Khulna University of Engineering and Technology
Department of Computer Science and Engineering

Report On
"Codes for Limited Magnitude Error Correction in Multilevel Cell Memories"

Course Title: Soft Error Tolerance

Course Code: CSE 6581

Submitted To:

Dr. Md. Sheikh Sadi
Professor
Department of CSE

Submitted By:

Syed Mahathir Hasan
Roll No: 1907559
Department of CSE

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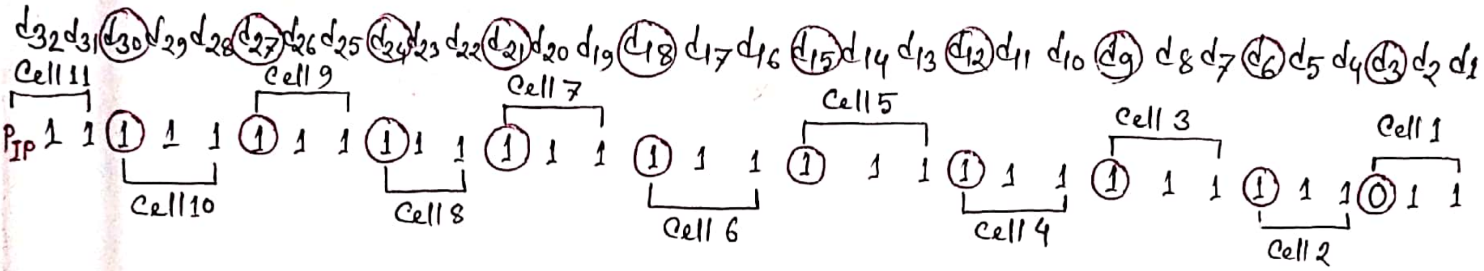
Combination of Interleaved Parity and SEC-DAEC Method (IP-DAEC)

As we were working with a 32 bit memory cell so the taken data length was 32. Each data cell consists of 3 bits, that means for 32 bit data there will be 11 data cells.

Input Data:



Interleaved Parity Calculation (P_{IP}):

Each cell consists of 3 bits. We will take "3rd bit" from each cell and "XOR" them to calculate P_{IP}.

$$\therefore P_{IP} = d_3 \oplus d_6 \oplus d_9 \oplus d_{12} \oplus d_{15} \oplus d_{18} \oplus d_{21} \oplus d_{24} \oplus d_{27} \oplus d_{30}$$

SEC-DAEC Parity Calculation:

Already we have taken 10 bits ($d_3, d_6, d_9, d_{12}, d_{15}, d_{18}, d_{21}, d_{24}, d_{27}, d_{30}$) from 32 bit data word for P_{IP} calculation.

$$\therefore \text{Remaining bits} = 32 \text{ bits} - 10 \text{ bits}$$
$$= 22 \text{ bits}.$$

By using these 22 bits, we will calculate the SEC-DAEC Parity (P_1 to P_6) with the help of (28, 22) H matrix. For 22 bit data 6 parity bits are needed. This is mentioned in "Table I" of this paper.

$$\begin{array}{c} \\ P_1 \leftarrow \\ P_2 \leftarrow \\ P_3 \leftarrow \\ P_4 \leftarrow \\ P_5 \leftarrow \\ P_6 \leftarrow \end{array} \quad H = \begin{array}{r} \begin{array}{cccccccccccccccccccccc} d_{32} & d_{31} & d_{29} & d_{28} & d_{26} & d_{25} & d_{23} & d_{22} & d_{20} & d_{19} & d_{17} & d_{16} & d_{14} & d_{13} & d_{11} & d_{10} & d_{8} & d_{7} & d_{5} & d_{4} & d_{2} & d_{1} \end{array} \\ \left[\begin{array}{cccccc|cccccccccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$$

Figure: H matrix of the (28,22) SEC-DAEC code

From here,

$$P_1 = d_2 \oplus d_4 \oplus d_5 \oplus d_7 \oplus d_{14} \oplus d_{16} \oplus d_{20} \oplus d_{22} \oplus d_{28} \oplus d_{29} \oplus d_{31} \oplus d_{32} \oplus d_{17}$$

$$P_2 = d_1 \oplus d_2 \oplus d_5 \oplus d_7 \oplus d_{11} \oplus d_{13} \oplus d_{14} \oplus d_{19} \oplus d_{22} \oplus d_{23} \oplus d_{25} \oplus d_{26} \oplus d_{28}$$

$$P_3 = d_2 \oplus d_5 \oplus d_7 \oplus d_8 \oplus d_{10} \oplus d_{19} \oplus d_{20} \oplus d_{23} \oplus d_{28} \oplus d_{32}$$

$$P_4 = d_2 \oplus d_7 \oplus d_{10} \oplus d_{13} \oplus d_{16} \oplus d_{20} \oplus d_{23} \oplus d_{26} \oplus d_{29}$$

$$P_5 = d_1 \oplus d_4 \oplus d_8 \oplus d_{11} \oplus d_{14} \oplus d_{17}$$

$$P_6 = d_1 \oplus d_4 \oplus d_7 \oplus d_{10} \oplus d_{13} \oplus d_{16} \oplus d_{19} \oplus d_{22} \oplus d_{25} \oplus d_{28} \oplus d_{31}$$

SEC-DAEC Syndrome S_1 S_2 S_3 S_4 S_5 S_6	Single Error (SE)
0 0 0 0 0 0	No errore
0 1 0 0 1 1	d_1
1 1 1 1 0 0	d_2
1 0 0 0 1 1	d_4
1 1 1 0 0 0	d_5
1 1 1 1 0 1	d_7
0 0 1 0 1 0	d_8
0 0 1 1 0 1	d_{10}
0 1 0 0 1 0	d_{11}
0 1 0 1 0 1	d_{13}
1 1 0 0 1 0	d_{14}
1 0 0 1 0 1	d_{16}
0 0 0 0 1 0	d_{17}
0 1 1 0 0 1	d_{19}
1 0 1 1 0 0	d_{20}
1 1 0 0 0 1	d_{22}
0 1 1 1 0 0	d_{23}
0 1 0 0 0 1	d_{25}
0 1 0 1 0 0	d_{26}
1 1 1 0 0 1	d_{28}
1 0 0 1 0 0	d_{29}
1 0 0 0 0 1	d_{31}
1 0 1 0 0 0	d_{32}
	Double Adjacent Error (DAE)
1 0 1 1 1 1	d_1 and d_2
0 1 1 0 1 1	d_4 and d_5
1 1 0 1 1 1	d_7 and d_8
0 1 1 1 1 1	d_{10} and d_{11}
1 0 0 1 1 1	d_{13} and d_{14}
0 0 0 1 1 1	d_{16} and d_{17}
1 1 0 1 0 1	d_{19} and d_{20}
1 0 1 1 0 1	d_{22} and d_{23}
0 0 0 1 0 1	d_{25} and d_{26}
0 1 1 1 0 1	d_{28} and d_{29}
0 0 1 0 0 1	d_{31} and d_{32}

Scanned with CamScanner

Input Data:

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

For this data SEC-DAEC Parity ($P_1 P_2 P_3 P_4 P_5 P_6$) is = 1 1 0 1 0 1

and Interleaved Parity P_{IP} = 1

For P_{IP} calculation 10 bits are used so remaining bits = 32 − 10 = 22 bits.

Remaining bits are –

d_{32}	d_{31}	d_{29}	d_{28}	d_{26}	d_{25}	d_{23}	d_{22}	d_{20}	d_{19}	d_{17}	d_{16}	d_{14}	d_{13}	d_{11}	d_{10}	d_8	d_7	d_5	d_4	d_2	d_1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Single Error (SE):

Error On "d_1":

$P_1 P_2 P_3 P_4 P_5 P_6$ = 1 1 0 1 0 1

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 0 1 0 0 1 1, refers to the first column i.e "d_1" column. d_1 was 1 at the beginning. After error it became "0". Now this "0" will be flipped to "1" to get the real value.

Error On "d_2":

$P_1 P_2 P_3 P_4 P_5 P_6$ = 1 1 0 1 0 1.

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 1 1 1 1 0 0, refers the 2nd column i.e "d_2" column of H matrix. 0 will be flipped to 1.

Error On "d_4":

$P_1 P_2 P_3 P_4 P_5 P_6$ = 1 1 0 1 0 1

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 1 0 0 0 1 1, refers the "d_4" column of H matrix and 0 will be flipped to 1.

Error On "d_5":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 1\ 1\ 0\ 0\ 0$ refers to the "d_5" column of the H matrix and 0 will be flipped to 1.

Error On "d_7":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 1\ 1\ 1\ 0\ 1$ refers to the "d_7" column of the H matrix and 0 will be flipped to 1.

Error On "d_8":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 0\ 1\ 0\ 1\ 0$ refers to the "d_8" column of the H matrix and 0 will be flipped to 1.

Error On "d_{10}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 0\ 1\ 1\ 0\ 1$ refers to the "d_{10}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{11}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 0\ 0\ 1\ 0$ refers to the "d_{11}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{13}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 0\ 1\ 0\ 1$ refers to the "d_{13}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{14}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 1\ 0\ 0\ 1\ 0$ refers to the "d_{14}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{16}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 0\ 1\ 0\ 1$ refers to the "d_{16}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{17}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 0\ 0\ 1\ 0$ refers to the "d_{17}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{19}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 1\ 0\ 0\ 1$ refers to the "d_{19}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{20}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 1\ 1\ 0\ 0$ refers to the "d_{20}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{22}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 1\ 0\ 0\ 0\ 1$ refers to the "d_{22}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{23}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\,1\,1\,1\,0\,0$ refers to the "d_{23}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{25}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\,1\,0\,0\,0\,1$ refers to the "d_{25}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{26}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\,1\,0\,1\,0\,0$ refers to the "d_{26}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{28}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\,1\,1\,0\,0\,1$ refers to the "d_{28}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{29}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\,0\,0\,1\,0\,0$ refers to the "d_{29}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{31}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\,1\,0\,1\,0\,1$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\,0\,0\,0\,0\,1$ refers to the "d_{31}" column of the H matrix and 0 will be flipped to 1.

Error On "d_{32}":

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 1\ 0\ 0\ 0$ refers to the "d_{32}" column of the H matrix and 0 will be flipped to 1.

Double Adjacent Error (DAE):

Input Data:

d_{32}	d_{31}	d_{30}	d_{29}	d_{28}	d_{27}	d_{26}	d_{25}	d_{24}	d_{23}	d_{22}	d_{21}	d_{20}	d_{19}	d_{18}	d_{17}	d_{16}	d_{15}	d_{14}	d_{13}	d_{12}	d_{11}	d_{10}	d_9	d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1

Here, Interleaved Parity, $P_{IP} = 1$

SEC-DAEC Parity, $P_1\ P_2\ P_3\ P_4\ P_5\ P_6 = 1\ 1\ 0\ 1\ 0\ 1$.

DAE on d_1 & d_2:

$d_2\ d_1 = 1\ 1$ after error $d_2\ d_1 = 0\ 0$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 1\ 1\ 1\ 1$ refers the XOR operation between "d_1" and "d_2" column in the H matrix. $d_2\ d_1 = 0\ 0$ will be $d_2\ d_1 = 1\ 1$ by bit flipping.

DAE on "d_1 & d_2" and affects "d_3":

$d_3\ d_2\ d_1 = 0\ 1\ 1$ after error $d_3\ d_2\ d_1 = 1\ 0\ 0$

Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 1\ 1\ 1\ 1$ refers the XOR operation between "d_1" and "d_2" column.

IP Syndrome, $SIP = 1$

Error Correction:

Error Pattern $\oplus$ SIP value $= 1\ 0\ 0 \oplus 1 = 0\ 1\ 1$

DAE of d_4 & d_5 :

$d_5\, d_4 = 1\,1$ after error $d_5\, d_4 = 0\,0$.

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6 = 0\,1\,1\,0\,1\,1$ refers to the XOR operation between d_5 and d_4 column of the H matrix. $d_5\, d_4 = 0\,0$ will be $d_5\, d_4 = 1\,1$ by bit flipping.

DAE on $d_4\, d_5$ and affects d_6 :

$d_6\, d_5\, d_4 = 1\,1\,1$ after error $d_6\, d_5\, d_4 = 0\,0\,0$

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6 = 0\,1\,1\,0\,1\,1$ refers to the XOR operation between d_5 and d_4 column of the H matrix.

IP Syndrome, $SIP = 1$

Error Correction :

Error Pattern $\oplus$ SIP value $= 0\,0\,0 \oplus 1 = 1\,1\,1$.

DAE on d_7 & d_8 :

$d_8\, d_7 = 1\,1$ after error $d_8\, d_7 = 0\,0$

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6 = 1\,1\,0\,1\,1\,1$ refers to the XOR operation between d_8 and d_7 column of the H matrix. $d_8\, d_7 = 0\,0$ will be $d_8\, d_7 = 1\,1$ by bit flipping.

DAE on d_7, d_8 and affects d_9 :

$d_9\, d_8\, d_7 = 1\,1\,1$ after error $d_9\, d_8\, d_7 = 0\,0\,0$

Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6 = 1\,1\,0\,1\,1\,1$ refers to the XOR operation between d_8 and d_7 column of the H matrix.

IP Syndrome, $SIP = 1$

Error Correction :

Error Pattern $\oplus$ SIP value $= 0\,0\,0 \oplus 1 = 1\,1\,1$

DAE on d_{11}, d_{10} :

$d_{11}\ d_{10} = 1\ 1$ after error $d_{11}\ d_{10} = 0\ 0$.
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 1\ 1\ 1\ 1$ refers to the XOR operation between d_{11} and d_{10} column of the H matrix. $d_{11}\ d_{10} = 0\ 0$ will be $d_{11}\ d_{10} = 1\ 1$ by bit flipping.

DAE on d_{11}, d_{10} and affects d_{12} :

$d_{12}\ d_{11}\ d_{10} = 1\ 1\ 1$ after error $d_{12}\ d_{11}\ d_{10} = 0\ 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 1\ 1\ 1\ 1$ refers to the XOR operation between d_{11} and d_{10} column of the H matrix.
IP Syndrome, $SIP = 1$.

Error Correction :

Error Pattern $\oplus$ SIP value $= 0\ 0\ 0 \oplus 1 = 1\ 1\ 1$.

DAE on d_{14}, d_{13} :

$d_{14}\ d_{13} = 1\ 1$ after error $d_{14}\ d_{13} = 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 0\ 1\ 1\ 1$ refers to the XOR operation between d_{14} and d_{13} column of the H matrix. $d_{14}\ d_{13} = 0\ 0$ will be $d_{14}\ d_{13} = 1\ 1$ by bit flipping.

DAE on d_{14}, d_{13} and affects d_{15} :

$d_{15}\ d_{14}\ d_{13} = 1\ 1\ 1$ after error $d_{15}\ d_{14}\ d_{13} = 0\ 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 1\ 0\ 0\ 1\ 1\ 1$ refers to the XOR operation between d_{14} and d_{13} column of the H matrix.
IP Syndrome, $SIP = 1$.

Error Correction :

Error Pattern $\oplus$ SIP value $= 0\ 0\ 0 \oplus 1 = 1\ 1\ 1$.

DAE on d_{17}, d_{16} :

$d_{17}\, d_{16} = 1\,1$ after error $d_{17}\, d_{16} = 0\,0$
Syndrome, $S_1\, S_2\, S_3\, S_4\, S_5\, S_6 = 0\,0\,0\,1\,1\,1$ refers to the XOR operation between d_{17} and d_{16} column of the H matrix. $d_{17}\, d_{16} = 0\,0$ will be $d_{17}\, d_{16} = 1\,1$ by bit flipping.

DAE on d_{17}, d_{16} and affects d_{18} :

$d_{18}\, d_{17}\, d_{16} = 1\,1\,1$ after error $d_{18}\, d_{17}\, d_{16} = 0\,0\,0$
Syndrome, $S_1\, S_2\, S_3\, S_4\, S_5\, S_6 = 0\,0\,0\,1\,1\,1$ refers to the XOR operation between d_{17} and d_{16} column of the H matrix.
IP Syndrome, SIP = 1

Error Correction :

Error pattern $\oplus$ SIP value $= 0\,0\,0 \oplus 1 = 1\,1\,1$

DAE on d_{20}, d_{19} :

$d_{20}\, d_{19} = 1\,1$ after error $d_{20}\, d_{19} = 0\,0$
Syndrome, $S_1\, S_2\, S_3\, S_4\, S_5\, S_6 = 1\,1\,0\,1\,0\,1$ refers to the XOR operation between d_{20} and d_{19} column of the H matrix. $d_{20}\, d_{19} = 0\,0$ will be $d_{20}\, d_{19} = 1\,1$ by bit flipping.

DAE on d_{20}, d_{19} and affects d_{21} :

$d_{21}\, d_{20}\, d_{19} = 1\,1\,1$ after error $d_{21}\, d_{20}\, d_{19} = 0\,0\,0$
Syndrome, $S_1\, S_2\, S_3\, S_4\, S_5\, S_6 = 1\,1\,0\,1\,0\,1$ refers to the XOR operation between d_{20} and d_{19} column of the H matrix.
IP Syndrome, SIP = 1.

Error Correction :

Error Pattern $\oplus$ SIP value $= 0\,0\,0 \oplus 1 = 1\,1\,1$.

DAE on d_{23}, d_{22} :

$d_{23}\ d_{22}$ = 1 1 after error $d_{23}\ d_{22}$ = 0 0
Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 1 0 1 1 0 1 refers to the XOR operation between d_{23} and d_{22} column of the H matrix. $d_{23}\ d_{22}$ = 0 0 will be $d_{23}\ d_{22}$ = 1 1 by bit flipping.

DAE on d_{23}, d_{22} and affects d_{24} :

$d_{24}\ d_{23}\ d_{22}$ = 1 1 1 after error $d_{24}\ d_{23}\ d_{22}$ = 0 0 0
Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 1 0 1 1 0 1 refers to the XOR operation between d_{23} and d_{22} column of the H matrix.
IP Syndrome, SIP = 1.

Error Correction :

Error Pattern ⊕ SIP value = 0 0 0 ⊕ 1 = 1 1 1.

DAE on d_{26}, d_{25} :

$d_{26}\ d_{25}$ = 1 1 after error $d_{26}\ d_{25}$ = 0 0
Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 0 0 0 1 0 1 refers to the XOR operation between d_{26} and d_{25} column of the H matrix. $d_{26}\ d_{25}$ = 0 0 will be $d_{26}\ d_{25}$ = 1 1 by bit flipping.

DAE on d_{26}, d_{25} and affects d_{27} :

$d_{27}\ d_{26}\ d_{25}$ = 1 1 1 after error $d_{27}\ d_{26}\ d_{25}$ = 0 0 0
Syndrome, $S_1 S_2 S_3 S_4 S_5 S_6$ = 0 0 0 1 0 1 refers to the XOR operation between d_{26} and d_{25} column of the H matrix.
IP Syndrome, SIP = 1

Error Correction :

Error Pattern ⊕ SIP value = 0 0 0 ⊕ 1 = 1 1 1

DAE on d_{29}, d_{28} :

$d_{29}\ d_{28} = 1\ 1$ after error $d_{29}\ d_{28} = 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 1\ 1\ 0\ 1$ refers to the XOR operation between d_{29} and d_{28} column of the H matrix. $d_{29}\ d_{28} = 00$ will be $d_{29}\ d_{28} = 1\ 1$ by bit flipping.

DAE on d_{29}, d_{28} and affects d_{30} :

$d_{30}\ d_{29}\ d_{28} = 1\ 1\ 1$ after error $d_{30}\ d_{29}\ d_{28} = 0\ 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 1\ 1\ 1\ 0\ 1$ refers to the XOR operation between d_{29} and d_{28} column of the H matrix.
IP Syndrome, SIP = 1

Error Correction :

Error Pattern $\oplus$ SIP value $= 000 \oplus 1 = 1\ 1\ 1$.

DAE on d_{32}, d_{31} :

$d_{32}\ d_{31} = 1\ 1$ after error $d_{32}\ d_{31} = 0\ 0$
Syndrome, $S_1\ S_2\ S_3\ S_4\ S_5\ S_6 = 0\ 0\ 1\ 0\ 0\ 1$ refers to the XOR operation between d_{32} and d_{31} column of the H matrix.
$d_{32}\ d_{31} = 0\ 0$ will be $d_{32}\ d_{31} = 1\ 1$ by bit flipping.

Uncorrectable Error:

An error will be uncorrectable when SEC-DAEC syndrome bits (S_1 to S_6) are zero but IP syndrome bit (SIP) is non-zero. Our input data was 32 bits. In this 32 bit data, there are 10 bits which are vulnerable to this uncorrectable error. Those bits are - d_3, d_6, d_9, d_{12}, d_{15}, d_{18}, d_{21}, d_{24}, d_{27} and d_{30}. Error in one of these bits are detectable (by SIP value) but not correctable. Decoder will send this received data which is erroneous as output.

Input Data:

d_{30} d_{27} d_{24} d_{21} d_{18} d_{15} d_{12} d_9 d_6 d_3

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Error in d_3:

Output:

d_3

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Erroneous Value

Error in d_6:

Output:

d_6

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1

Erroneous Value

Error in d_9:

Output:

d_9

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{12} :

Output:

d_{12}

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{15} :

Output:

d_{15}

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{18} :

Output:

d_{18}

1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{21} :

Output:

d_{21}

1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value.

Error in d_{24} :

Output:

d_{24}

1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{27} :

Output:

d_{27}

1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Error in d_{30} :

Output: d_{30}

1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1

Erroneous Value

Number of Detected Error :

Single Error detection = 22

Double Adjacent Error detection = 11

Detection error in 3rd bit in specific data cell = 10 (no change in other two bits)

Detection error in 3rd bit wrt other two bits = 10

Total number of detected error = 22 + 11 + 10+10= ~~43~~ 53

Number of Corrected Error :

Can correct all "Single Error (SE)" i.e 22

Can correct all "Double Adjacent Error (DAE) i.e 11.

Can correct 3rd bit error which changed wrt other two bits i.e 10

Total number of corrected error = 22+11+10 = 43.

Redundancy :

$$\text{Redundancy} = \frac{\text{Check bit}}{\text{Data bit}} \times 100\%$$

$$= \frac{7}{32} \times 100\%$$

$$= 21.88\%$$