Assignment 4 - Extra non-submission tasks **Kim Hammar**
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Constraint Programming Due Date: 2 June 2017

Golomb Rulers

Symmetry breaking

The problem has a symmetry of a "reflection". The markers are already ordered by the model but the reflection symmetries can still occur, i.e two symmetrical solutions to $n = 3$:

$\{0, 1, 3\} \iff \{0, 2, 3\}$

These symmetries can be broken by constraining that distance between mark 1 m_1 and mark 2 m_2 ($d_{2,1}$) is less than distance between mark n and mark $n - 1$, $d_{n,n-1}$.

Propagation levels

IPL_VAL (value propagation, wait until variable assigned before propagating) is less efficient for this model than IPL_DOM (domain consistent propagation) according to my measurements. I.e more propagation instead of more search is preferred here.

IPL_BND and IPL_DOM give same propagation/search and same results. This is probably because bounds propagation is also domain consistent in this particular case due to the ordering.

Redundant constraint for stronger propagation

Another observation is that $d_{i,j}$ must be at least the sum of the first $j - i$ integers. Explain why! Use this insight as a lower bound for the $d_{i,j}$. Note: The sum of the first n integers is $n(n + 1)/2$.

The insight about this implied constraint is far from intuitive and is explained by the paper but basically it is because all distances are constrainted to be different and the ordering of the markers.

Linear Arithmetic

$$x - 2 \times y = 0 \iff x - y - y = 0$$

$$x - 2 \times y = 0$$
$$x - 2y = 0$$

$$x - y - y = 0$$

$$x - 2y = 0$$

$p \in constraint \equiv x - 2y = 0$

$$p(s) = \begin{cases} x \mapsto \{n \in s(x) | \exists m \in s(y), n = 2m\} \\ y \mapsto \{n \in s(y) | \exists m \in s(x), n = \frac{m}{2}\} \end{cases}$$

$s_0 = \{x \mapsto \{1, \ldots, 4\}, y \mapsto \{0, \ldots, 4\}\}$

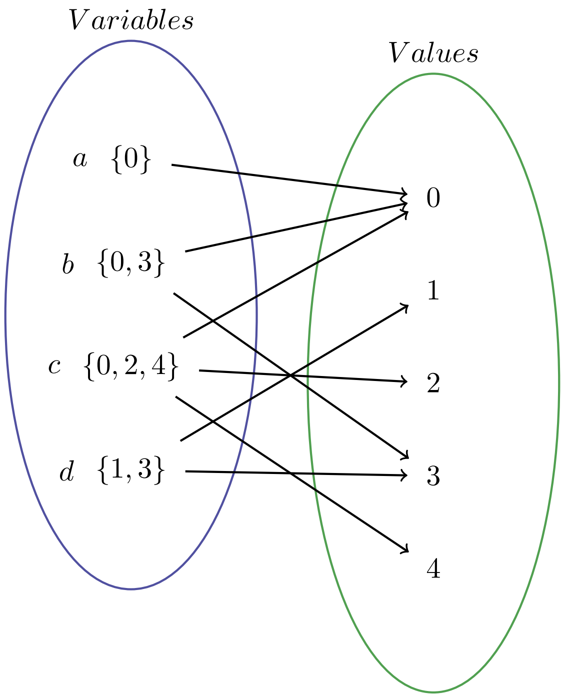
$p(s_0) = s_1 = \{x \mapsto \{2, 4\}, y \mapsto \{1, 2\}\}$

p is at fixpoint at s_1.

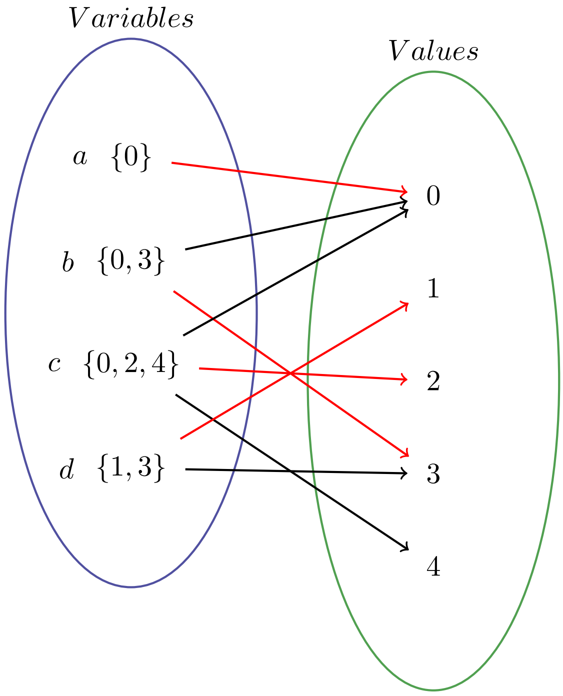
Understanding Régin's Algorithm

Example propagation with Régin's algorithm of contraint $distinct(s)$ where $s = \{a \mapsto \{0\}, b \mapsto \{0, 3\}, c \mapsto \{0, 2, 4\}, d \mapsto \{1, 3\}\}$

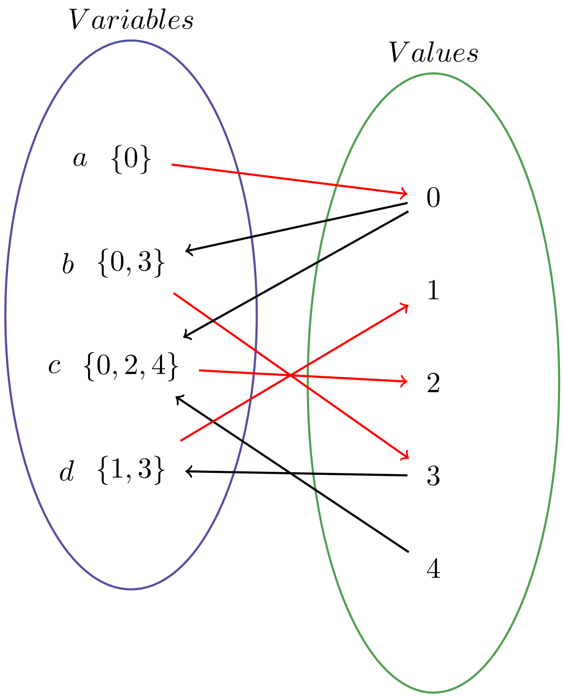
Variable value graph of s:



Find an initial matching:



Orient edges for simplicity:



Find alternating paths and mark edges traversed through the path

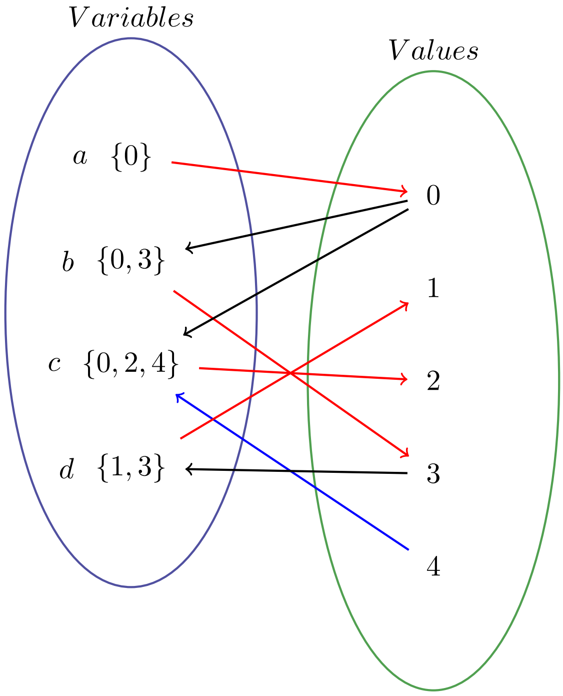
Start from free node and visit one node at a time on the path and we find the following alternating even paths starting at free node:

Free nodes are: $\{4\}$

Even alternating path starting at 4 of length 2:

$4 \rightarrow c \rightarrow 2$

Mark edge $(4, c)$ (edge $(c, 2)$ already part of matching):

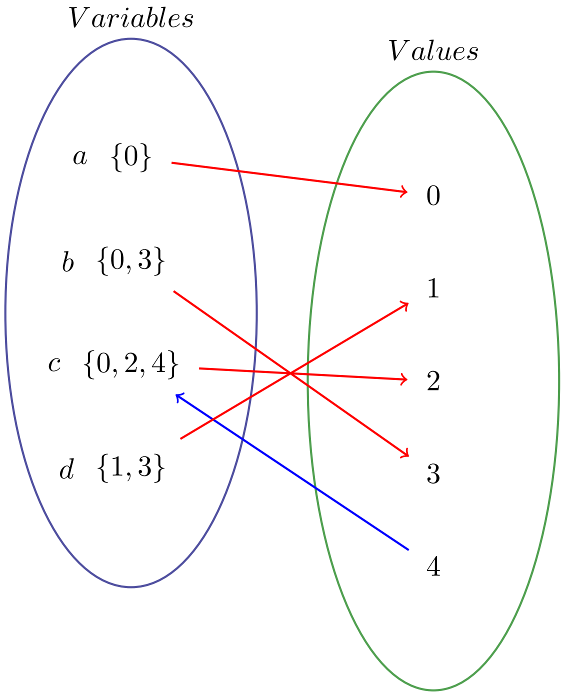


Find even alternating cycles and mark edges through the path

There are no such cycles in this graph.

The cycles can be computed by first finding strongly connected components in the graph (strongly connected component is a component where there are pairwise paths from every node in the component). Mark the edges connecting the component. (Notice that there is no restriction of cycles starting at free nodes so that's why this technique will mark all edges in all even alternating cycles).

Remove all unmarked edges from the graph:



Now the algorithm is terminated and we have computed the store as result of propagation:

$p(s) = s' = \{a \mapsto \{0\} b \mapsto \{3\} c \mapsto \{2, 4\} d \mapsto \{1\}\}$