Recovering large-scale incomplete traffic speed via tensor completion

Xinyu Chen, Master's student
Zhaocheng He, professor
Jiawei Wang, Master's student
Yuhuan Lu, Master's student

Research Center of Intelligent Transportation System
Sun Yat-Sen University

Background

- Traffic data can be collected from a number of mobile and stationary sensors nowadays.
- However, the problem of *missing data* is inevitable due to communication malfunctions and transmission distortions.
- ***Example***
  - *Traffic volume matrix with five sensors and four 15-minute time windows.*

$$X=\begin{bmatrix} ? & 99 & 449 & 517 \\ ? & ? & 412 & ? \\ 192 & ? & 697 & 687 \\ 185 & ? & 699 & 657 \\ 164 & 68 & ? & ? \end{bmatrix}\in\mathbb{R}^{5\times 4}$$

- ***Question***
  - *How to estimate unknown $x_{11},x_{21},x_{22},\cdots,x_{53},x_{54}$ according to the observations $x_{12}=99,x_{13}=449,x_{14}=517,\cdots,x_{51}=164,x_{52}=68$?*

Matrix Decomposition

- ***Model***
  - *For (i,j)-th entry of $X \in \mathbb{R}^{m\times n}$, if we assume*

k-th factor related to the *i-th sensor*

$$x_{ij} \approx \sum_{k=1}^{r} u_{ik} v_{jk}$$

k-th factor related to the *j-th time window*

then the ***factor matrices*** $U \in \mathbb{R}^{m\times r}$ and $V \in \mathbb{R}^{n\times r}$ can be learned by solving

$$J = \frac{1}{2}\|S * (X - UV^T)\|_F^2$$

an *indicator* for observed entries

- ***With regularization***

$$J = \frac{1}{2}\|S * (X - UV^T)\|_F^2 + \frac{\lambda}{2}(\|U\|_F^2 + \|V\|_F^2)$$

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- $*$: the *Hadamard product* of matrices (or tensors) with same size.
- $\|\cdot\|_F^2$: the sum of squared entries.
- λ: the parameter of regularization term.

Matrix Decomposition

- ***A simple example***

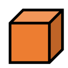
$$\begin{bmatrix} 15 & 10 \\ 7 & 12 \\ \boxed{\mathbf{12} \quad \mathbf{20}} \\ 9 & 21 \\ 8 & 18 \end{bmatrix} \times \begin{bmatrix} \boxed{\begin{matrix}\mathbf{4} \\ \mathbf{7}\end{matrix}} & 5 & 11 & 20 \\ & 1 & 28 & 23 \end{bmatrix} = \begin{bmatrix} ? & 85 & 445 & 530 \\ ? & ? & 413 & ? \\ \mathbf{188} & ? & 692 & 700 \\ 183 & ? & 687 & 663 \\ 158 & 58 & ? & ? \end{bmatrix}$$

$$\begin{bmatrix} \boxed{\mathbf{15} \quad \mathbf{10}} \\ 7 & 12 \\ 12 & 20 \\ 9 & 21 \\ 8 & 18 \end{bmatrix} \times \begin{bmatrix} 4 & \boxed{\begin{matrix}\mathbf{5} \\ \mathbf{1}\end{matrix}} & 11 & 20 \\ 7 & & 28 & 23 \end{bmatrix} = \begin{bmatrix} ? & \mathbf{85} & 445 & 530 \\ ? & ? & 413 & ? \\ 188 & ? & 692 & 700 \\ 183 & ? & 687 & 663 \\ 158 & 58 & ? & ? \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15 & 10 \\ 7 & 12 \\ \boxed{\mathbf{12} \quad \mathbf{20}} \\ 9 & 21 \\ 8 & 18 \end{bmatrix} \times \begin{bmatrix} 4 & \boxed{\begin{matrix}\mathbf{5} \\ \mathbf{1}\end{matrix}} & 11 & 20 \\ 7 & & 28 & 23 \end{bmatrix} = \begin{bmatrix} ? & \mathbf{85} & 445 & 530 \\ ? & ? & 413 & ? \\ \mathbf{188} & \mathbf{80} & 692 & 700 \\ 183 & ? & 687 & 663 \\ 158 & 58 & ? & ? \end{bmatrix}$$

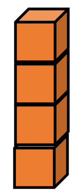
What is tensor?

- ***Intuition***: $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_d}$



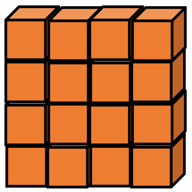
Scalar

$d = 1$



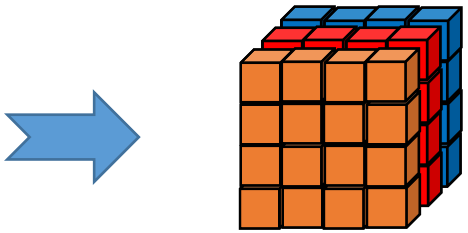
Vector
(first-order tensor)

$d = 2$



Matrix
(second-order tensor)

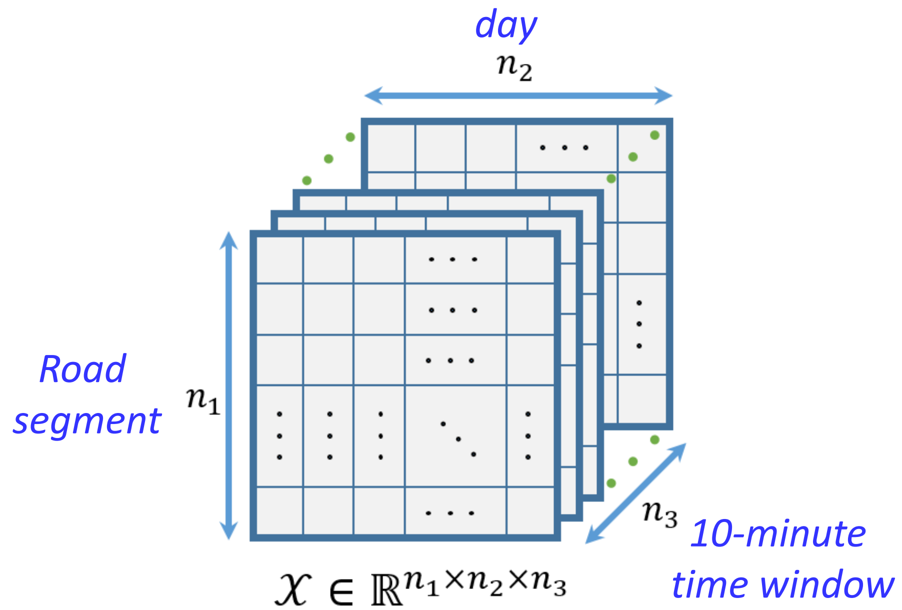
$d = 3$



Third-order tensor
(arranged by slices/matrices)

What is tensor?

- ***Traffic speed dataset***
  - *214 road segments*
  - *61 days (from Aug. 1 to Sep. 30, 2016)*
  - *144 10-minute time windows*
  - *Million scale & city-wide*: $214 \times 61 \times 144 \approx 1.88 \times 10^6$



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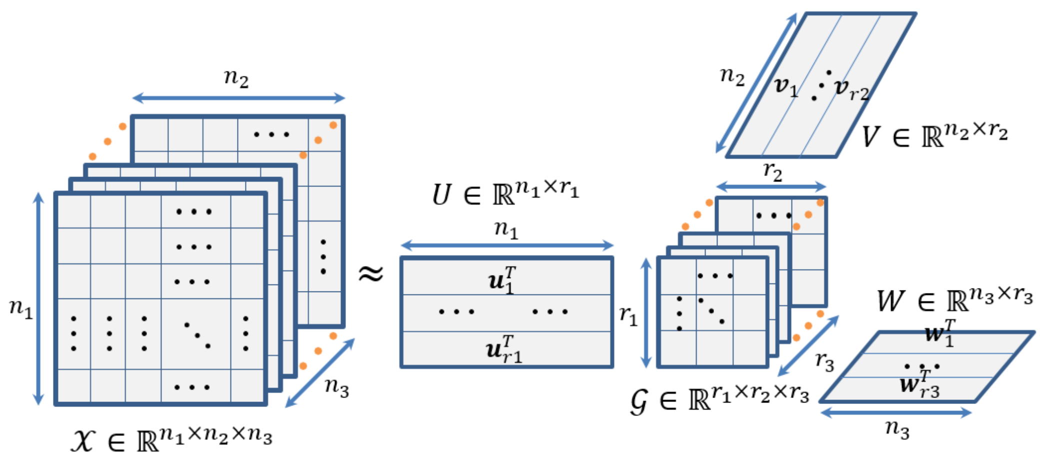
The traffic speed dataset is released by the *Communications Commission of Guangzhou Municipality, China*.

Tensor Decomposition

- ***Tucker Decomposition (Tukcer, 1966)***
  - *Decomposes a given tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ into a core tensor $\mathcal{G} \in \mathbb{R}^{r_1 \times r_2 \times r_3}$ and factor matrices $U \in \mathbb{R}^{n_1 \times r_1}$, $V \in \mathbb{R}^{n_2 \times r_2}$ and $W \in \mathbb{R}^{n_3 \times r_3}$ in a sequence.*

$$\mathcal{X} \approx \mathcal{G} \times_1 U \times_2 V \times_3 W$$



- ***Non-constraint optimization*** Decomposition based approximation

an *indicator* for observed entries

$$J = \frac{1}{2} \| \mathcal{S} * (\mathcal{X} - \mathcal{G} \times_1 U \times_2 V \times_3 W) \|_F^2 + \frac{\lambda}{2} (\|\mathcal{G}\|_F^2 + \|U\|_F^2 + \|V\|_F^2 + \|W\|_F^2)$$

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- $\times_q$: the *tensor-matrix multiplication* or *modal-q product* between tensor and matrix.
- λ: the parameter of regularization term.

Tensor Decomposition

- ***Non-constraint optimization***

an *indicator* for observed entries

$$J=\frac{1}{2}\|\mathcal{S}*(\mathcal{X}-\mathcal{G}\times_1 U\times_2 V\times_3 W)\|_F^2+\frac{\lambda}{2}(\|\mathcal{G}\|_F^2+\|U\|_F^2+\|V\|_F^2+\|W\|_F^2)$$

- ***Gradient Descent Method (GDM)***

$$U\Leftarrow(1-\alpha\lambda)U+\alpha(\mathcal{S}*\mathcal{E})_{(1)}(W\otimes V)\mathcal{G}_{(1)}^T$$

$$V\Leftarrow(1-\alpha\lambda)V+\alpha(\mathcal{S}*\mathcal{E})_{(2)}(W\otimes U)\mathcal{G}_{(2)}^T$$

$$W\Leftarrow(1-\alpha\lambda)W+\alpha(\mathcal{S}*\mathcal{E})_{(3)}(V\otimes U)\mathcal{G}_{(3)}^T$$

$$\mathcal{G}\Leftarrow(1-\alpha\lambda)\mathcal{G}+\alpha\cdot\mathcal{E}\times_1 U^T\times_2 V^T\times_3 W^T$$

- ***Challenge*: *non-convex optimization!***
  - *For any (i,j,k)-th entry, the estimation is formulated by*

$$x_{ijk}\approx\sum_{m=1}^{r_1}\sum_{n=1}^{r_2}\sum_{l=1}^{r_3}g_{mnl}u_{im}v_{jn}w_{kl}$$

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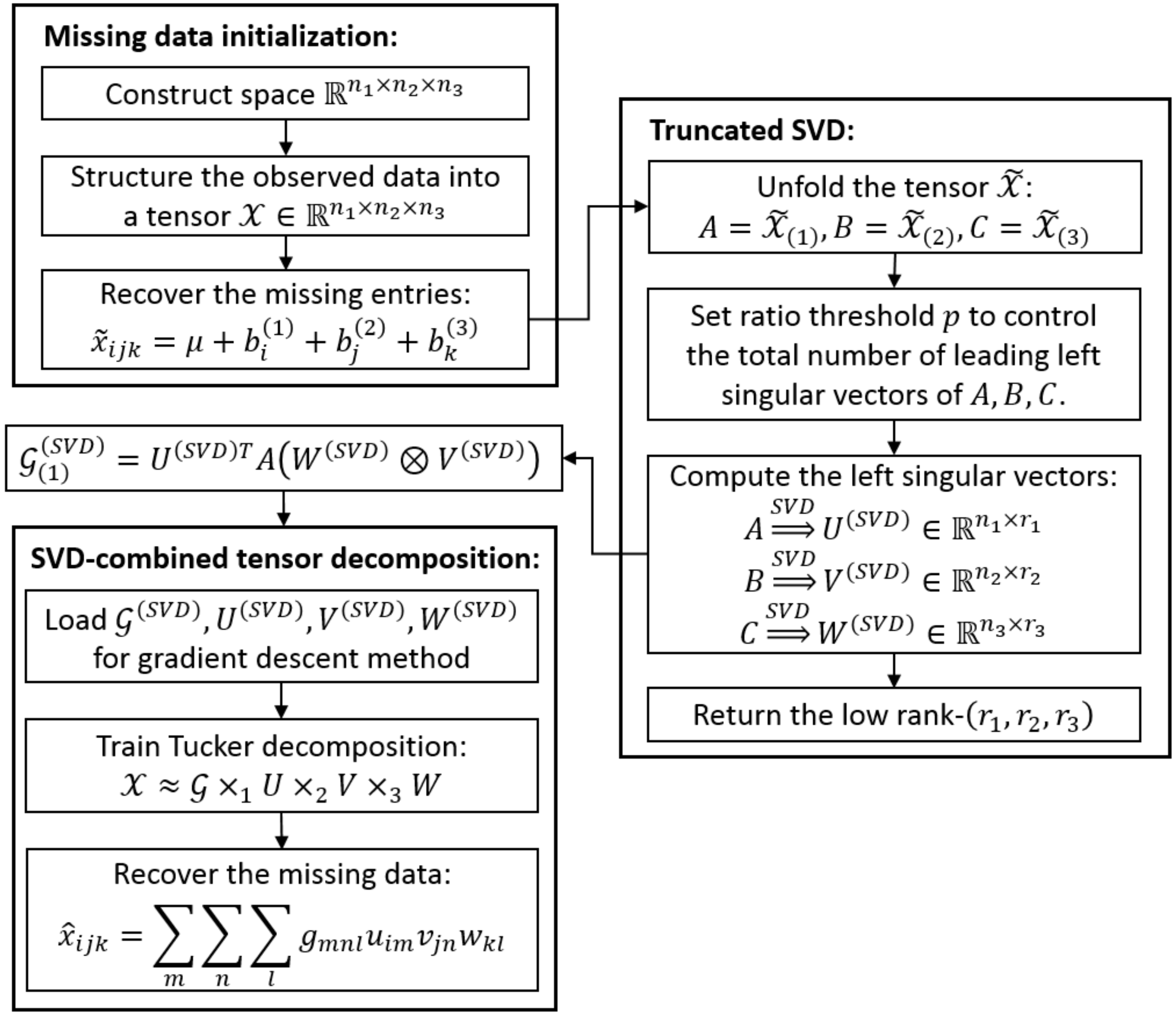
- $\otimes$: Kronecker product.
- α: *learning rate* of gradient descent method.
- $\mathcal{X}_{(q)}$: mode-q unfolding of tensor $\mathcal{X}$.

Modeling framework



- $b_i^{(1)}, b_j^{(2)}, b_k^{(3)}$: *biases* of any x_{ijk} over μ (mean) along each mode.
- ***SVD***: singular value decomposition.

HaLRTC

- ***High accuracy low rank tensor completion (HaLRTC, Liu et al., 2013)***
  - ***Low rank matrix completion*** (*the minimum rank solution can be recovered by solving a convex problem, i.e., the minimization of the trace norm*)

Non-convex

$$\min_{\hat{X}} \operatorname{rank}(\hat{X}) \quad \text{s.t.}\ S * \hat{X} = S * X$$

$\Leftrightarrow$

Convex

$$\min_{\hat{X}} \|\hat{X}\|_* \quad \text{s.t.}\ S * \hat{X} = S * X$$

Trace norm: the sum of singular values

  - ***Low rank tensor completion***

$$\min_{\hat{\mathcal{X}},\mathcal{B}_1,\mathcal{B}_2,\mathcal{B}_3} \frac{1}{3}\left(\|\mathcal{B}_{1(1)}\|_* + \|\mathcal{B}_{2(2)}\|_* + \|\mathcal{B}_{3(3)}\|_*\right)$$

$$\text{s.t.} \begin{cases} \mathcal{S} * \hat{\mathcal{X}} = \mathcal{S} * \mathcal{X} \\ \hat{\mathcal{X}} = \mathcal{B}_q, q = 1,2,3 \end{cases}$$

  - *The rank is a powerful tool to capture some types of global information.*

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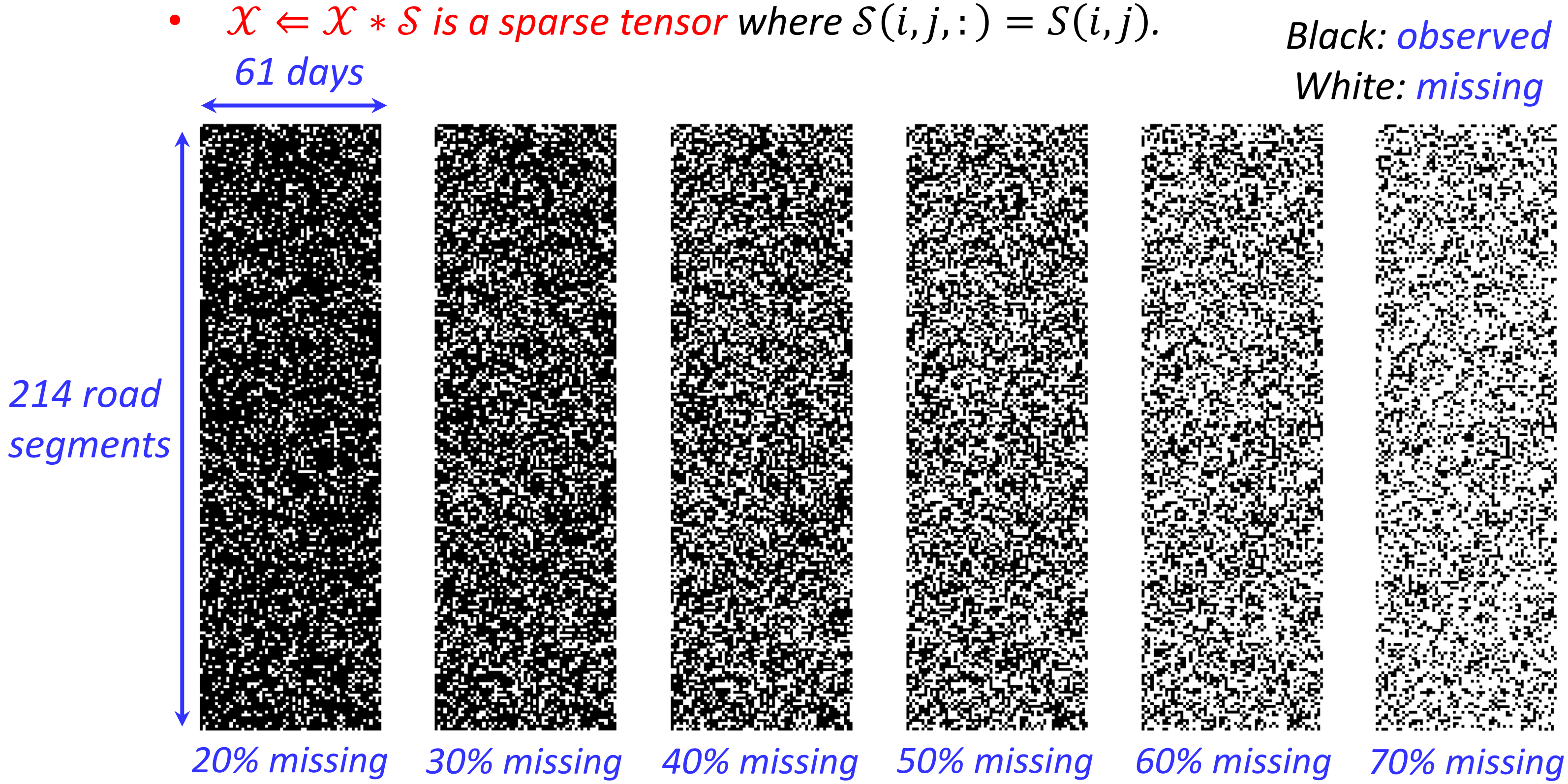
Liu, J., Musialski, P., Wonka, P., Ye, J., 2013. Tensor completion for estimating missing values in visual data. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(1): 208-220.

Empirical studies

- ***Fiber-like missing***
  - *$S \in \mathbb{R}^{214\times 61}$ is a binary matrix where $s_{ij} = round(a_{ij} + 0.5 - \theta)$ and $a_{ij} \sim Uniform(0,1)$.*
  - *$\mathcal{X} \Leftarrow \mathcal{X} * \mathcal{S}$ is a sparse tensor where $\mathcal{S}(i,j,:) = S(i,j)$.*



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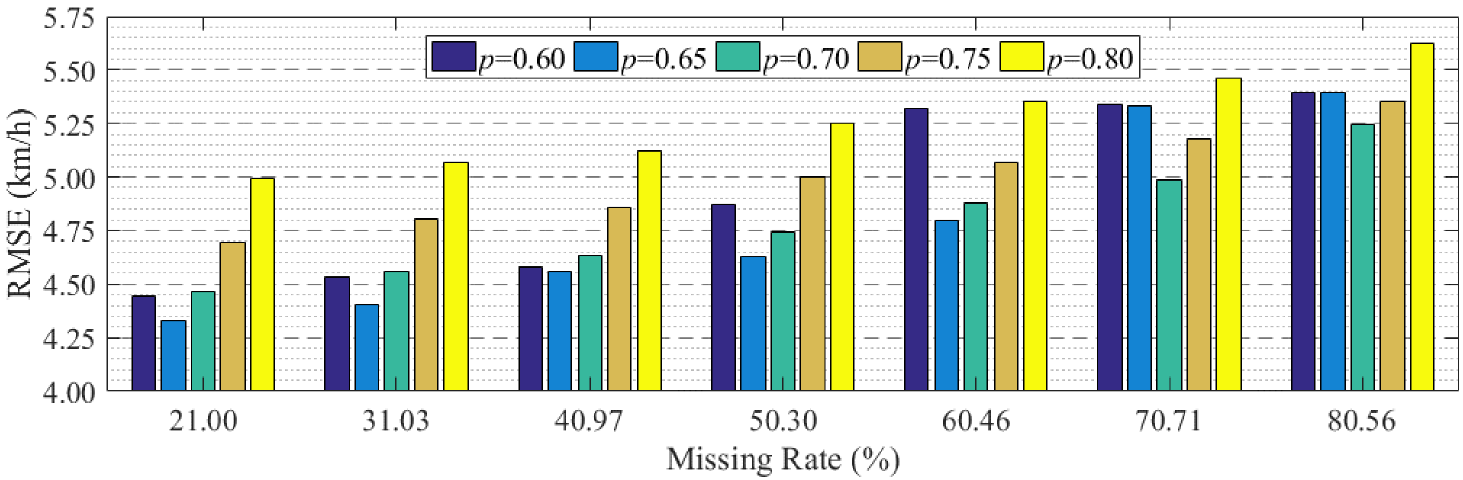
- θ is ***a variable to condition the missing***, for example, if $\theta = 0.7$, then $s_{ij} = round(a_{ij} - 0.2)$ and approximate 70% entries of S are 0.

Empirical studies

- ***Determination of core tensor size***
  - *In the framework, p is the ratio threshold of singular values for each unfolding.*
  - *The number of leading singular vectors for unfolding $\tilde{\mathcal{X}}_{(q)}$ is $r_q, q = 1,2,3$.*
  - *The core tensor size is $r_1 \times r_2 \times r_3$.*

Sensitivity test over final results



- *While missing rate ranging from 20% to 60%, the best threshold is $p = 0.65$.*
- *For heavier missing, the best threshold is $p = 0.70$.*

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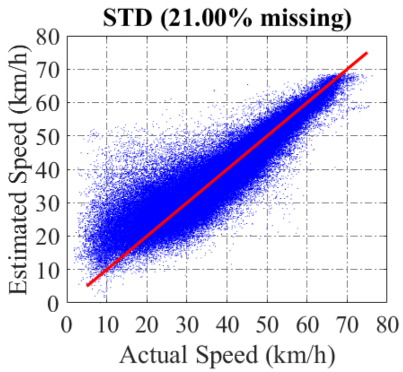
- RMSE: *root mean square error*.

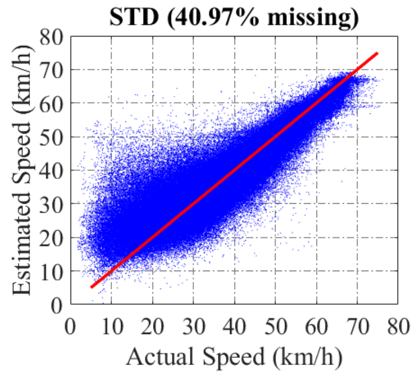
Empirical studies

- ***Overall performance*** (*missing rate is 20%, 40% and 60%*)



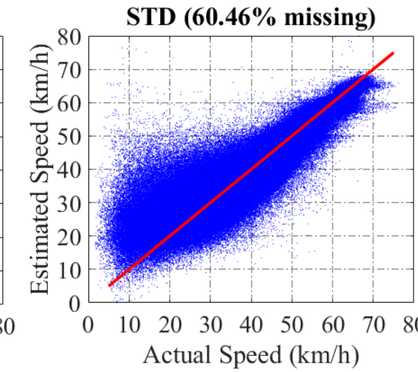
$RMSE = 4.33\text{km/h}$

$MRE = 7.29\%$



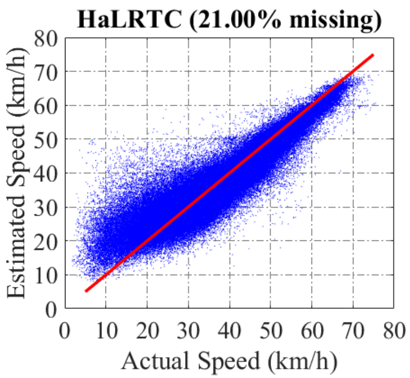
$RMSE = 4.56\text{km/h}$

$MRE = 7.66\%$



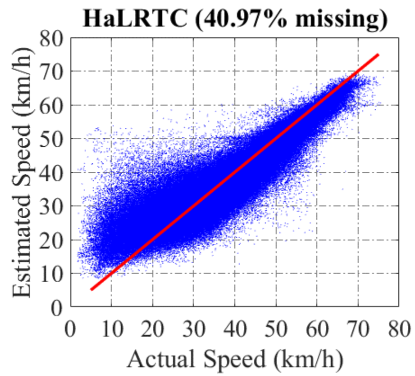
$RMSE = 4.80\text{km/h}$

$MRE = 8.12\%$



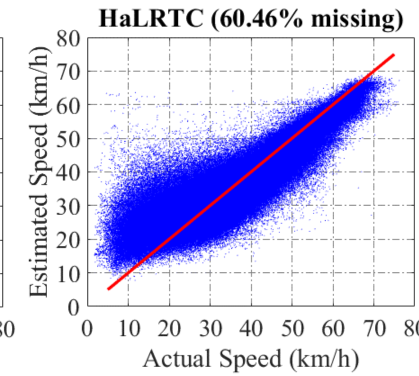
$RMSE = 4.21\text{km/h}$

$MRE = 7.58\%$



$RMSE = 4.38\text{km/h}$

$MRE = 8.05\%$



$RMSE = 4.69\text{km/h}$

$MRE = 8.90\%$

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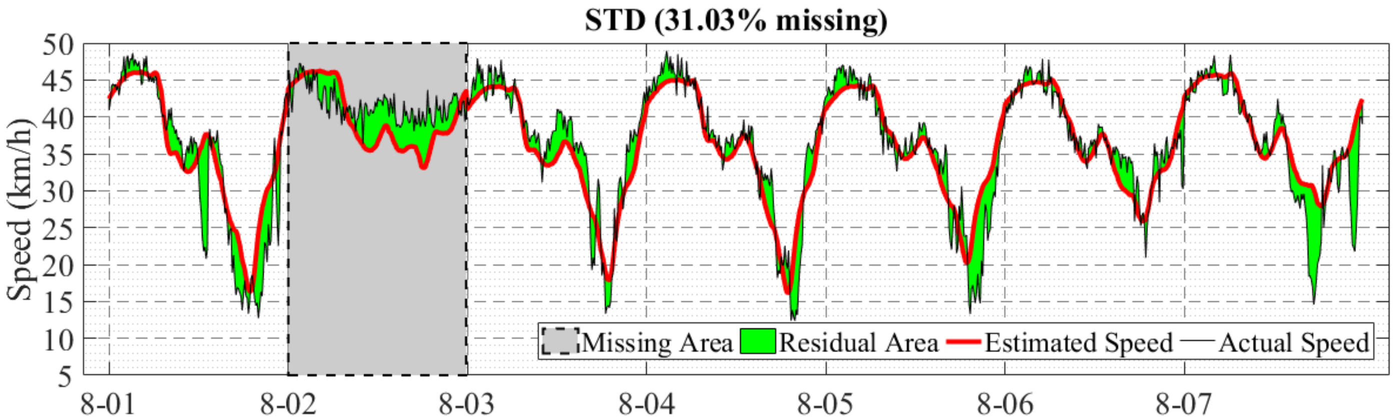
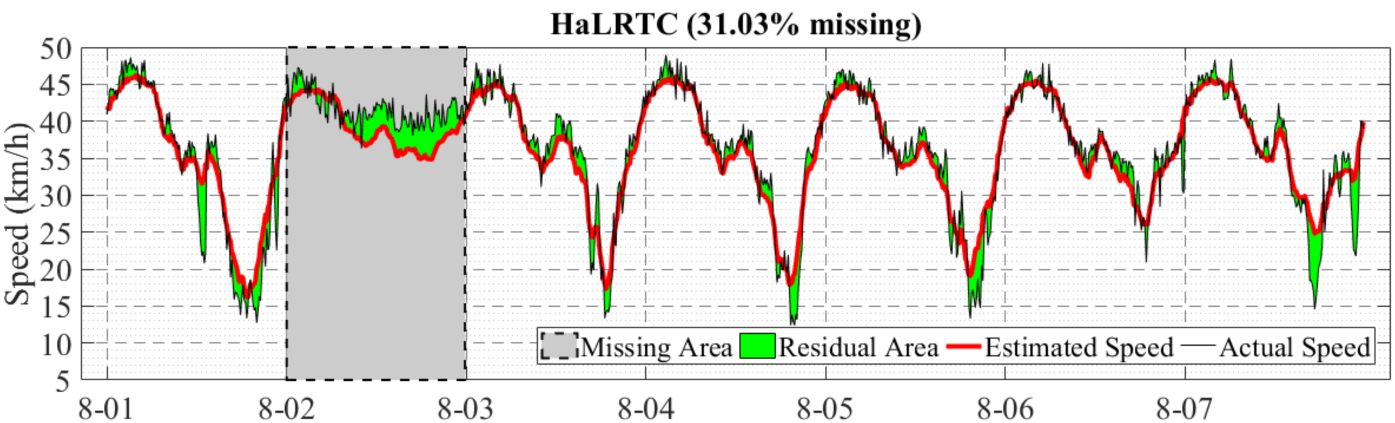
- RMSE: *root mean square error*, **km/h**.
- MRE: *mean relative error*, **%**.

Empirical studies

- ***Time-series analysis*** (*traffic speed of a road segment under the 30% missing*)





The observations of Aug. 2, 2016 are completely lost

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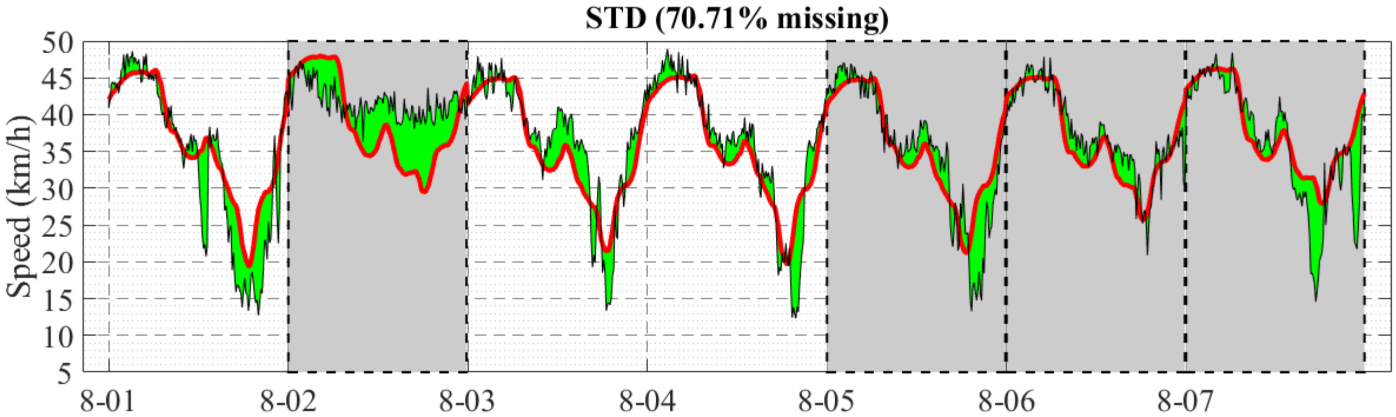
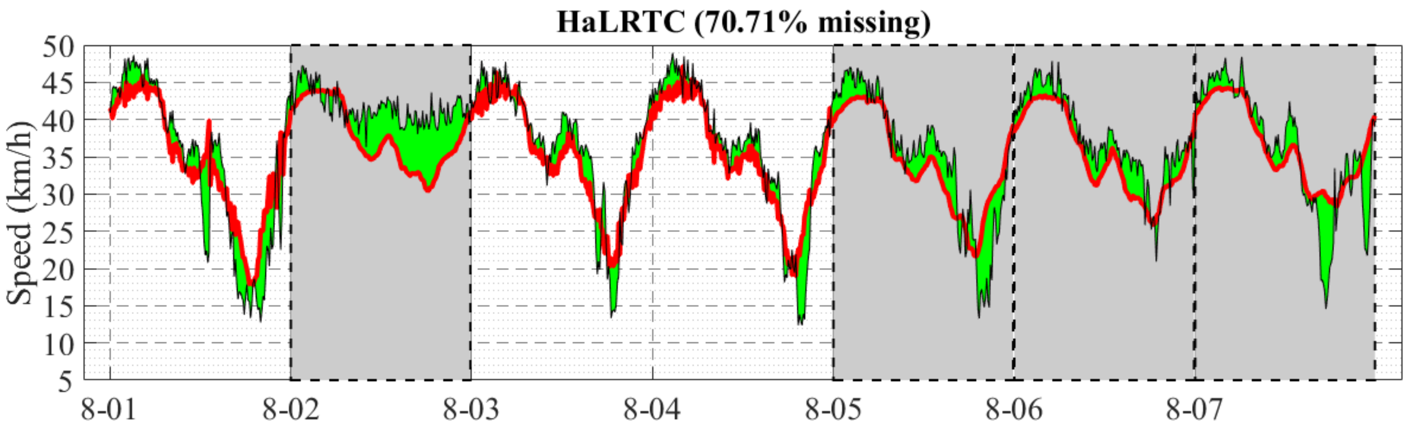
Empirical studies

- ***<u>Time-series analysis</u>*** (*traffic speed of a road segment under the 70% missing*)





The observations of four days are completely lost

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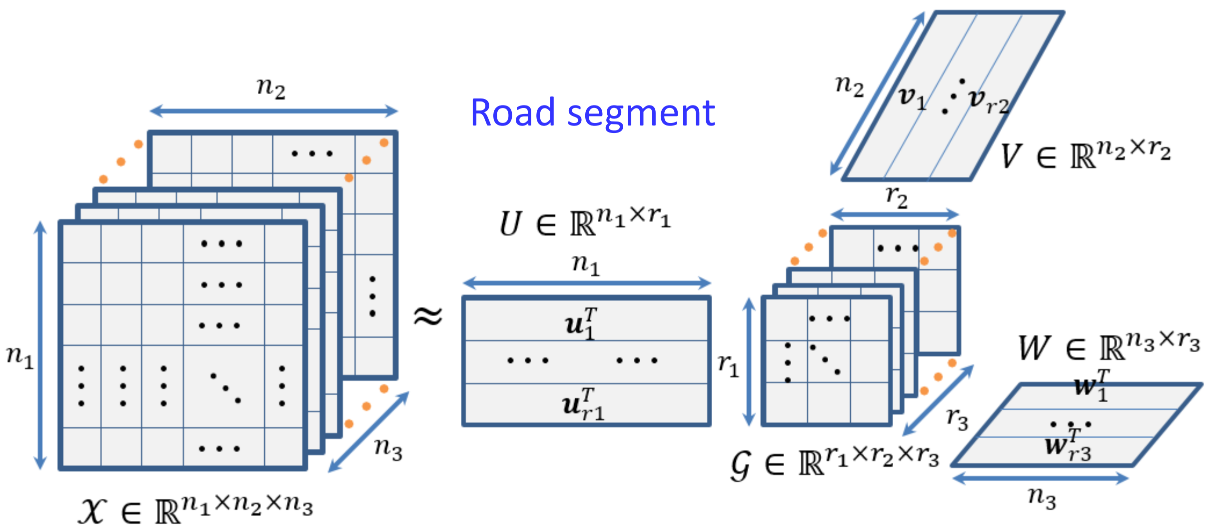
Empirical studies

- ***Interpretability*** *(Tucker decomposition)*

day

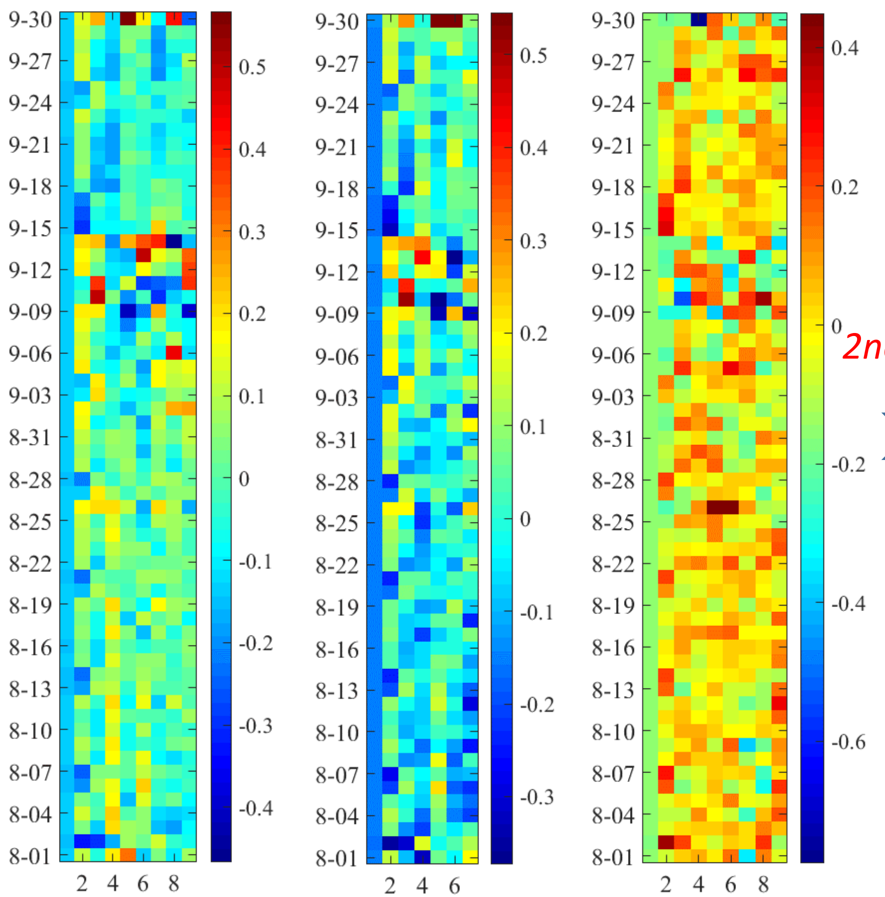


Time window

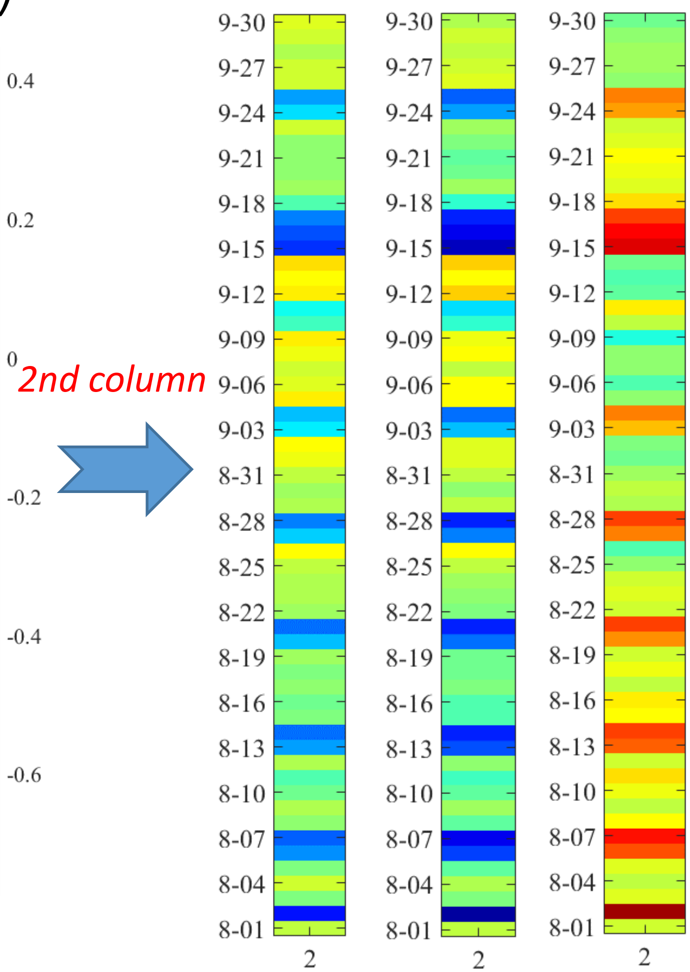
$$\mathcal{X} \approx \mathcal{G} \times_1 U \times_2 V \times_3 W$$

Empirical studies

- ***Interpretation*** (*factor matrix* $V \in \mathbb{R}^{n_2 \times r_2}$)

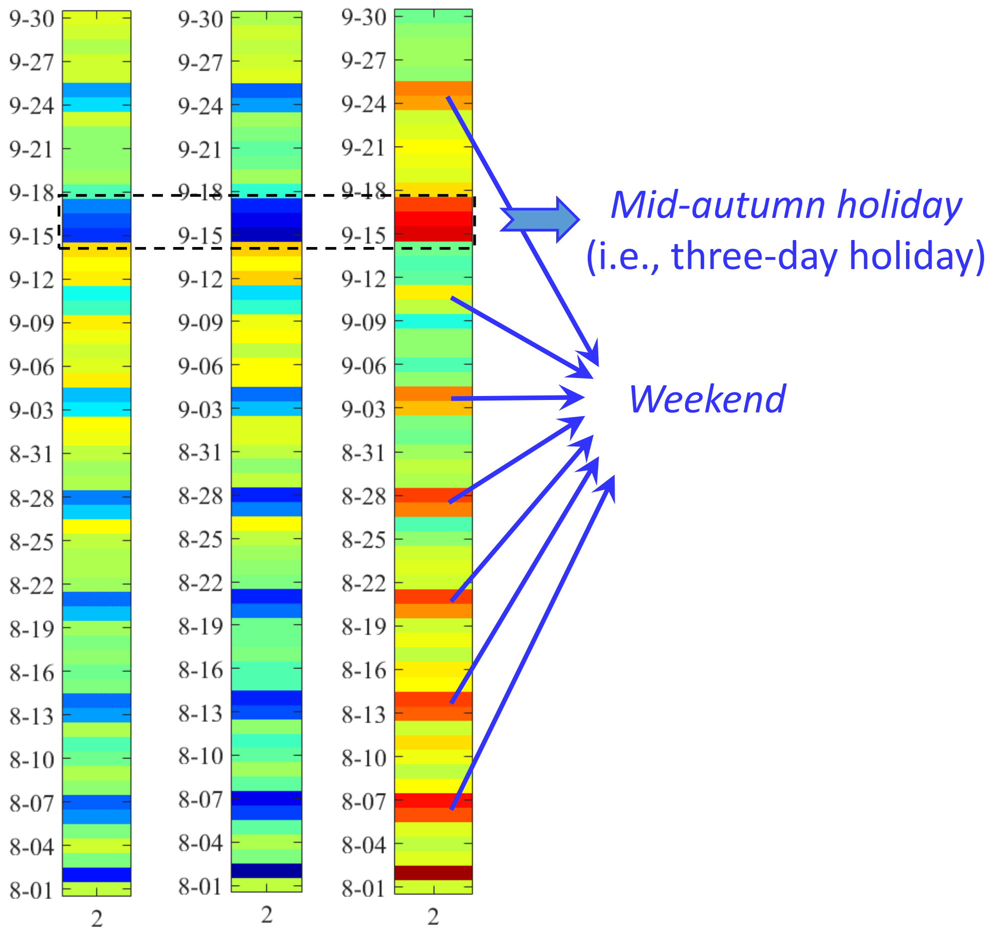


30% missing 50% missing 70% missing



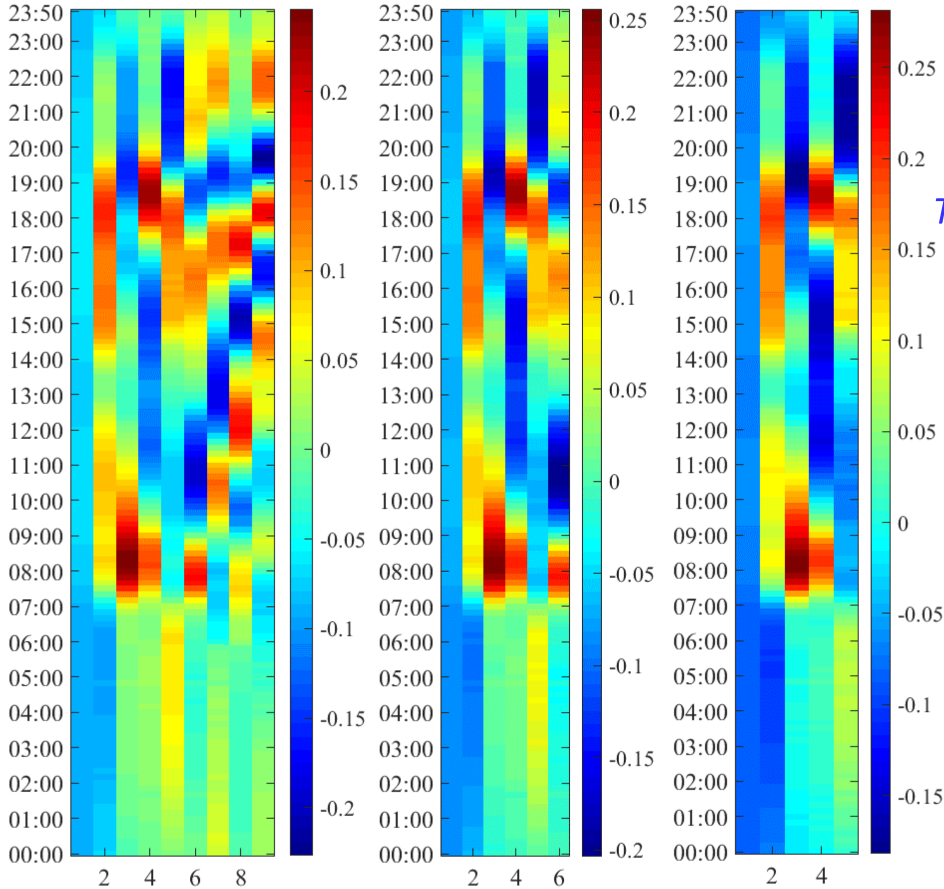
Empirical studies

- ***Interpretation*** (*factor matrix* $V \in \mathbb{R}^{n_2 \times r_2}$)



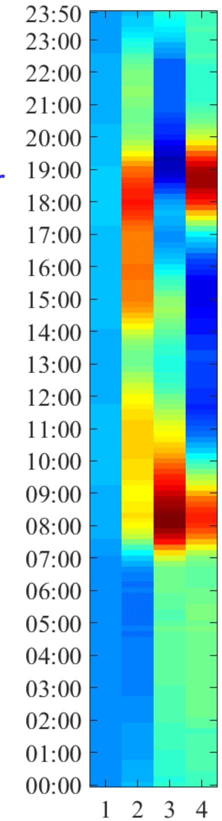
Empirical studies

- ***Interpretation*** (*factor matrix* $W \in \mathbb{R}^{n_3 \times r_3}$)



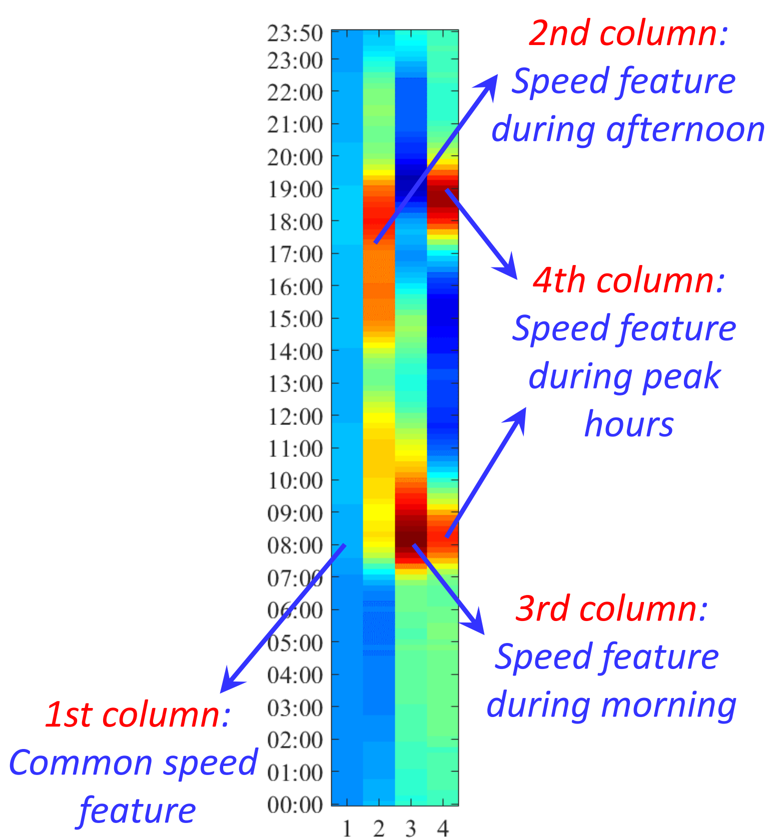
30% missing *50% missing* *70% missing*

They have similar leading vectors (first 4 columns)



Empirical studies

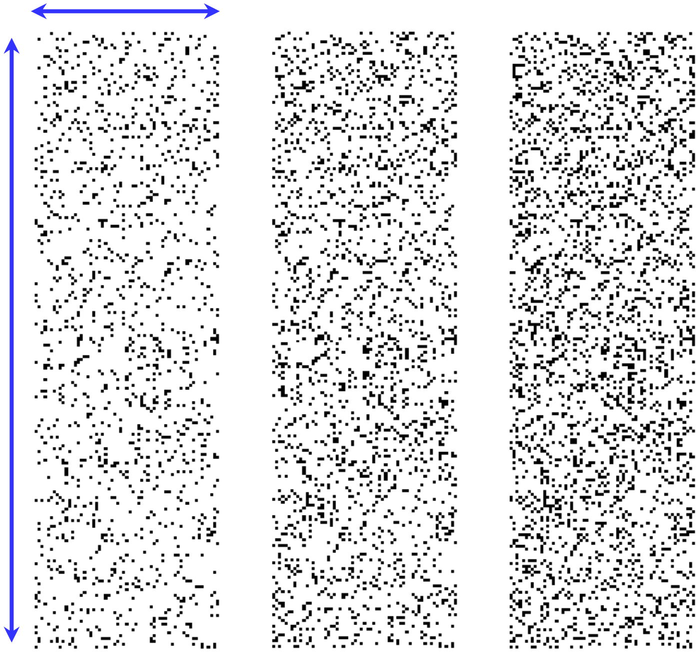
- ***Interpretation*** (*factor matrix* $W \in \mathbb{R}^{n_3 \times r_3}$)



- The result also indicate that ***7:00~9:00*** is ***morning peak hours***, and ***17:30~19:30*** is ***afternoon peak hours***.

Empirical studies

- ***Deal with heavy missing*** (*missing rates are 80%, 85% and 90%*)
  - *Two requirements: during 61 days,* ***observations of each road segment cannot be completely lost****; for each day,* ***at least one speed of road segment is observed****.*
  - $S \in \mathbb{R}^{214 \times 61}$*: the columns and rows must have at least one non-zero entry.*



The STD outperforms HaLRTC!

missing rate	***Measures***	***STD***	***HaLRTC***
80%	RMSE	5.25	5.56
	MRE	9.04	11.71
85%	RMSE	5.45	6.25
	MRE	9.46	13.95
90%	RMSE	5.58	10.00
	MRE	9.70	26.37

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- RMSE: *root mean square error*, **km/h**.
- MRE: *mean relative error*, **%**.

Appendix

- ***STD Algorithm*** *Decomposition based approximation*

Algorithm: SVD-combined Tensor Decomposition (**STD**)

1. **Input**: incomplete tensor $\mathcal{X}$, binary tensor $\mathcal{S}$
2. Set the learning rate α, the regularization parameter λ, and ε
3. $\mathcal{G} = \mathcal{G}^{(SVD)}, U = U^{(SVD)}, V = V^{(SVD)}, W = W^{(SVD)}$
4. $\mathcal{X}^{(0)} = \mathcal{G} \times_1 U \times_2 V \times_3 W$
5. $\mathcal{E} = \mathcal{S} * (\mathcal{X} - \mathcal{G} \times_1 U \times_2 V \times_3 W)$
6. $U^+ = (1 - \alpha\lambda)U + \alpha(\mathcal{S} * \mathcal{E})_{(1)}(W \otimes V)\mathcal{G}_{(1)}^T$
7. $V^+ = (1 - \alpha\lambda)V + \alpha(\mathcal{S} * \mathcal{E})_{(2)}(W \otimes U)\mathcal{G}_{(2)}^T$
8. $W^+ = (1 - \alpha\lambda)W + \alpha(\mathcal{S} * \mathcal{E})_{(3)}(V \otimes U)\mathcal{G}_{(3)}^T$
9. $\mathcal{G}^+ = (1 - \alpha\lambda)\mathcal{G} + \alpha \cdot \mathcal{E} \times_1 U^T \times_2 V^T \times_3 W^T$
10. Update $U \Leftarrow U^+, V \Leftarrow V^+, W \Leftarrow W^+, \mathcal{G} \Leftarrow \mathcal{G}^+$
11. $\mathcal{X}^{(1)} = \mathcal{G} \times_1 U \times_2 V \times_3 W$
12. Check the convergence condition, $\left\|\mathcal{X}^{(1)} - \mathcal{X}^{(0)}\right\|_F^2 < \varepsilon$
13. **while** not (convergence) **do**
14. $\quad \mathcal{X}^{(0)} = \mathcal{X}^{(1)}$, execute step 5-12
15. **Return** $\mathcal{G}, U, V, W, \widehat{\mathcal{X}} = \mathcal{X}^{(1)}$

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Appendix

- ***HaLRTC Algorithm*** *Low-rank approximation*

Algorithm: High Accuracy Low Rank Tensor Completion (**HaLRTC**)

1. **Input**: $\widehat{\mathcal{X}}$ with $\mathcal{S} * \widehat{\mathcal{X}} = \mathcal{S} * \mathcal{X}$ and $(1-\mathcal{S}) * \widehat{\mathcal{X}} = 0$, ρ, K
2. Set $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3 \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ as additive tensors with all entries are 0
3. **for** $k = 0$ to K **do**
4. $\quad \mathcal{B}_q = \text{fold}_q\left\{D_{\frac{1}{3\rho}}\left(\widehat{\mathcal{X}}_{(q)} + \frac{1}{\rho}\mathcal{Y}_{q(q)}\right)\right\}, q = 1,2,3$
5. $\quad \widehat{\mathcal{X}} = (1-\mathcal{S}) * \left[\frac{1}{3}\sum_{q=1}^{3}\left(\mathcal{B}_q - \frac{1}{\rho}\mathcal{Y}_q\right)\right] + \mathcal{S} * \mathcal{X}$
6. $\quad \mathcal{Y}_q = \mathcal{Y}_q - \rho\left(\mathcal{B}_q - \widehat{\mathcal{X}}\right), q = 1,2,3$
7. **Output**: $\widehat{\mathcal{X}}$

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Thanks for your listening!