 1 to 2 seconds in the acceleration time.

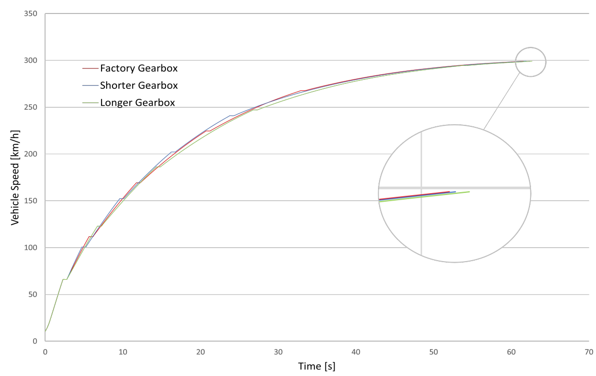


Figura 3. Sensitivity analysis results

5 Evaluation

Both problems were crafted utilizing MATLAB as the primary tool. The methodology followed a sequential process, starting with the assignment of values to the parameters, leading up to the comprehensive modeling of the problem. This included the evaluation of the objective functions and constraints.

In terms of constraints, penalties have been implemented to sanction the objective functions whenever the constraints weren't satisfied. Once the final value of the objective function was determined, an automatic generation of graphical results was initiated. This not only allowed to visualize the established solution but also granted insights into the evolution of the results obtained to eventually reach the final result.

6 Optimization Algorithm

Simulated Annealing is an optimization method, that takes inspiration from the natural process of annealing in metallurgy where a material is heated, then gradually cooled to minimize defects. Applied to optimization, this principle aids in exploring a solution space effectively.

Initially, a solution and a temperature parameter is established, often starting high to represent a state of high entropy. The solution then undergoes minor disturbances to generate potential new solutions nearby in the solution space, essentially heating the system and enhancing exploration.

New solutions are assessed using the objective function. If better, they replace the current solution. However, even if they're worse, they may still be accepted depending on the system's temperature, thus avoiding premature convergence on local minima. This represents the annealing or cooling phase, where the temperature gradually reduces over iterations, and acceptance of inferior solutions becomes less likely, encouraging a shift from exploration to exploitation.

The probabilistic nature of this algorithm demands that multiple optimizations are performed.

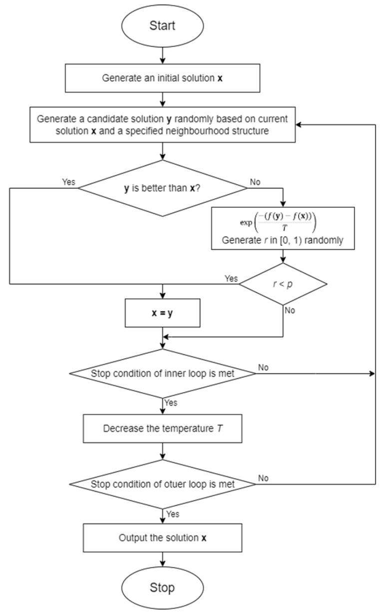


Figura 4. Simulated Annealing Flow Chart [4]

7 Results

7.1 Optimization of Cooling Tower Design for Minimized Return on Investment Time

The simulated annealing optimization approach delivered some interesting results for the cooling tower height optimization problem. With the aim to minimize the return on investment (ROI) time for the power plant, the optimal solution found was a tower height of exactly 100.00 meters. The total investment associated with this solution came out to be $287,311,259.62.

Further examination of this solution indicated a ROI time of 11.09 years. This result is appealing, suggesting that the total investment made in constructing the power plant could be recouped in a little over a decade. Naturally, because of the probabilistic nature of simulated annealing, it was then decided that the algorithm would run for 5 cycles in order to validate its consistency.

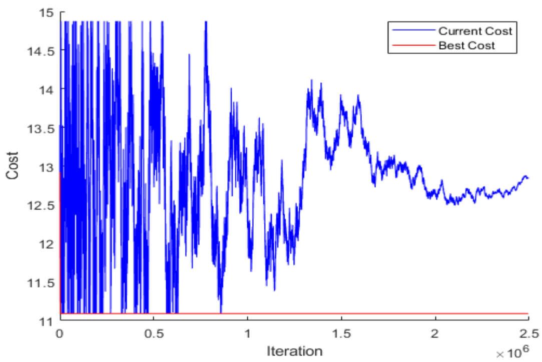


Figura 5. Best height progression

The best results obtained can be consulted in figure 5. It is quite noticeable how the unrestrained liberty of the algorithm in the early stages of the optimization cycle, allows it to find the optimal value almost immediately.

That being said, there are certain details to bear in mind while interpreting these results. While the model is comprehensive, it is not exhaustive. Certain parameters which could potentially influence the optimization problem might have been omitted due to a lack of data or other constraints. Like for example the fact that the amount of heat that needed to be removed, 500 MW, might just be too low to represent a tangible constraint. Another example is the fact that many parameters were interpreted as a constant that varies in direct or indirect relation to the height

Therefore, while the results provide a somewhat realistic estimate for the minimum ROI time, they may not completely reflect reality. Further refinements to the model, incorporating additional variables and constraints, would be necessary for a more accurate prediction of the ROI time for the power plant.

7.2 Minimization of Time to Accelerate from an Initial Velocity to an End Velocity

The availability of the factory gear ratios of the vehicle presents a notable advantage in addressing this optimization problem. Similar to the sensitivity analysis discussed in section 4.2, these gear ratios serve as valuable reference that facilitate the comparison and evaluation of the optimized results. With this in mind, the focus now shifts to the presentation of the outcome obtained for minimizing the time required to accelerate the vehicle from 10 to 305 km/h.

Considering the probabilistic nature of the selected optimization algorithm, multiple runs were performed with slight adjustments to the algorithm's parameters. The subsequent analysis focuses on the best outcome achieved during the tuning process. Table 3 presents the respective parameters that led to the shortest acceleration time. The number of iterations represents the count of evaluations carried out prior to decreasing the temperature based on the cooling factor.

Tabela 3. Algorithm settings

Parameter	Value
Initial temperature	50
Final temperature	0.01
Cooling rate	0.98
Iterations	500
Starting solution	[0.5 0.5 0.5 0.5]

Figure 6 provides a visual representation of the optimization process's progression throughout the iterations, offering valuable insights into the algorithm's behaviour. The blue curve demonstrates that when the temperature is high, the algorithm is permitted to explore the solution space, occasionally accepting sub-optimal solutions. As the temperature decreases, the algorithm gradually converges towards the minimum value of the objective function which represents the best solution obtained during the process (red curve).

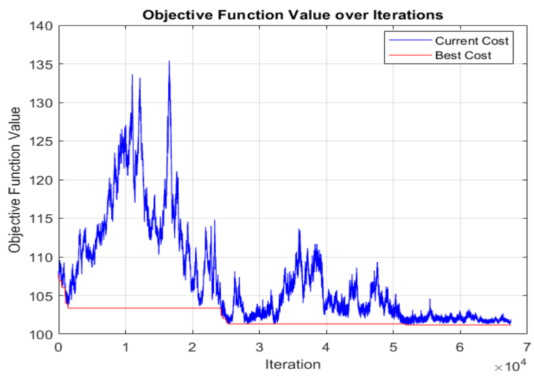


Figura 6. Evolution of the optimization process

Upon examination of the best optimization run, it was found that the time to accelerate was slightly better than the time achieved using the factory gear ratios. To further analyse this improvement, table 4 presents a comparison between the resulting optimal gear ratios and the factory ratios. Moreover, the table includes the corresponding values of the objective function for each set of input variables, providing a comprehensive view of the optimization outcomes.

The marginal improvement (approximately 1.4%) can be attributed to the performance-oriented nature

Tabela 4. Comparison between factory and optimal values

GEARS	II	III	IV	V	Time
Factory	0.417	0.632	0.839	1.000	**102.6**
Optimal	0.356	0.512	0.700	0.938	**101.1**

of the Alfa Romeo Giulia Quadrifoglio. It is plausible that the gearbox of this car was specifically designed with performance factors in mind, which may have limited the potential for significant improvements. Yet some differences in the ratios are expected, as the design of a gearbox for a road car takes into account various other factors, such as fuel consumption.

As a bonus, the algorithm was also tested, for the same velocity interval, on the 8-speed automatic transmission version of the Alfa Romeo. In order to accommodate the increased number of variables in this scenario, which amounts to six (excluding the known first and eighth gears), slight adjustments were made to the objective function as well as the parameters of the algorithm. In this case, the reference acceleration time achieved with the factory gear ratios is 98.3 seconds, slightly shorter than the previous scenario. This difference can be attributed to both the reduced gear change time (obtained with the automatic transmission) and the improved utilization of the power curve during acceleration. The best optimization run yielded a value which was again marginally better than the reference, stopping the clock at 97.5 s. The optimization process evolution is showed in figure 7.

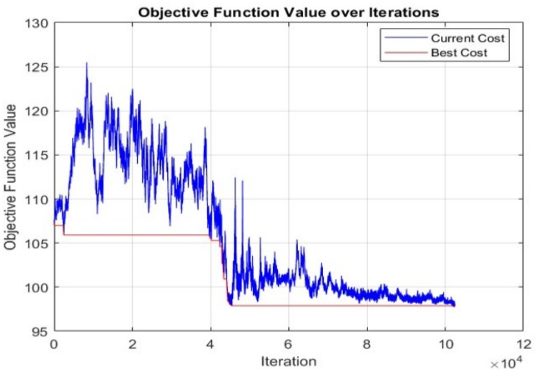


Figura 7. Evolution of the optimization process

8 Conclusions

In conclusion, simulated Annealing has proven to be a highly effective optimization algorithm in addressing the two problems presented. However, achieving optimal results requires careful parameter tuning due to the probabilistic nature of the algorithm. While simulated annealing does not guarantee finding the absolute minimum, the obtained results showed realistic and improved values.

A few things can and must be stated about these two problems. While formulating the problem of optimizing the time for the return on investment some of the parameters and variables were taken into account as constants that vary linearly with the changes in height, that doesn't represent that well the actual scenario.

Regarding the second optimization problem, while it may not be particularly captivating from a pure engineering standpoint, it proved to be a valuable learning opportunity from an academic perspective. The author perceived it as an exciting challenge that contributed to a deeper understanding of vehicles mechanics and provided a platform to strengthen and expand programming skills.

A Benchmark 2023

A.1 Benchmark Explanation

This optimization exercise revolves around the optimization of the mass of a truss structure such as it was explained in the paper by João Oliveira and António Campos [5]. A two-fold approach was used to optimize the truss structure. The first part of the optimization process concentrates on minimizing the area of each segment of the truss individually. While the second part involves the adaptive alteration of the coordinates of certain nodes in the truss (nodes 2, 3, and 5). Varying the nodes' coordinates potentially impacts the truss's overall shape and the stress distribution within the structure. This adjustment allows us to explore a variety of configurations to find the optimal design that can withstand the imposed loads, while not exceeding the admissible stress limit.

A.2 Implementation

The simulated annealing algorithm was utilized for optimization, drawing inspiration from the annealing process in metallurgy. Starting with a random solution, it makes minor changes to find the best solution. Its temperature parameter begins high, allowing acceptance of varied solutions, then lowers over time, only accepting improved solutions, thereby avoiding local minima. The diagram showing the logic behind simulated annealing algorithm is shown in paragraph 6, figure 4.

In the truss optimization process, the objective is to identify designs that minimize mass while satisfying load and stress requirements. Through multiple iterations, the optimization algorithm converges towards an optimal solution.

A.3 Results

Simulated annealing was applied to two optimization problems, requiring careful parameter selection. Perturbations were adjusted based on the current solution's magnitude. Start and end temperatures of 30 and 0.1, respectively, were selected, determined through experimentation. A step-type cooling schedule (shown in figure 8) was used, gradually reducing temperature over iterations, with a cooling rate of 0.95, balancing accuracy and run time. This allowed a balance between exploration (searching new solution regions) and exploitation (refining promising solutions), ensuring efficient convergence towards optimal solutions.

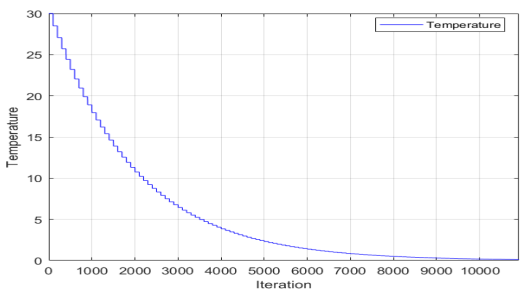


Figura 8. Utilised cooling schedule

Three independent runs were carried out using the parameters described earlier to ensure the reliability and robustness of the results. Figure 9 illustrates the convergence of the best cost values over the course of the optimization process for all three runs. Run one, which achieved the lowest mass of the system at 9.96 kg, also highlights the accepted solution throughout the optimization process, visually represented in red. This plot, in particular, provides valuable insights into the algorithm's exploration of the search space and the corresponding fluctuations in the objective function value as the algorithm occasionally accepts sub optimal solutions. The results for each run where later tested to ensure that the constraints on the admissible stresses were not violated. A visual representation of the optimized structure is also shown in figure 10.

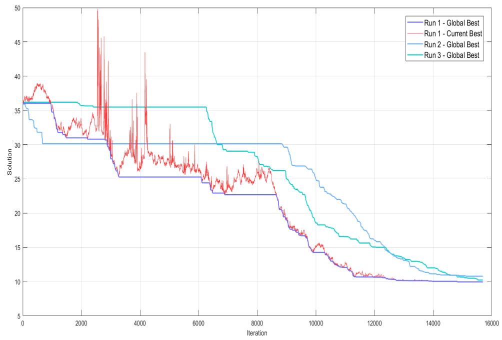


Figura 9. Optimization results for the first approach

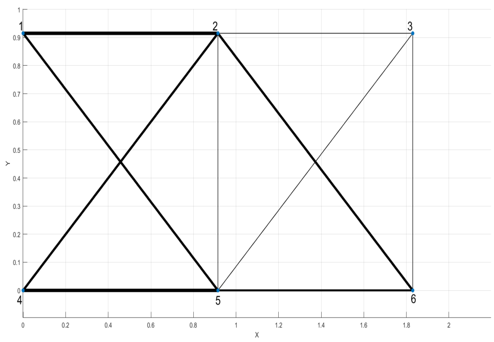


Figura 10. Visual representation of the optimized areas of the structure

For the second approach, three runs were conducted using the same parameters introduced previously. This time, an additional constraint was introduced to prevent the three variable nodes from being too close to each other. A plot depicting the behavior of the best cost values over the iterations is provided in figure 11, showcasing the progress of the optimization process. Run 1 exhibited the most favorable outcome, achieving a mass of 24.72 kg. Furthermore, figure 12 illustrates the resultant shape of the optimized structure.

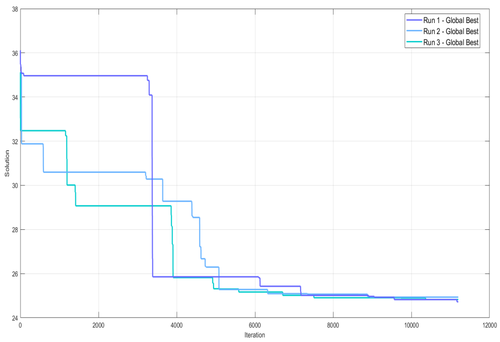


Figura 11. Optimization results for the first approach

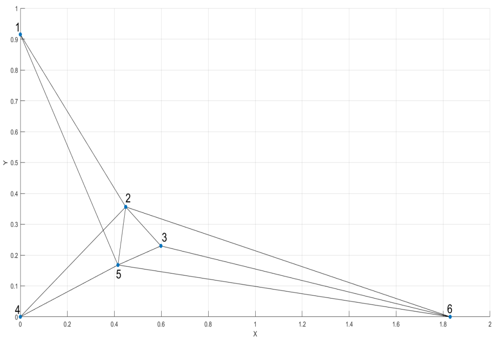


Figura 12. Optimization results for the first approach

As a bonus, the algorithm was tested on a more intricate structure comprising 36 bars, showcasing its re-usability and scalability. Similar to the previous approaches, three runs were conducted and verified according to the constraints. A plot depicting the optimization process for each run is provided in figure 13. Remarkably, in run one, the algorithm yielded the most favorable outcome, achieving an optimized mass of 127.2 kg. The exploration process for this run is also highlighted in red.

Figura 13. Optimization results for the 36 bar problem

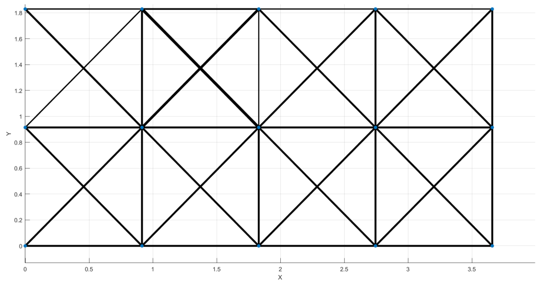


Figura 14. Visual representation of the optimized areas for the 36 bar structure

In conclusion, the implementation of the simulated annealing algorithm for the optimization of the 2D 10 bar truss structure has proven to be effective in generating realistic and consistent results. While the algorithm provided optimized values for the objective function, it is important to note that these values may not represent the absolute minimum due to the nature of simulated annealing, which explores the search space based on probabilistic criteria. Nonetheless, the algorithm demonstrated its versatility by successfully accommodating different approaches, such as considering bar areas and node coordinates as decision variables. Furthermore, its re-usability was showcased through the successful application on a more complex structure consisting of 36 bars.

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