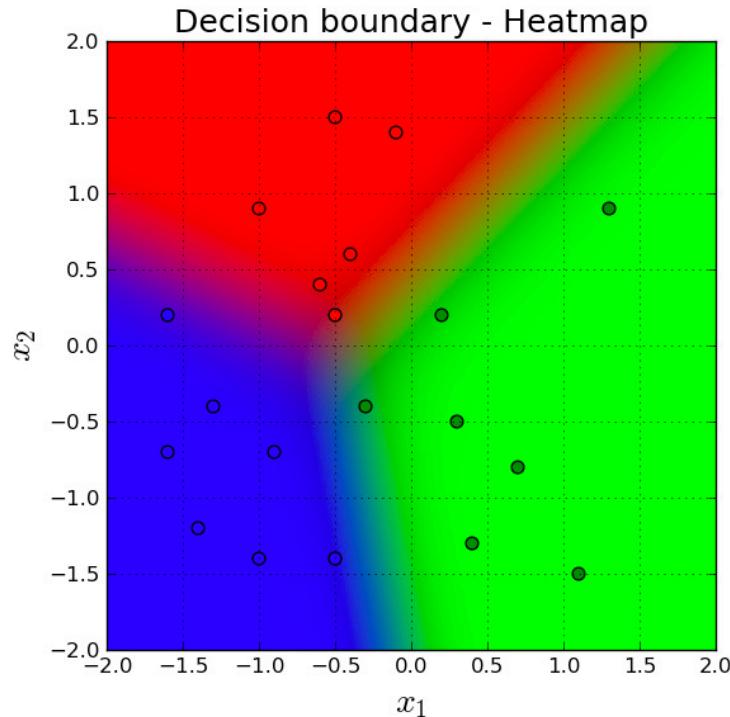




# Softmax 回归





# 回归 vs 分类

- 回归估计一个连续值
- 分类预测一个离散类别

MNIST: 手写数字识别 (10类)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	8	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9

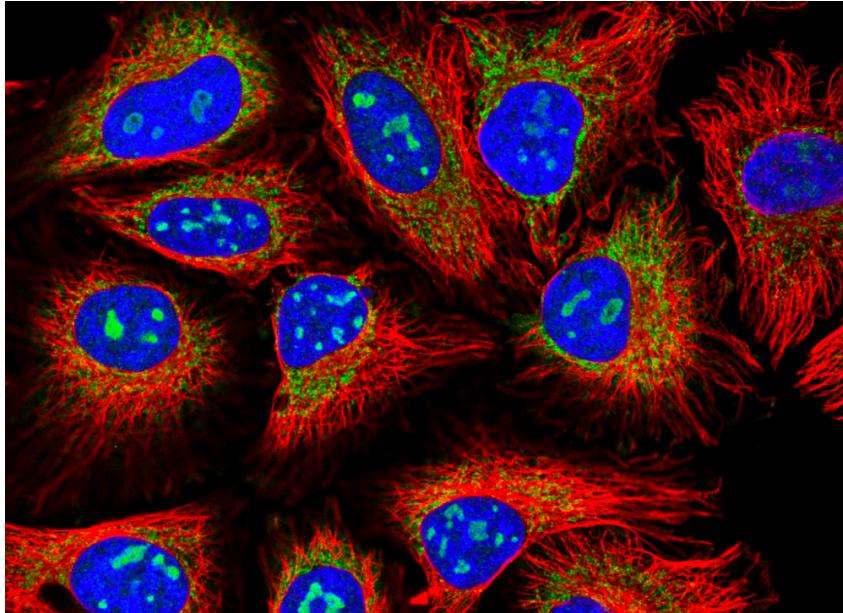
ImageNet: 自然物体分类 (1000类)





# Kaggle 上的分类问题

将人类蛋白质显微镜图片分成28类



- 0. Nucleoplasm
- 1. Nuclear membrane
- 2. Nucleoli
- 3. Nucleoli fibrillar
- 4. Nuclear speckles
- 5. Nuclear bodies
- 6. Endoplasmic reticu
- 7. Golgi apparatus
- 8. Peroxisomes
- 9. Endosomes
- 10. Lysosomes
- 11. Intermediate fila
- 12. Actin filaments
- 13. Focal adhesion si
- 14. Microtubules
- 15. Microtubule ends
- 16. Cytokinetic bridg

<https://www.kaggle.com/c/human-protein-atlas-image-classification>



# Kaggle 上的分类问题

将恶意软件分成9个类别



<https://www.kaggle.com/c/malware-classification>

# Kaggle 上的分类问题



将恶意的 Wikipedia 评论分成 7 类

comment_text	toxic	severe_toxic	obscene
Explanation\nWhy the edits made under my user...	0	0	0
D'aww! He matches this background colour I'm s...	0	0	0
Hey man, I'm really not trying to edit war. It...	0	0	0
"\nMore\nI can't make any real suggestions on ...	0	0	0
You, sir, are my hero. Any chance you remember...	0	0	0

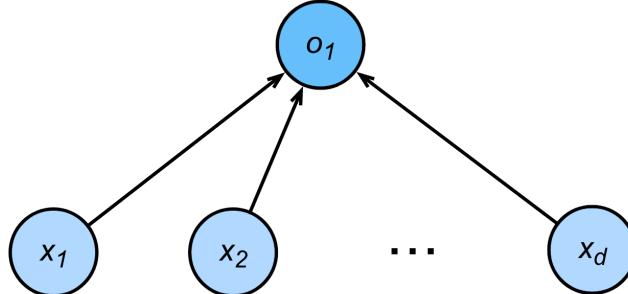
<https://www.kaggle.com/c/jigsaw-toxic-comment-classification-challenge>



# 从回归到多类分类

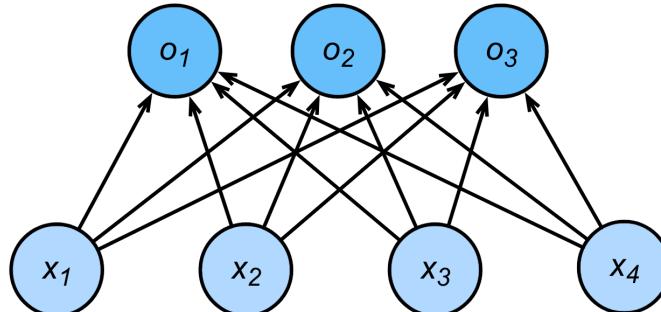
## 回归

- 单连续数值输出
- 自然区间  $\mathbb{R}$
- 跟真实值的区别作为损失



## 分类

- 通常多个输出
- 输出  $i$  是预测为第  $i$  类的置信度





# 从回归到多类分类 – 均方损失

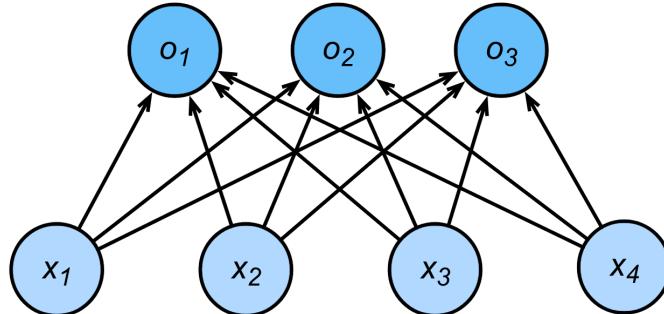
- 对类别进行一位有效编码

$$\mathbf{y} = [y_1, y_2, \dots, y_n]^\top$$

$$y_i = \begin{cases} 1 & \text{if } i = y \\ 0 & \text{otherwise} \end{cases}$$

- 使用均方损失训练
- 最大值最为预测

$$\hat{y} = \operatorname{argmax}_i o_i$$





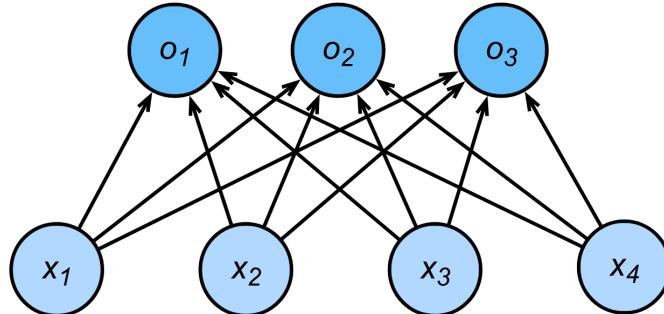
# 从回归到多类分类 – 无校验比例

- 对类别进行一位有效编码
- 最大值最为预测

$$\hat{y} = \operatorname{argmax}_i o_i$$

- 需要更置信的识别正确类 (大余量)

$$o_y - o_i \geq \Delta(y, i)$$





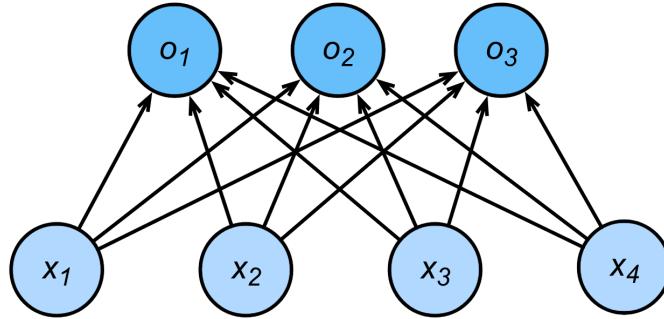
# 从回归到多类分类 – 校验比例

- 输出匹配概率（非负，和为 1）

$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{o})$$

$$\hat{y}_i = \frac{\exp(o_i)}{\sum_k \exp(o_k)}$$

- 概率  $\mathbf{y}$  和  $\hat{\mathbf{y}}$  的区别作为损失





# Softmax 和交叉熵损失

- 交叉熵常用来衡量两个概率的区别  $H(\mathbf{p}, \mathbf{q}) = \sum_i -p_i \log(q_i)$
- 将它作为损失

$$l(\mathbf{y}, \hat{\mathbf{y}}) = - \sum_i y_i \log \hat{y}_i = - \log \hat{y}_y$$

- 其梯度是真实概率和预测概率的区别

$$\partial_{o_i} l(\mathbf{y}, \hat{\mathbf{y}}) = \text{softmax}(\mathbf{o})_i - y_i$$



# 总结

- Softmax 回归是一个多类分类模型
- 使用 Softmax 操作子得到每个类的预测置信度
- 使用交叉熵来衡量预测和标号的区别