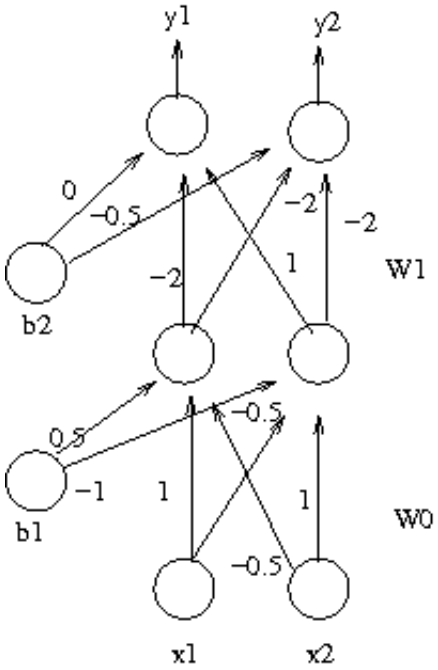
Neural Network Computation

The Example Network

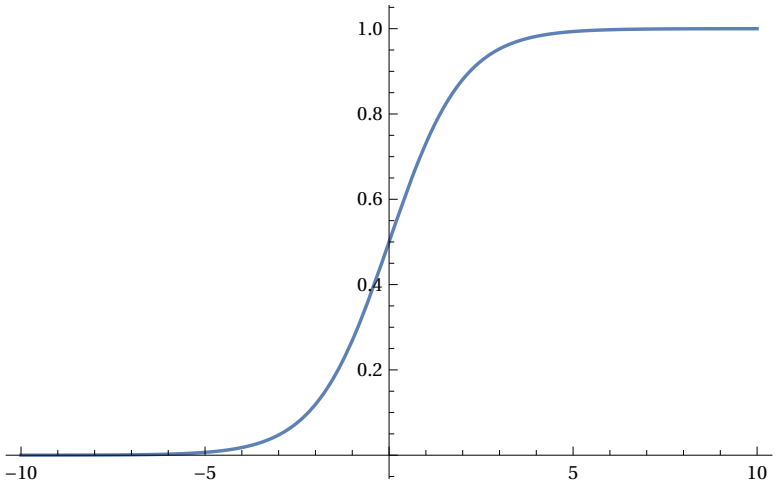
In[817]:=
```
labels = {{1,1}};
x = {{1,1}};
W0 = {{1,-0.5},{-0.5,1}};
W1 = {{-2,-2}, {1, -2}};
b1 = {-0.5, -1};
b2 = {0,-0.5};
```



Example Feed-Forward Neural network with initialized weights.

Hidden and Output Unit Activations

In[824]:=
```
S[x_]:= 1/(1 + E^(-x));
Plot[S[x], {x,-10,10}];
```



Sigmoid activation function

First Layer Computations

In[826]:=
```
z0 = Transpose[W0].Transpose[x] + b1;
a0 = S[z0];
```

$$\begin{pmatrix} 0. \\ -0.5 \end{pmatrix}$$

$$z_0 = W_0^{\mathsf{T}}\, x + b_1$$

$$\begin{pmatrix} 0.5 \\ 0.377541 \end{pmatrix}$$

$$a_0 = S(z_0)$$

Second Layer Computations

In[828]:=
```
z1 = Transpose[W1].a0 + b2;
a1 = LogisticSigmoid[z1];
```

$$\begin{pmatrix} -0.622459 \\ -2.25508 \end{pmatrix}$$

$$z0 = W_1^{\mathsf{T}}\, a_0 + b_2$$

$$\begin{pmatrix} 0.349222 \\ 0.0949121 \end{pmatrix}$$

$$a_1 = S(z_1)$$

Error

In[830]:=
```
Err = RootMeanSquare[(Transpose[a1] - y)]^2;
```

$$\begin{pmatrix} 0.423512 \\ 0.819184 \end{pmatrix}$$

$$\text{Err} = \sum_i (a_{1_i} - y_i)^2$$

Derivatives

$$\frac{\partial E}{\partial a_1} = (a_1 - y)$$

$$\frac{\partial a_1}{\partial z_1} = S(z_1) * (1 - S(z_1))$$

$$\frac{\partial z_1}{\partial W_1} = a_0$$

$$\frac{\partial z_1}{\partial b_2} = 1$$

$$\frac{\partial z_1}{\partial a_0} = W_1$$

$$\frac{\partial a_0}{\partial z_0} = S(z_0) * (1 - S(z_0))$$

$$\frac{\partial z_0}{\partial W_0} = x$$

$$\frac{\partial z_0}{\partial b_1} = 1$$

In[831]:=
```
dEa1 = (Transpose[y] - a1);
da1z1 = S[z1] * (1-S[z1]);
dz1W1 = a0;
dz1b2 = 1;
dz1a0 = W1;
da0z0 = S[z0] * (1-S[z0]);
dz0W0 = x;
dz0b1 = 1;
```

$$\frac{\partial E}{\partial W_1} = \frac{\partial E}{\partial a_1} * \frac{\partial a_1}{\partial z_1} \otimes \frac{\partial z_1}{\partial W_1}$$

$$\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial a_1} * \frac{\partial a_1}{\partial z_1} * \frac{\partial z_1}{\partial b_2}$$

$$\frac{\partial E}{\partial W_0} = \frac{\partial E}{\partial a_1} * \frac{\partial a_1}{\partial z_1} * \frac{\partial z_1}{\partial a_0} * \frac{\partial a_0}{\partial z_0} \otimes \frac{\partial z_0}{\partial W_0}$$

$$\frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial a_1} * \frac{\partial a_1}{\partial z_1} * \frac{\partial z_1}{\partial a_0} * \frac{\partial a_0}{\partial z_0} * \frac{\partial z_0}{\partial b_1}$$

```
dEW1 = Outer[Times, ArrayReshape[dEa1*da1z1, {2}], ArrayReshape[Transpose[dz1W1],{2}]];
dEb2 = dEa1*da1z1*dz1b2;
dEW0 = Outer[Times, ArrayReshape[dz1a0.dEa1*da1z1*da0z0, {2}], ArrayReshape[dz0W0,{2}]];
dEb1 = dz1a0.dEa1*da1z1*da0z0*dz0b1;
```

$$\begin{pmatrix} 0.0739498 & 0.0558382 \\ 0.0388752 & 0.029354 \end{pmatrix}$$

$$\frac{\partial E}{\partial W_1}$$

$$\begin{pmatrix} 0.1479 \\ 0.0777505 \end{pmatrix}$$

$$\frac{\partial E}{\partial b_2}$$

$$\begin{pmatrix} -0.176798 & -0.176798 \\ -0.0234056 & -0.0234056 \end{pmatrix}$$

$$\frac{\partial E}{\partial W_0}$$

$$\begin{pmatrix} -0.176798 \\ -0.0234056 \end{pmatrix}$$

$$\frac{\partial E}{\partial b_1}$$

Weight Updates

$$\eta = 0.1$$

$$\Delta_{w_1} = -\eta \frac{\partial E}{\partial W_1}$$

$$\Delta_{b_2} = -\eta \frac{\partial E}{\partial b_2}$$

$$\Delta_{w_0} = -\eta \frac{\partial E}{\partial W_0}$$

$$\Delta_{b_1} = -\eta \frac{\partial E}{\partial b_1}$$

In[843]:=
```
n = 0.1;
W1 = W1 - n*dEW1;
b2 = b2 - n*dEb2;
W0 = W0 - n*dEW0;
b1 = b1 - n*dEb1;
```

$$\begin{pmatrix} -2.00739 & -2.00558 \\ 0.996112 & -2.00294 \end{pmatrix}$$

W_1

$$\begin{pmatrix} -0.01479 \\ -0.507775 \end{pmatrix}$$

b_2

$$\begin{pmatrix} 1.01768 & -0.48232 \\ -0.497659 & 1.00234 \end{pmatrix}$$

W_0

$$\begin{pmatrix} -0.48232 \\ -0.997659 \end{pmatrix}$$

b_1

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