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# Propagator Rewriting

For a correct rewrite of a propagator $p$ to another propagator $p'$ on a store $s$, the following must hold:

$$p(s') = p'(s') \quad \forall s' \text{ where } s' \leq s$$

I.e a propagator can be rewritten to another (simpler) propagator if the propagators behave the same on all stronger stores than the current store. Typical usage of propagator rewriting is to simplify a propagator when a store $s$ is reached, where complex propagation is no longer necessary due to the reduced variable domains.

To rewrite a single propagator $p$ to a set of $Q$ simpler propagators it is necessary for correctness that the following holds:

$$p(s') = \bigcap q(s') \quad \forall q \in Q \quad \forall s' \text{ where } s' \leq s$$

A modification to the propagation algorithm to support propagator rewriting might be to allow propagators to send an extra status message: $\{rewrite, Q\}$ and the algorithm can then replace $p$ with the propagators in $Q$. The correctness of the relation between $Q$ and $p$ would be up to the propagator itself. I.e you add the following line to the algorithm:

**`if ms = {rewrite, Q} then`** $EP := (EP - \{ep\}) \cup Q$

# An Addition Propagator

## Propagator Definition

Propagator $p_+ \in c \equiv x + y = z$.

$$p_+(s) = \begin{cases} x \mapsto \{n \in s(x) | \min s(z) - \max s(y) \leq n \leq \max s(z) - \min s(y)\} \\ y \mapsto \{n \in s(y) | \min s(z) - \max s(x) \leq n \leq \max s(z) - \min s(x)\} \\ z \mapsto \{n \in s(z) | \min s(x) + \min s(y) \leq n \leq \max s(x) + \max s(y)\} \end{cases}$$

## Idempotence

**Theorem 0.1.** *The propagator $p_+$ is not idempotent.*

*Proof.* If $p_+$ is idempotent then $p_+(p_+(s)) = p_+(s) \quad \forall s$, Counterexample:

$$s = \{x \mapsto \{0, 4\}, y \mapsto \{0, 4\}, z \mapsto \{0, 1, 2\}\}$$

$$p_+(s) = s' = \{x \mapsto \{0\}, y \mapsto \{0\}, z \mapsto \{0,1,2\}\}$$
$$p_+(s') = s'' = \{x \mapsto \{0\}, y \mapsto \{0\}, z \mapsto \{0\}\}$$

$$\therefore s' \neq s'' \quad Q.E.D$$

□

### Subsumption

Detect subumption:

$$\begin{gathered} p_+(s) = \textbf{let } s' = p_+(s), \quad s'' = p_+(s') \\ \textbf{if } |s'(x)| = 1 \wedge |s'(y)| = 1 \wedge |s'(z)| = 1 \wedge s'(x) + s'(y) = s'(z) \\ \textbf{then } \langle subsumed, s' \rangle \\ \textbf{else if } s'' = s' \quad \langle fix, s' \rangle \\ \textbf{else if } s'' \neq s' \quad \langle nofix, s' \rangle \end{gathered}$$

### Rewriting

Rewrite into a simpler propagator:

Assume for instance that propagation have computed $s(x) = 1$ then $p_+$ can be rewritten to the following simpler (more efficient) propagator:

$$p_+(s) = \begin{cases} y \mapsto \{n \in s(y) | \min s(z) - 1 \leq n \leq \max s(z) - 1\} \\ z \mapsto \{n \in s(z) | \min s(y) + 1 \leq n \leq \max s(y) + 1\} \end{cases}$$