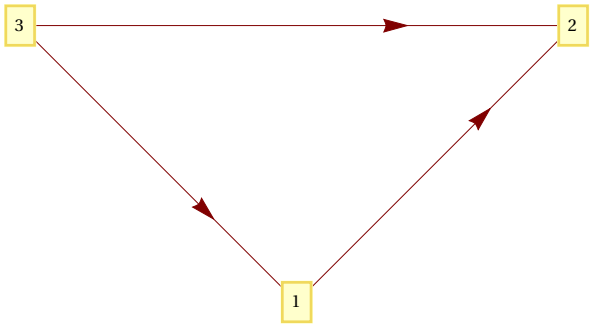
HITS Linear Algebra Formulation

The Example Graph

In[271]:=
```
g = {1 → 2, 3→2 , 3→1};
LayeredGraphPlot[g, Left, VertexLabeling→True]
```

Out[272]=



The Adjacency/Link Matrix

In[273]:=
```
A = AdjacencyMatrix[g];
MatrixForm[A];
MatrixForm[Transpose[A]];
```

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

A

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

A^T

Initialization

In[276]:=
```
n = VertexCount[g];
a = ConstantArray[1/Sqrt[n],n];
h = ConstantArray[1/Sqrt[n],n];
MatrixForm[a];
MatrixForm[h];
```

$$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

a

$$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

h

Iterative Recurrence Formula

$$h = \lambda \mathrm{A}.a$$

$$a = \mu \mathrm{A}^T.h$$

$$\lambda = \frac{1}{\sum_i h_i}$$

$$\mu = \frac{1}{\sum_i a_i}$$

In[281]:=
```
iterations = 100;
For[i=0,i<iterations,i++,
lambda = 1/Total[h];
mu = 1/Total[a];
h = lambda*N[A.a];
a = mu*N[Transpose[A].h];
];
```

The Resulting Authority and Hub scores

In[283]:=
```
MatrixForm[a];
MatrixForm[h];
```

$$\begin{pmatrix} 0.366514 \\ 0.593032 \\ 0. \end{pmatrix}$$

a

$$\begin{pmatrix} 0.384773 \\ 0. \\ 0.622576 \end{pmatrix}$$

h

Comparison With Mathematica's Built-in Function

In[285]:=
```
{a1,h1} = HITSCentrality[g];
MatrixForm[a1];
MatrixForm[h1];
```

$$\begin{pmatrix} 0.381966 \\ 0.618034 \\ 0. \end{pmatrix}$$

a

$$\begin{pmatrix} 0.618034 \\ 0. \\ 1. \end{pmatrix}$$

h

Comparison with Principal EigenVectors

a should be the principal eigenvector of $A^T A$ and h should be the principal eigenvector of AA^T

In[288]:=
```
{AV1, AEV1} = Eigensystem[Transpose[A].A,1];
{HV1, HEV1} = Eigensystem[A.Transpose[A],1];
MatrixForm[AEV1];
MatrixForm[HEV1];
```

$$\left(\frac{1}{2}\left(-1+\sqrt{5}\right) \quad 1 \quad 0 \right)$$

Principal Eigenvector of $A^T A$

$$\left(\frac{1}{2}\left(-1+\sqrt{5}\right) \quad 0 \quad 1 \right)$$

Principal Eigenvector of AA^T

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