$$
\begin{aligned}
\boldsymbol{\Sigma}^{-1} &= \left(\boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{\top}\right)^{-1} \\
&= \left(\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\right)^{-1} \\
&= \left(\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \lambda_1\cos\alpha & \lambda_1\sin\alpha \\ -\lambda_2\sin\alpha & \lambda_2\cos\alpha \end{bmatrix}\right)^{-1} \\
&= \left(\begin{bmatrix} \lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha & \lambda_1\cos\alpha\sin\alpha - \lambda_2\cos\alpha\sin\alpha \\ \lambda_1\cos\alpha\sin\alpha - \lambda_2\cos\alpha\sin\alpha & \lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha \end{bmatrix}\right)^{-1} \\
&= \left(\begin{bmatrix} \lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha & (\lambda_1-\lambda_2)\cos\alpha\sin\alpha \\ (\lambda_1-\lambda_2)\cos\alpha\sin\alpha & \lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha \end{bmatrix}\right)^{-1} \\
&= \frac{\begin{bmatrix} \lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha & (\lambda_2-\lambda_1)\cos\alpha\sin\alpha \\ (\lambda_2-\lambda_1)\cos\alpha\sin\alpha & \lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha \end{bmatrix}}{\det\left\{\begin{bmatrix} \lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha & (\lambda_2-\lambda_1)\cos\alpha\sin\alpha \\ (\lambda_2-\lambda_1)\cos\alpha\sin\alpha & \lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha \end{bmatrix}\right\}}
\end{aligned}
$$

$$
\begin{aligned}
&\det\left\{\begin{bmatrix} \lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha & (\lambda_2-\lambda_1)\cos\alpha\sin\alpha \\ (\lambda_2-\lambda_1)\cos\alpha\sin\alpha & \lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha \end{bmatrix}\right\} \\
&= \left(\lambda_1\sin^2\alpha + \lambda_2\cos^2\alpha\right)\left(\lambda_1\cos^2\alpha + \lambda_2\sin^2\alpha\right) - (\lambda_2-\lambda_1)^2\cos^2\alpha\sin^2\alpha \\
&= \left(\lambda_1^2+\lambda_2^2\right)\sin^2\alpha\cos^2\alpha + \lambda_1\lambda_2\left(\sin^4\alpha+\cos^4\alpha\right) - \left(\lambda_1^2 - 2\lambda_1\lambda_2 + \lambda_2^2\right)\cos^2\alpha\sin^2\alpha \\
&= \lambda_1\lambda_2\left(\sin^4\alpha+\cos^4\alpha\right) + 2\lambda_1\lambda_2\cos^2\alpha\sin^2\alpha \\
&= \lambda_1\lambda_2\left(\sin^4\alpha + 2\cos^2\alpha\sin^2\alpha + \cos^4\alpha\right) \\
&= \lambda_1\lambda_2\left(\sin^2\alpha+\cos^2\alpha\right)^2 \\
&= \lambda_1\lambda_2
\end{aligned}
$$

$$\Downarrow$$

$$
\boxed{\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \frac{1}{\lambda_2}\sin^2\alpha + \frac{1}{\lambda_1}\cos^2\alpha & \frac{(\lambda_2-\lambda_1)}{\lambda_1\lambda_2}\cos\alpha\sin\alpha \\ \frac{(\lambda_2-\lambda_1)}{\lambda_1\lambda_2}\cos\alpha\sin\alpha & \frac{1}{\lambda_2}\cos^2\alpha + \frac{1}{\lambda_1}\sin^2\alpha \end{bmatrix}}
$$

$$
f(\boldsymbol{X}) \propto \exp\left\{-\frac{1}{2}(\boldsymbol{X}-\boldsymbol{\mu})^{\top}\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})\right\}
$$

$$\Downarrow$$

$$
\boxed{f\left(\begin{bmatrix} x_i \\ y_i \end{bmatrix}\right) \propto \exp\left\{-\frac{1}{2}\left(\begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} x_{\text{center}} \\ y_{\text{center}} \end{bmatrix}\right)^{\top}\boldsymbol{\Sigma}^{-1}\left(\begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} x_{\text{center}} \\ y_{\text{center}} \end{bmatrix}\right)\right\}}
$$