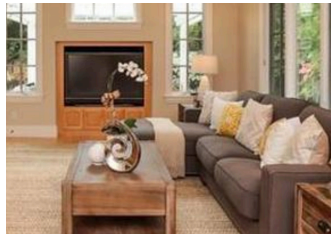
如何在美国买房

- 看中一个房，参观了解
- 估计一个价格，出价





标价

$5,498,000 Price | **7** Beds | **5** Baths | **4,865** Sq. Ft. $1130 / Sq. Ft.

Redfin Estimate: $5,390,037 | On Redfin: 15 days

预计价格

Virtual Tour
- Branded Virtual Tour
- Virtual Tour (External Link)

Parking Information
- Garage (Minimum): 2
- Garage (Maximum): 2
- Parking Description: Attached Garage, On Street
- Garage Spaces: 2

Multi-Unit Information
- # of Stories: 2

School Information
- Elementary School: El Carmelo
- Elementary School District: Palo
- Middle School: Jane Lathrop Sta
- High School: Palo Alto High
- High School District: Palo Alto U

Interior Features

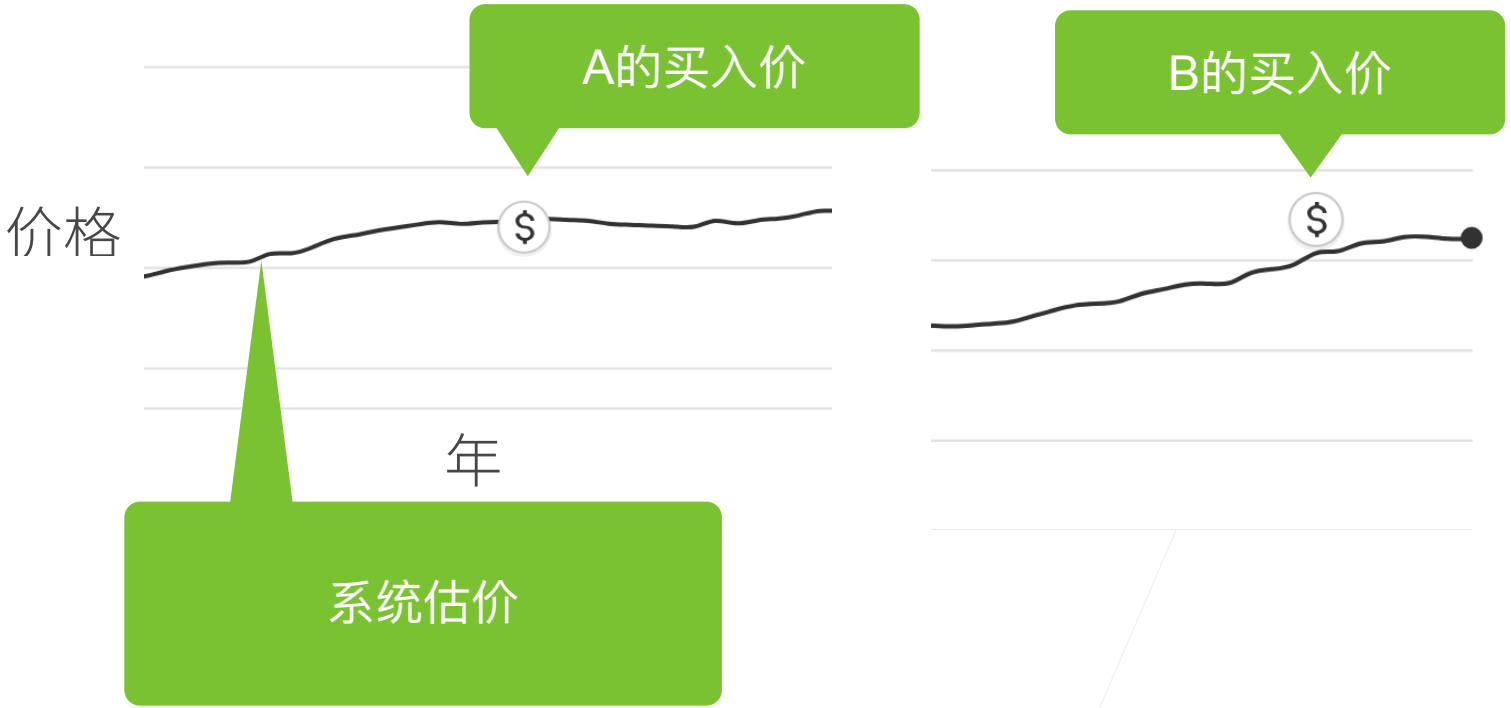
Bedroom Information
- # of Bedrooms (Minimum): 7
- # of Bedrooms (Maximum): 7

- Kitchen Description: Countertop Dishwasher, Garbage Disposal, H Island with Sink, Microwave, Ove

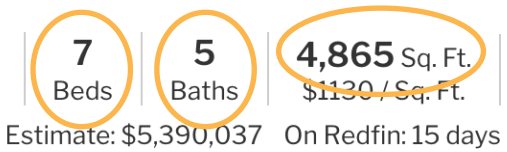
房价预测

很重要，这是真钱

$100K+ 差价



一个简化模型



- 假设 1：影响房价的关键因素是卧室个数，卫生间个数，和居住面积，记为 x_1, x_2, x_3
- 假设 2：成交价是关键因素的加权和

$$y = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

权重和偏差的实际值在后面决定

线性模型

- 给定 n 维输入 $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$
- 线性模型有一个 n 维权重和一个标量偏差

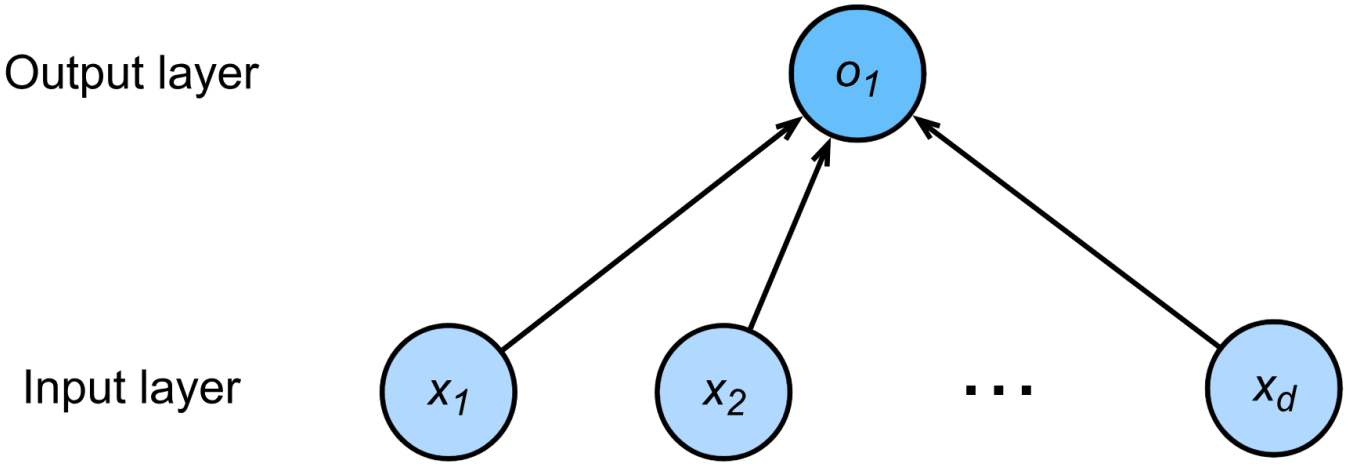
$$\mathbf{w} = [w_1, w_2, \ldots, w_n]^T, \quad b$$

- 输出是输入的加权和

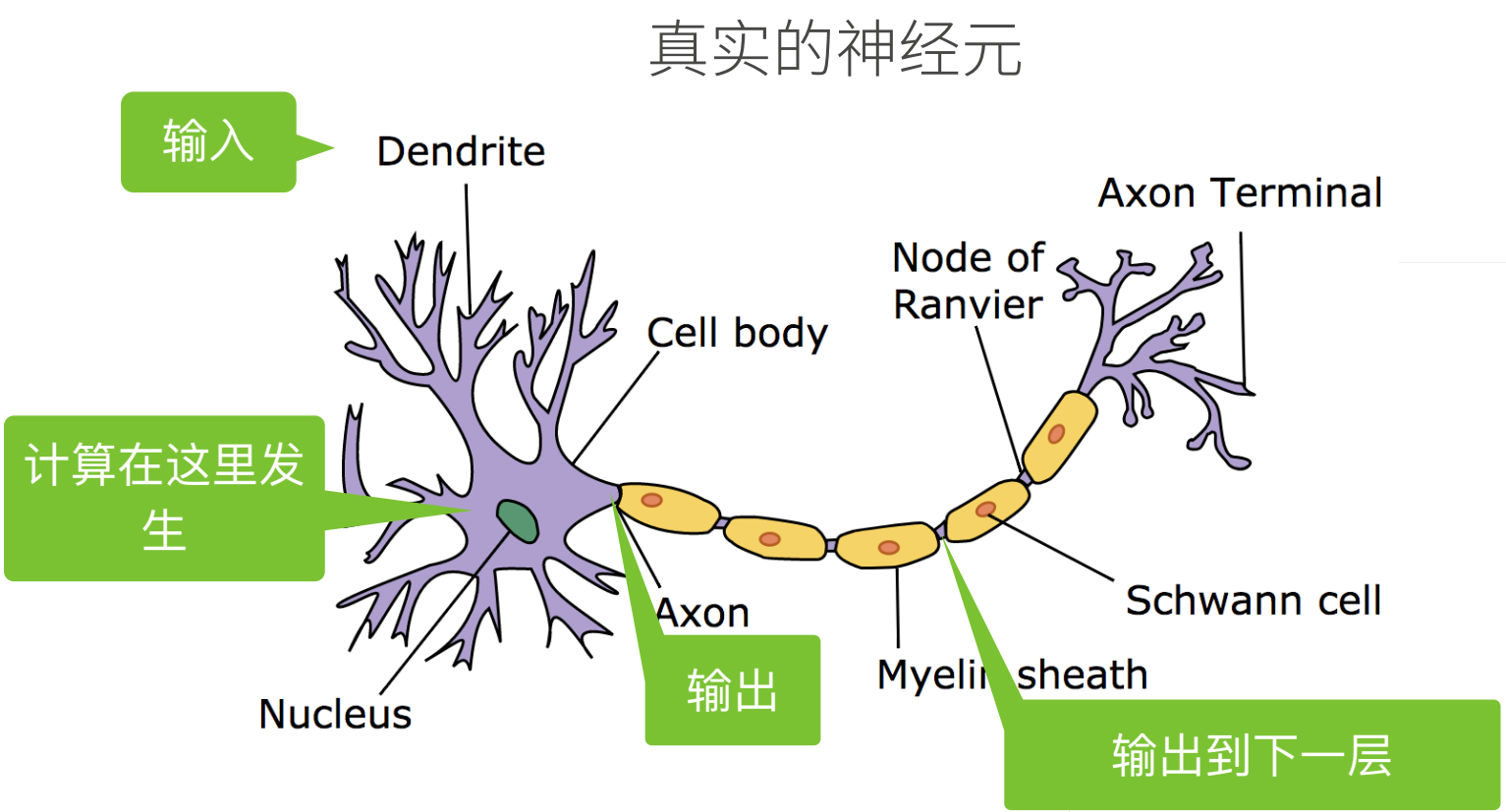
$$y = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + b$$

向量版本：$y = \langle \mathbf{w}, \mathbf{x} \rangle + b$

线性模型可以看做是单层神经网络



神经网络源于神经科学


真实的神经元
输入
Dendrite
Axon Terminal
Node of Ranvier
Cell body
计算在这里发生
Schwann cell
Axon
输出
Myelin sheath
Nucleus
输出到下一层

衡量预估质量

- 比较真实值和预估值，例如房屋售价和估价
- 假设 y 是真实值，$\hat{y}$ 是估计值，我们可以比较

$$\ell(y, \hat{y}) = \left(y - \hat{y}\right)^2$$

这个叫做平方损失

训练数据

- 收集一些数据点来决定参数值（权重和偏差），例如过去6个月卖的房子
- 这被称之为训练数据
- 通常越多越好
- 假设我们有 n 个样本，记

$$\mathbf{X} = \left[\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n\right]^T \quad \mathbf{y} = \left[y_0, y_1, \ldots, y_n\right]^T$$

参数学习

- 训练损失

$$\ell(\mathbf{X}, \mathbf{y}, \mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \langle \mathbf{x}_i, \mathbf{w}\rangle - b\right)^2 = \frac{1}{n}\left\| \mathbf{y} - \mathbf{X}\mathbf{w} - b \right\|^2$$

- 最小化损失来学习参数

$$\mathbf{w}^*, \mathbf{b}^* = \arg\min_{\mathbf{w},b} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}, b)$$

显示解

- 将偏差加入权重 $\mathbf{X} \leftarrow [\mathbf{X}, \mathbf{1}] \quad \mathbf{w} \leftarrow \begin{bmatrix} \mathbf{w} \\ b \end{bmatrix}$

$$\ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = \frac{1}{n} \left\| \mathbf{y} - \mathbf{X}\mathbf{w} \right\|^2 \qquad \frac{\partial}{\partial \mathbf{w}} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = \frac{2}{n} \left(\mathbf{y} - \mathbf{X}\mathbf{w} \right)^T \mathbf{X}$$

- 损失是凸函数，所以最优解满足

$$\frac{\partial}{\partial \mathbf{w}} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = 0$$

$$\Leftrightarrow \frac{2}{n} \left(\mathbf{y} - \mathbf{X}\mathbf{w} \right)^T \mathbf{X} = 0$$

$$\Leftrightarrow \mathbf{w}^* = \left(\mathbf{X}^T \mathbf{X} \right)^{-1} \mathbf{X}\mathbf{y}$$

总结

- 线性回归是对n维输入的加权，外加偏差
- 使用平方损失来衡量预测值和真实值的差异
- 线性回归有显示解
- 线性回归可以看做是单层神经网络