

# Efficient Extended Ant Colony Optimization for Capacitated Electric Vehicle Routing

2022.12

**Bo-Cheng Lin<sup>1</sup>, Xiao-Fang Liu<sup>\*1, 2</sup>, Yi Mei<sup>3</sup>**

<sup>1</sup>Institute of Robotics and Automatic Information System, College of Artificial Intelligence, Nankai University, Tianjin 30050, China

<sup>2</sup>Tianjin Key Laboratory of Intelligent Robotics, Nankai University, Tianjin 30050, China

<sup>3</sup>School of Engineering and Computer Science, Victoria University of Wellington, Wellington 6140, New Zealand

This work was supported by the National Natural Science Foundation of China (NSFC) under Grant 62103202. (Corresponding author: Xiao-Fang Liu)

# Electric Vehicle Routing



- **Strict CO<sub>2</sub> emissions targets**
- **Demand from markets**
- **Compulsory phasing out of traditional vehicles**
- **Long charging time**
- **Less charging and swapping stations**

# Variants of EVRP

- **Capacitated EVRP (CEVRP)**
- **EV RP with partial recharging (EVRPPR)**
- **EV RP with time window and recharging stations (EVRPTW)**
- ...

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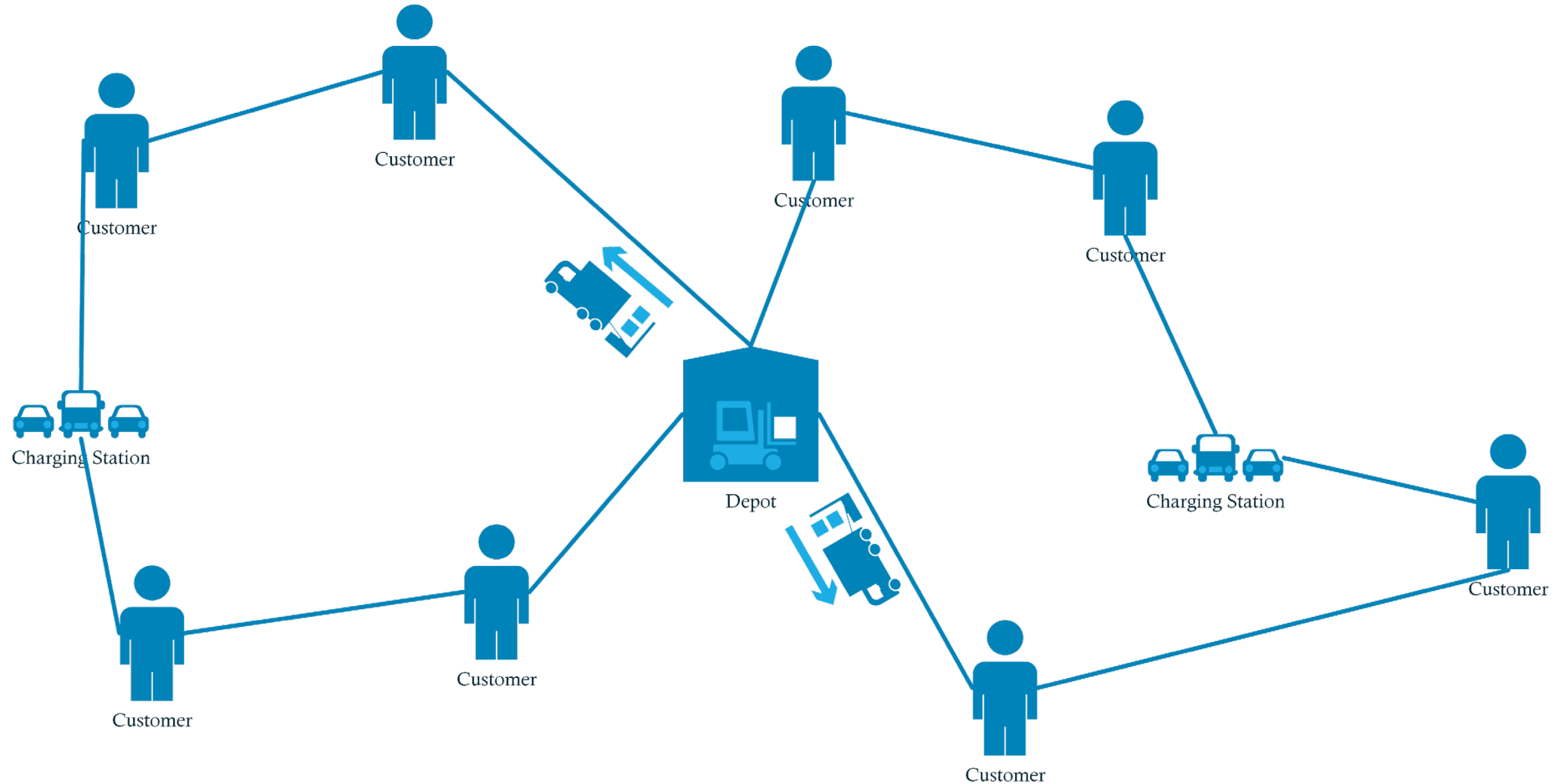
...

**have not been further studied for the complicated constraints in the new problems**

# Variants of EVRP

- **Capacitated EVRP (CEVRP)**  
**the fundamental problem of  
other variants**
- EVRP with partial recharging (EVRPPR)
- EVRP with time window and recharging stations (EVRPTW)
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# Capacitated Electric Vehicle Routing



# Solving CEVRPs

## Exact algorithms

- mixed integer linear programming (MILP)  
**may fail on large-scale instances**

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- Variable neighborhood search (VNS)
- Iterative local search (ILS)  
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- Ant colony optimization(ACO) algorithms  
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# CEVRP

## bi-level problem

Upper-level sub-problem

**Constructing customer service sequence without recharging**

**ACO**

Lower-level sub-problem

**Inserting charging stations into the customer service sequence**

**Heuristic**

# CEVRP

## bi-level problem

Upper-level sub-problem

**Constructing customer service  
sequence without recharging**

**connection between battery capacity  
and cargo capacity is diluted**

Lower-level sub-problem

**Inserting charging stations into  
the customer service sequence**



**Can the ants construct  
scheduling routing with charging stations and customers  
at the same time?**

**Pheromone matrixes in existing ACO methods for CEVRP only record route information between customers.**

**We extend the pheromone matrix to record the historical information between customers and charging stations.**

**A new probability distribution is designed for the roulette wheel selection strategy, which considers the remaining battery level and is able to guide ants to select a suitable charging station at a proper time.**

**So, we call it**

**EACO**

# Traditional roulette wheel selection strategy

Move from customer to next customer

pheromone value of travelling from i to j

$$p_{ij} = \frac{\varphi_{ij}^{\alpha} / d_{ij}^{\beta}}{\sum_{l \in I_s} \varphi_{il}^{\alpha} / d_{il}^{\beta}}$$

the distance between i and j



# New roulette wheel selection strategy

Move from customer/recharging station to next customer/recharging station

$$p_{i \rightarrow j} = \frac{\frac{\left[ \varphi_{ij} \left( \left( \frac{kQ}{Rq_i} \right)^\gamma \tau_j + (1 - \tau_j) \right) \right]^\alpha}{d_{ij}^\beta}}{\sum_{l \in I'} \left( \frac{\left[ \varphi_{il} \left( \left( \frac{kQ}{Rq_i} \right)^\gamma \tau_l + (1 - \tau_l) \right) \right]^\alpha}{d_{il}^\beta} \right)}$$

$$\tau_j \in \{0, 1\}$$

# New roulette wheel selection strategy

Move from customer/recharging station to next customer/recharging station

the maximum battery capacity of each EV

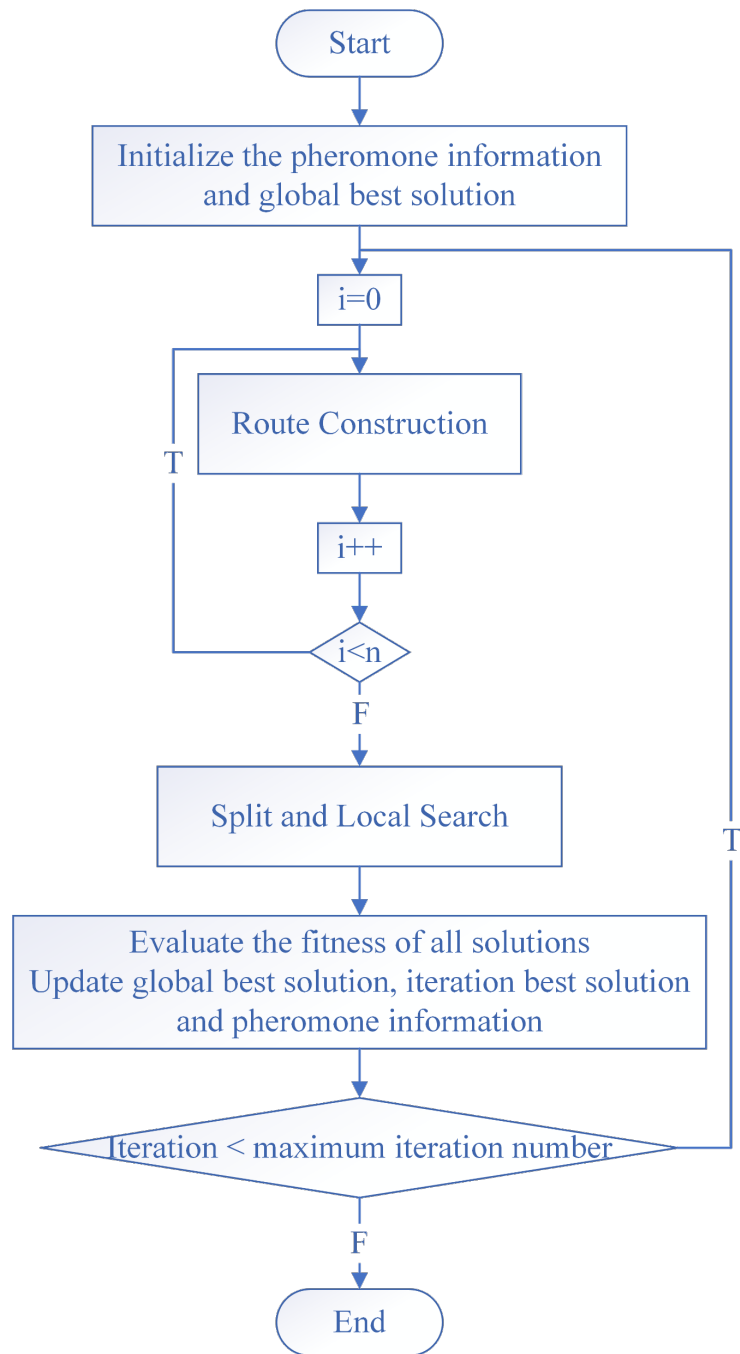
$$p_{i \rightarrow j} = \frac{\frac{\left[ \varphi_{ij} \left( \left( \frac{kQ}{Rq_i} \right)^\gamma \tau_j + (1 - \tau_j) \right) \right]^\alpha}{d_{ij}^\beta}}{\sum_{l \in I'} \left( \frac{\left[ \varphi_{il} \left( \left( \frac{kQ}{Rq_i} \right)^\gamma \tau_l + (1 - \tau_l) \right) \right]^\alpha}{d_{il}^\beta} \right)}$$

a fixed parameter ranging from 0 to 1

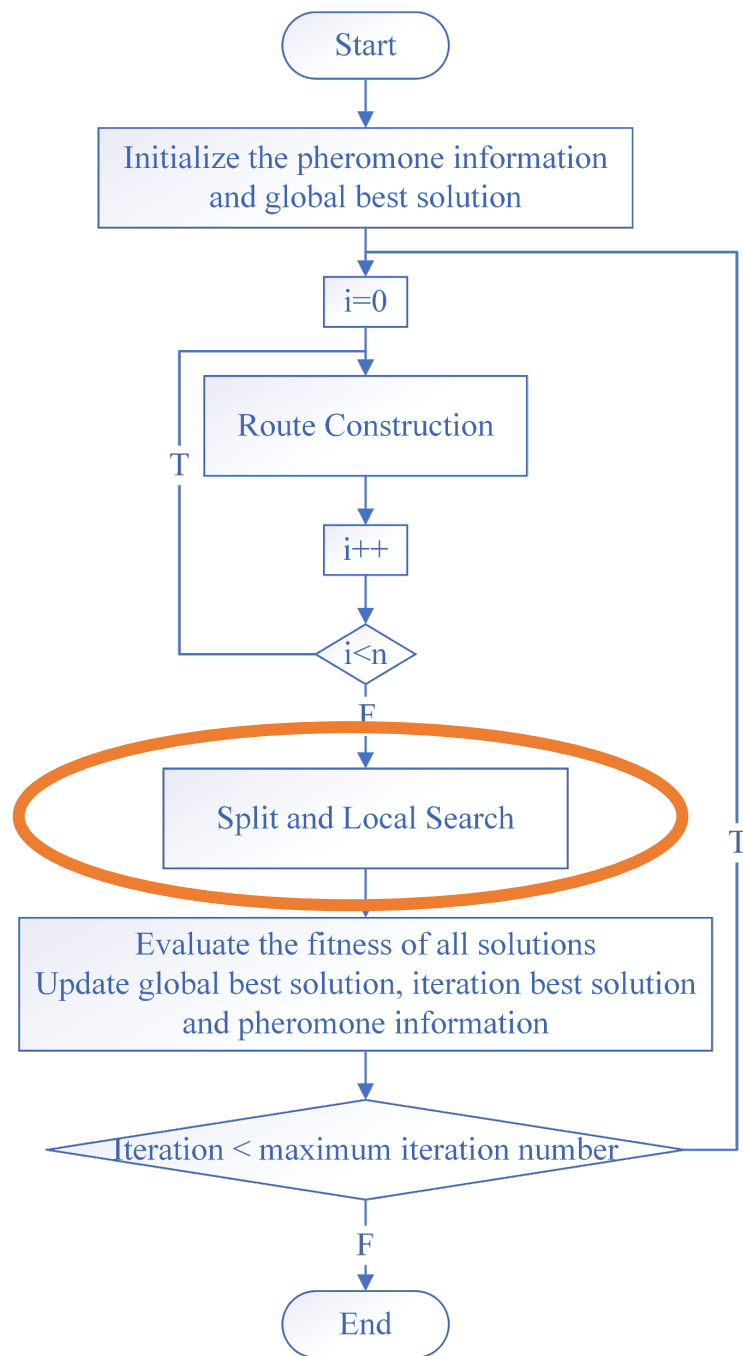
remaining battery level of an EV after it arrives at node i

$$\tau_j \in \{0, 1\}$$

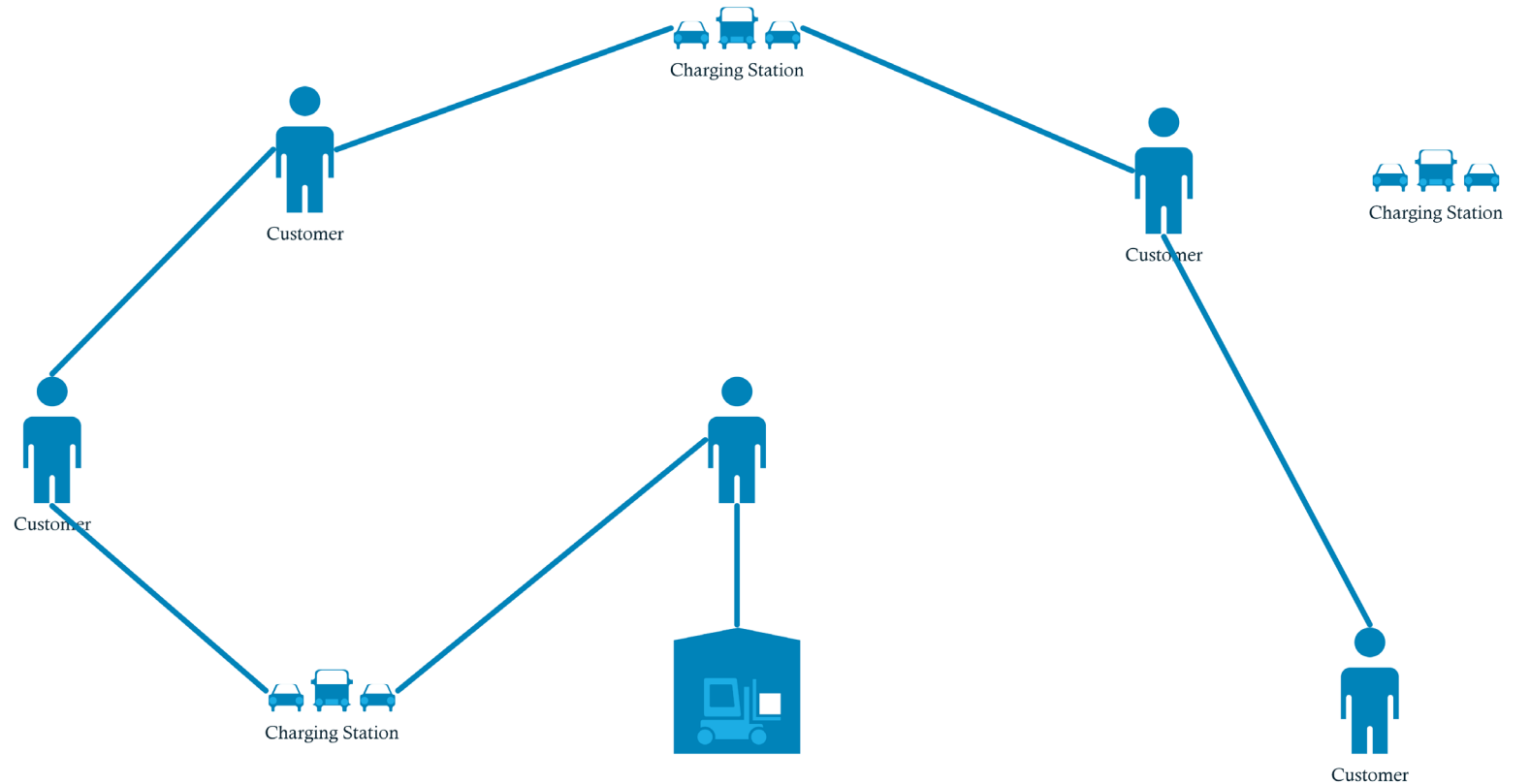
If j is the customer,  $\tau_j$  equals 0, otherwise equals 1

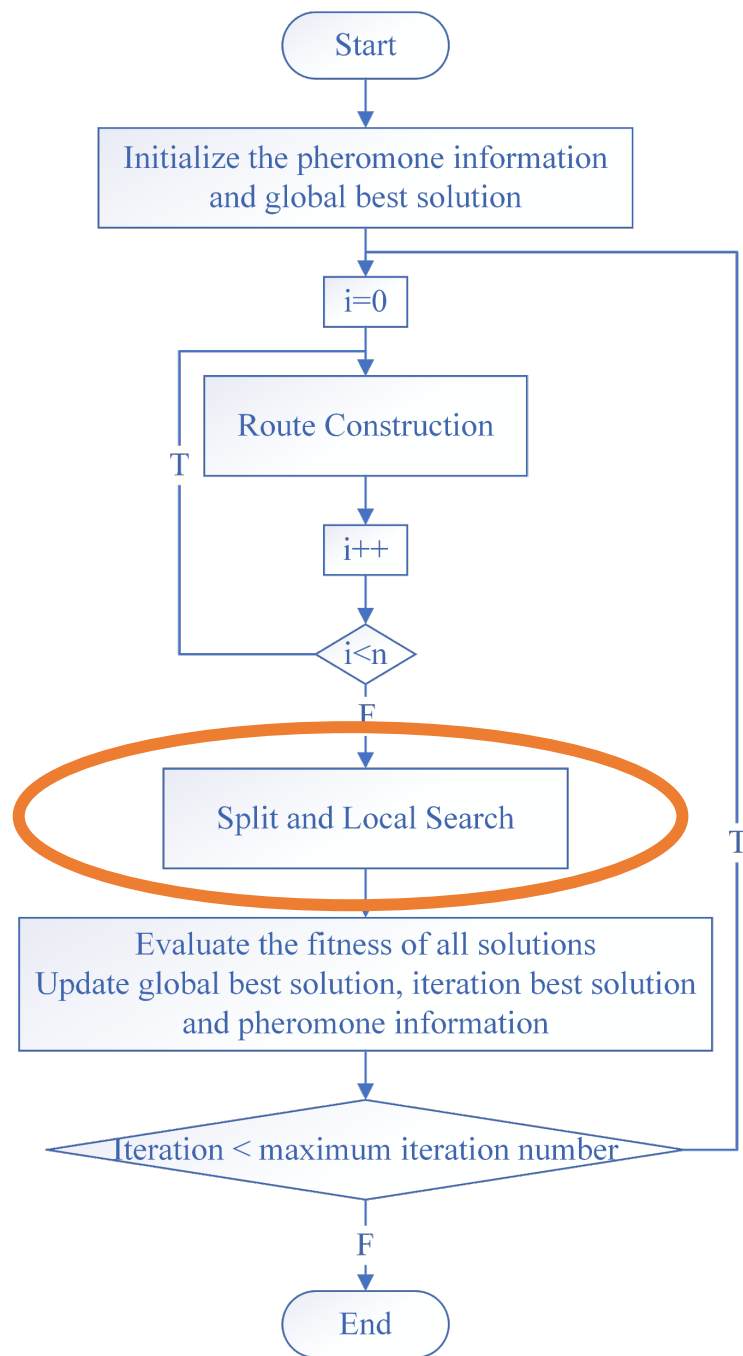


# Overall Process of EACO

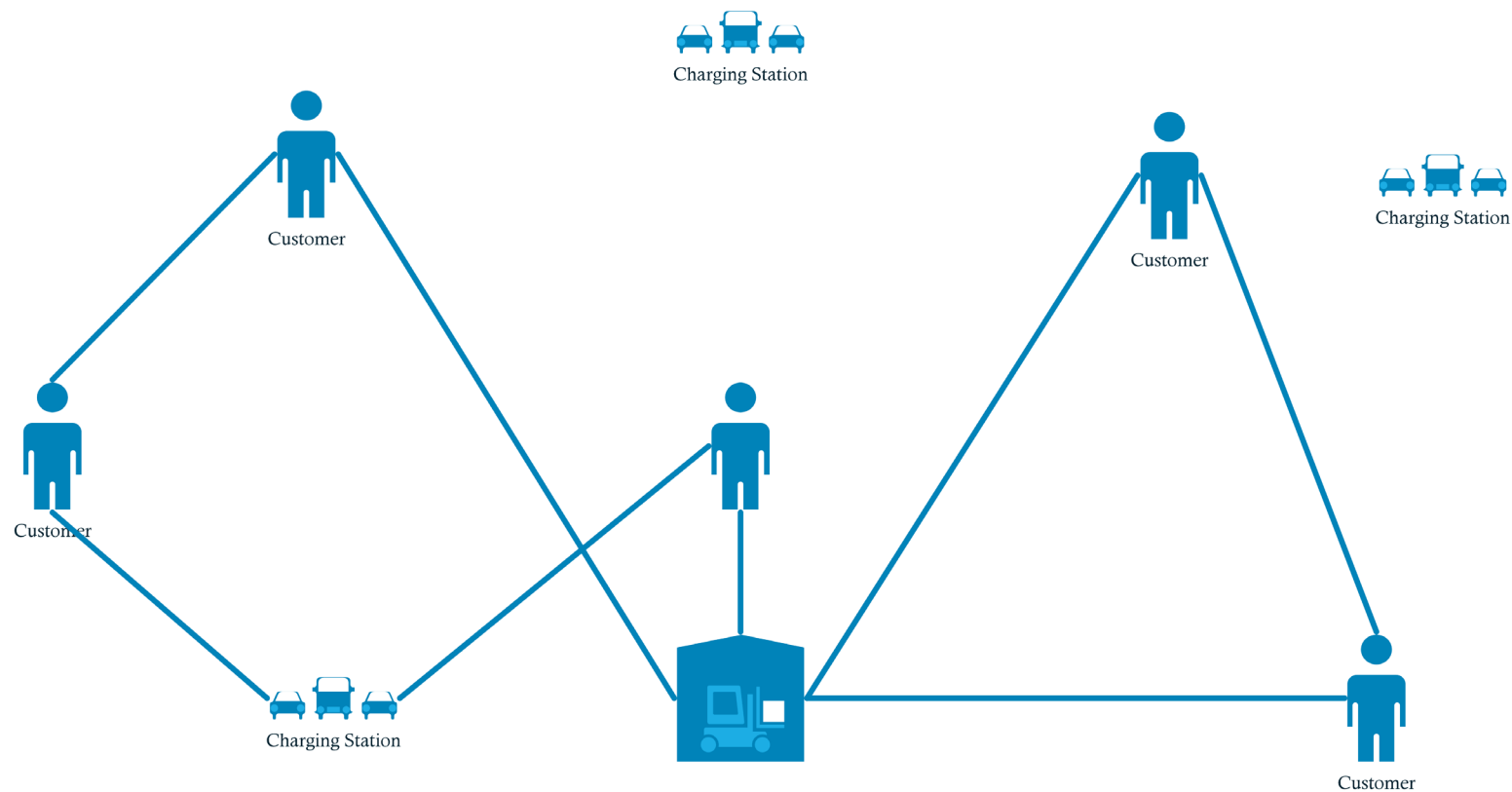


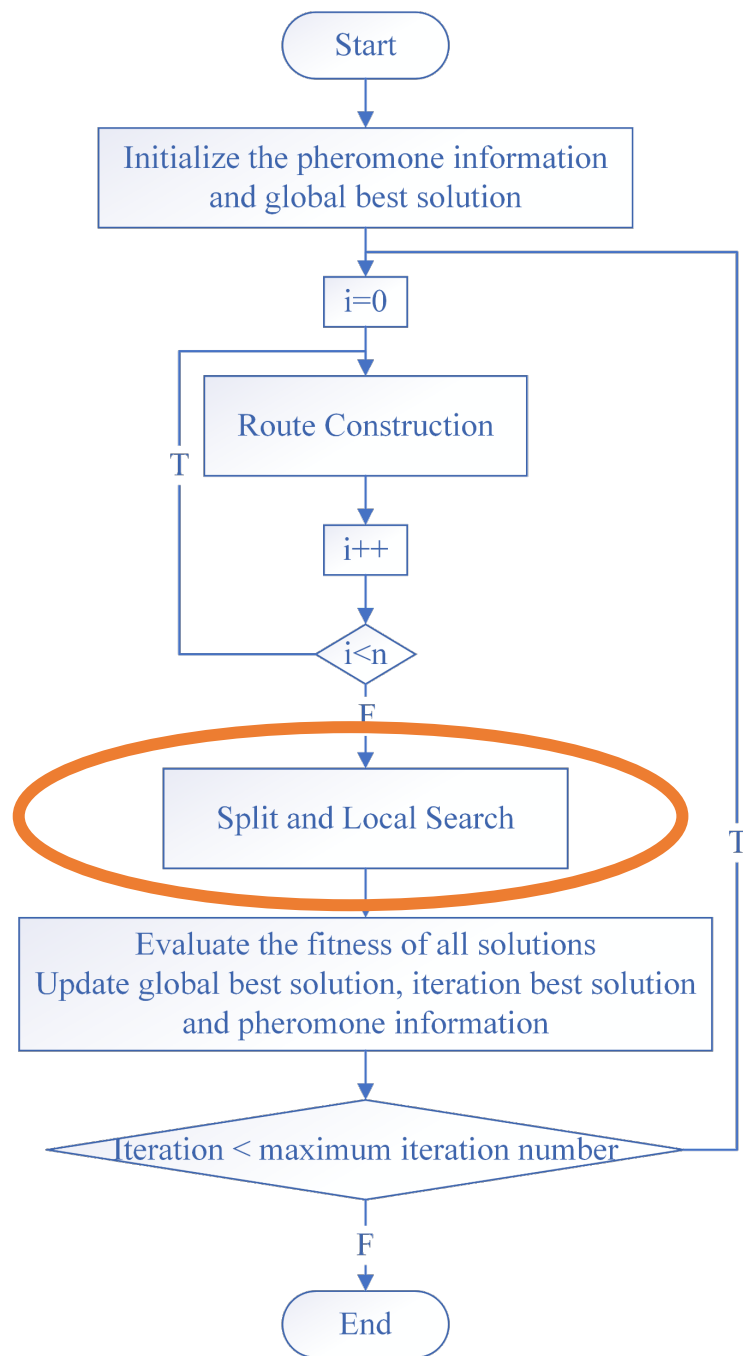
# Giant Tour



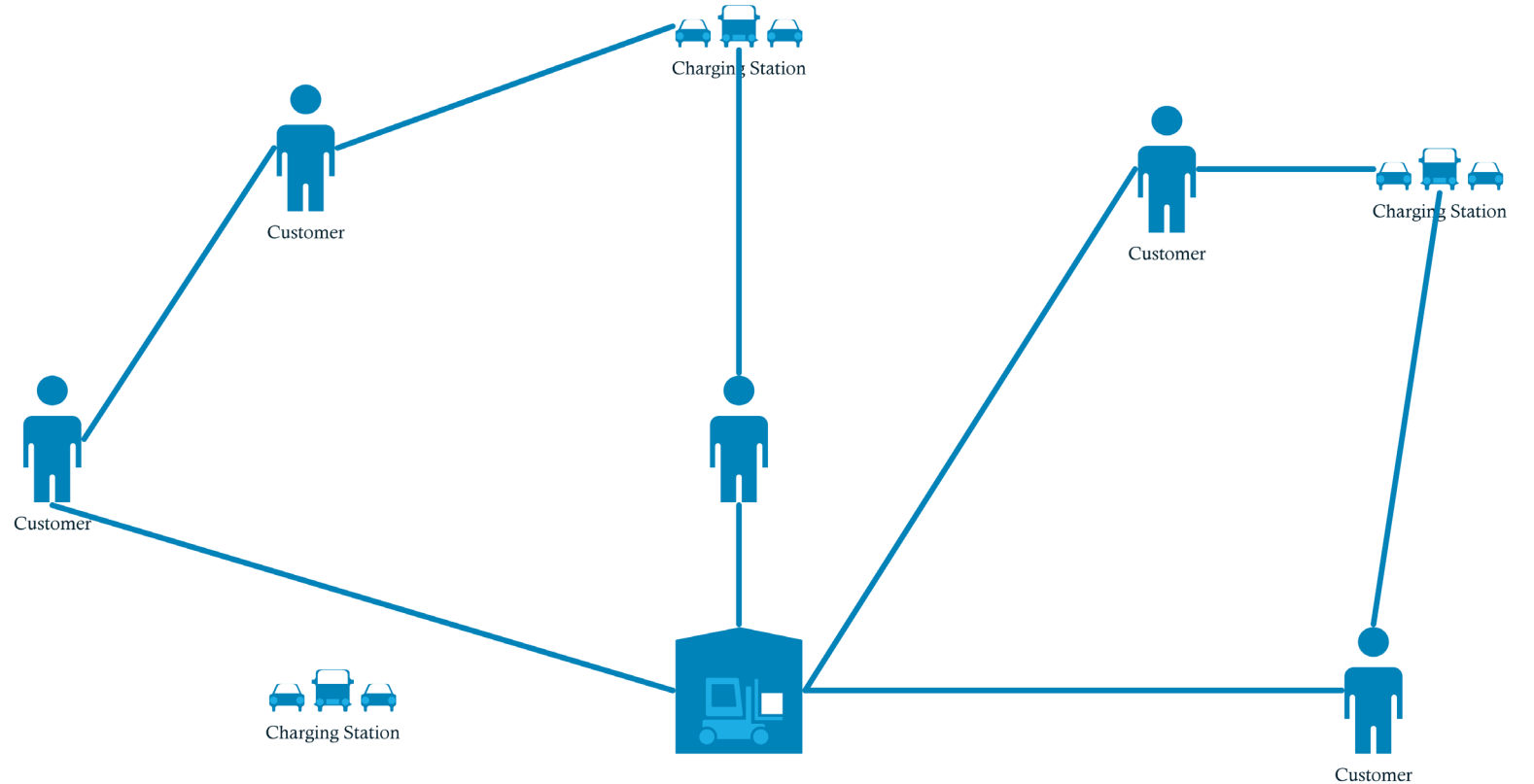


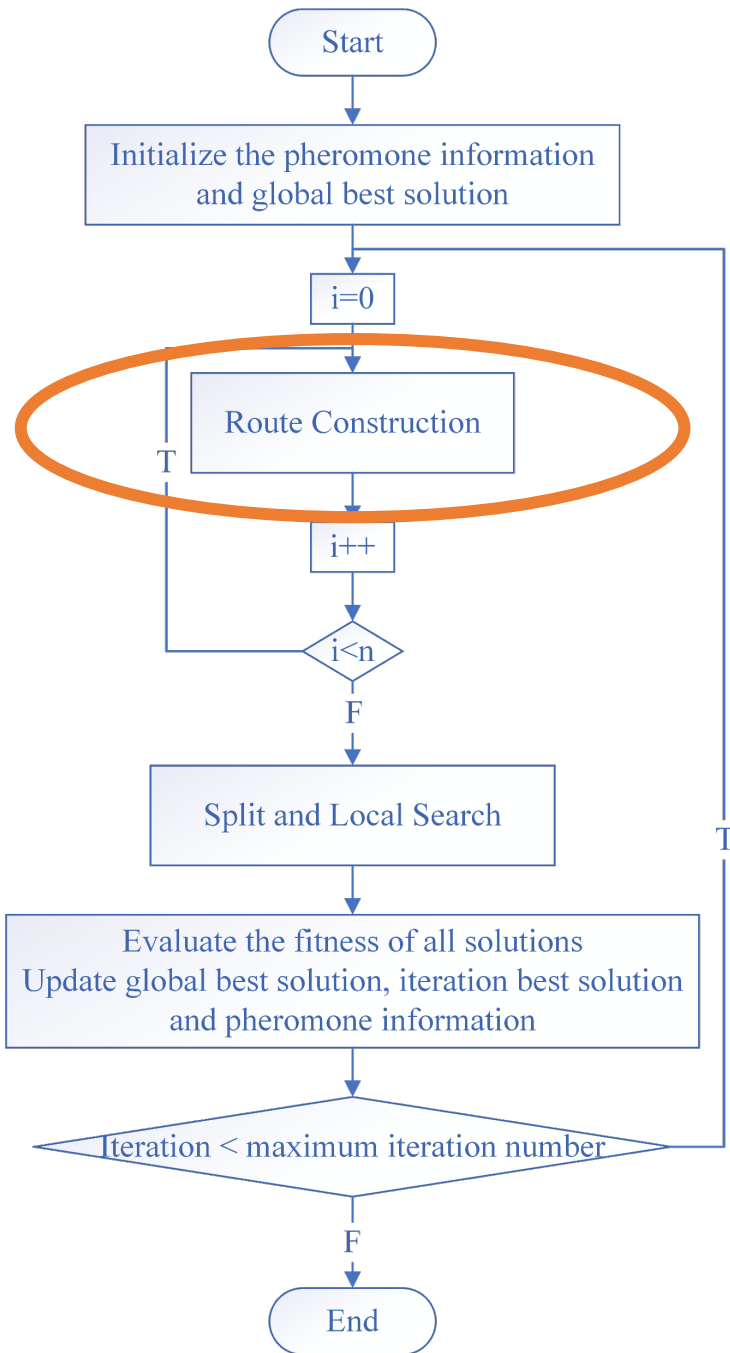
# Split





# Local Search






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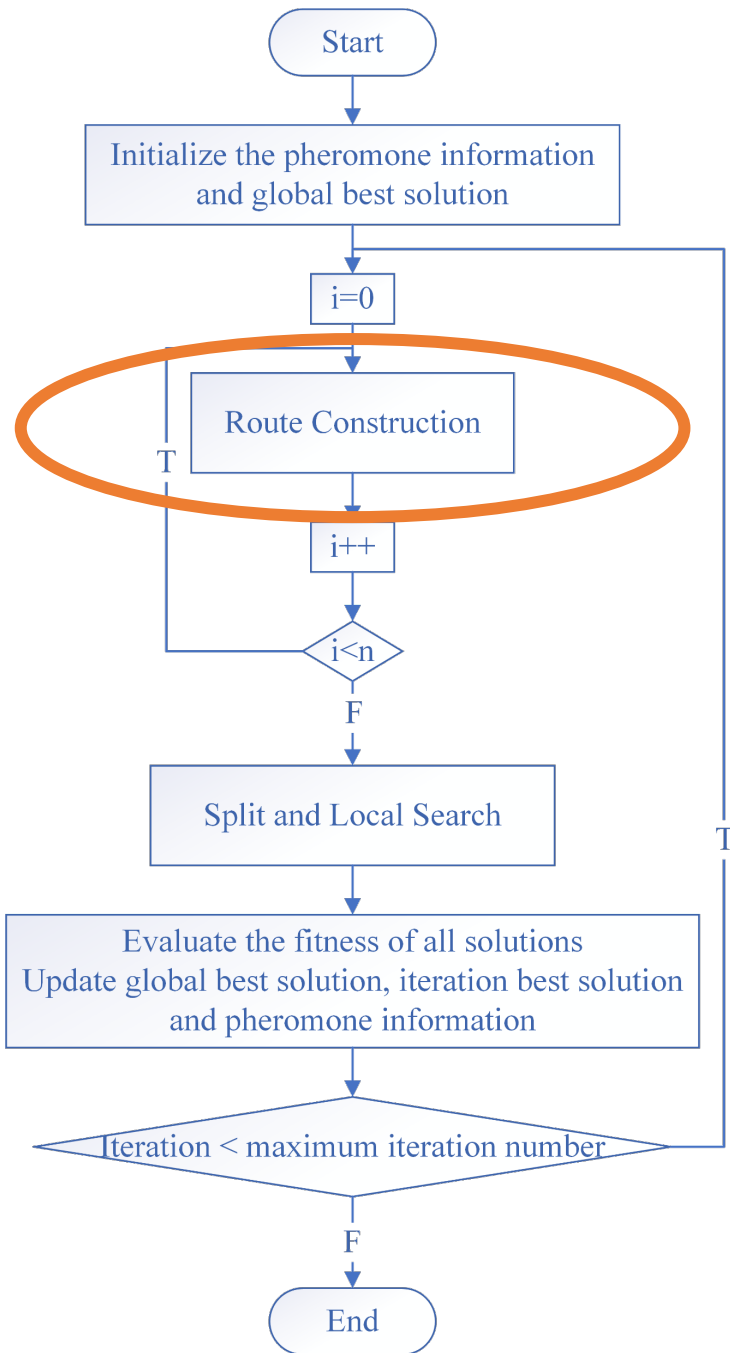
## Algorithm 1: Coarse Route Construction

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**Input:** Pheromone matrix  $\Phi$ , arc set  $E$ , node set  $\hat{I}$ , maximum battery capacity  $Q$

**Output:** a set of giant route  $T$

- 1 initialize  $T = \{0\}$ ,  $I' = I - \{0\}$  ;
  - 2 **while**  $I'$  is not empty **do**
  - 3     take the last node in  $T$  ;
  - 4      $j = roulette\_wheel(\hat{I}, i)$  ;
  - 5     append  $j$  to  $T$  ;
  - 6     remove  $j$  from  $I'$  ;
  - 7 **end**
-




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## Algorithm 1: Coarse Route Construction

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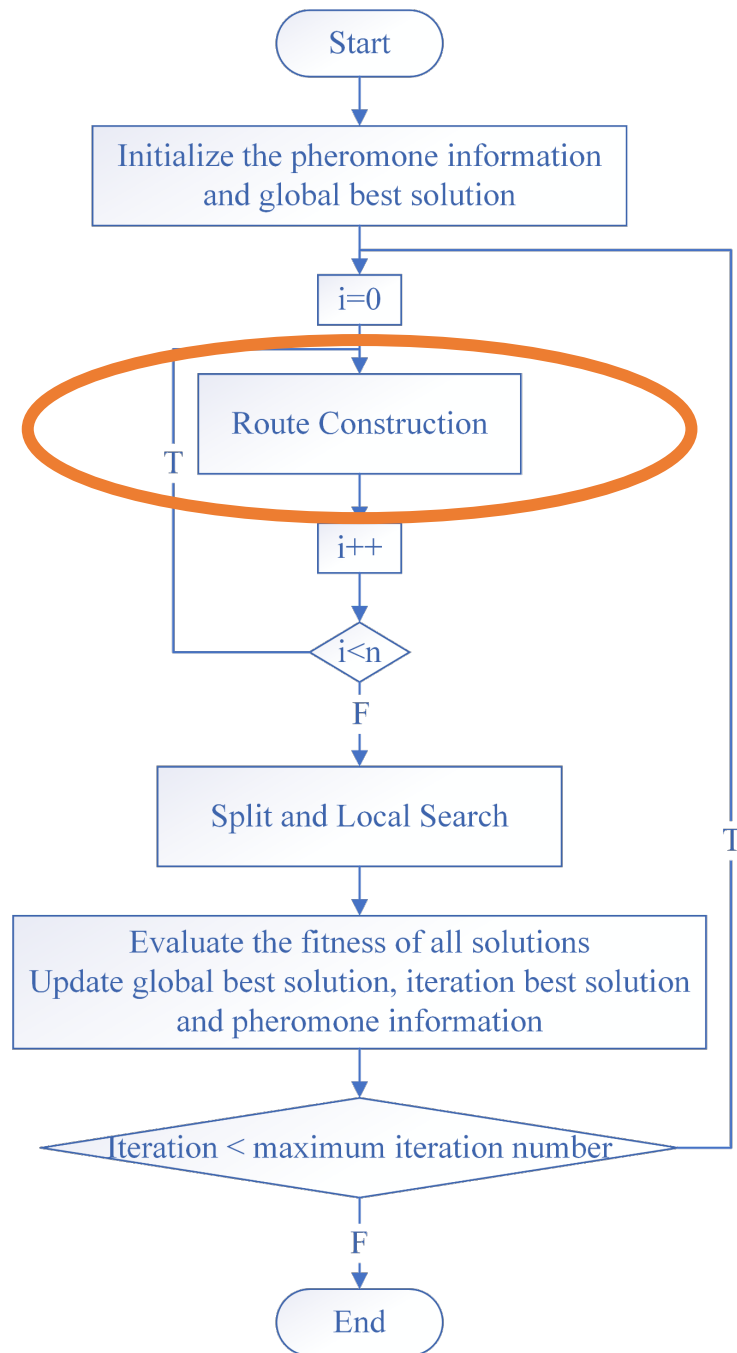
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  - 5     append  $j$  to  $T$  ;
  - 6     remove  $j$  from  $I'$  ;
  - 7 **end**
- 

**However, the giant route will be split by capacity constraints, which leads to the fact that the battery required for returning to a depot should be considered.**






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## Algorithm 2: Fine Route Construction

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**Input:** Pheromone matrix  $\Phi$ , arc set  $E$ , node set  $\hat{I}$ , maximum battery capacity  $Q$ , maximum capacity of cargo demand  $C$

**Output:** a set of giant route  $T$

```

1 initialize  $T = \{0\}$ ,  $I' = I - \{0\}$  ;
2 while  $I'$  is not empty do
3   take the last node in  $T$  ;
4   if  $Rc_i < average_{demand}$  then
5      $Rq_i = Q$  ;
6      $Rc_i = C$  ;
7   end
8    $j = roulette\_wheel(\hat{I}, i)$  ;
9   append  $j$  to  $T$  ;
10  remove  $j$  from  $I'$  ;
11 end
  
```

**we re-evaluate the remaining battery level in the fine route construction method to include the battery consumption for returning to depots.**

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**OK, let's compare EACO with SLMMAS.**

**SLMMAS inserts charging stations during the construction of a route. Whenever the next selected customer cannot be reached due to the electricity constraint, one charging station that brings minimum extra distance is visited before serving the customer.**

TABLE I  
SEVENTEEN INSTANCES OF CEVRP

Case	Customer	Station	Route
E22	21	8	4
E23	22	9	3
E30	29	6	4
E33	32	6	4
E51	50	5	5
E76	75	7	7
E101	100	9	8
X143	142	4	7
X214	213	9	11
X351	350	35	40
X459	458	20	26
X573	572	6	30
X685	684	25	75
X749	748	30	98
X819	818	25	171
X916	915	9	207
X1001	1000	9	43

# Benchmark

IEEE WCCI2020 competition

Small-scale

Medium-scale

Large-scale

Instance		EACO			SLMMAS	
		EACO-C		EACO-F	AS-S	AS-O
E22	best	<b>384.68</b>		<b>384.68</b>	<b>386.15</b>	385.44
	mean	385.62	=	<b>384.68</b>	<b>386.15</b>	385.44
	std.	2.10		0.00	0.00	0.00
E23	best	585.99		<b>579.07</b>	<b>582.61</b>	<b>582.61</b>
	mean	587.16	-	<b>584.03</b>	<b>582.88</b>	582.94
	std.	1.62		3.40	0.60	0.60
E30	best	513.63		<b>513.63</b>	<b>512.72</b>	514.42
	mean	514.27	-	<b>513.63</b>	<b>512.72</b>	516.11
	std.	1.41		0.00	0.00	0.61
E33	best	<b>846.79</b>		847.07	860.62	<b>859.25</b>
	mean	<b>850.44</b>	+	850.98	863.44	<b>860.06</b>
	std.	2.55		2.22	2.18	0.74
E51	best	<b>572.36</b>		573.26	551.19	<b>549.81</b>
	mean	578.27	-	<b>575.14</b>	<b>559.34</b>	564.93
	std.	5.73		1.82	5.60	8.70
E76	best	748.04		<b>746.79</b>	744.07	<b>724.24</b>
	mean	756.67	=	<b>752.54</b>	<b>754.03</b>	762.09
	std.	13.23		8.62	8.43	13.23
E101	best	915.00		<b>904.38</b>	<b>872.98</b>	891.77
	mean	926.67	-	<b>908.39</b>	<b>885.28</b>	904.62
	std.	11.74		4.67	10.09	4.64
X143	best	<b>16434.32</b>		16465.70	<b>16885.63</b>	17238.76
	mean	<b>16619.80</b>	+	16627.03	<b>17099.20</b>	17985.91
	std.	217.13		108.45	271.24	415.20
X214	best	12160.14		<b>12088.35</b>	12197.75	<b>11421.43</b>
	mean	12369.90	=	<b>12197.89</b>	12256.69	<b>12023.72</b>
	std.	187.91		74.21	41.90	129.22
X351	best	29116.15		<b>28750.95</b>	<b>27749.48</b>	31927.71
	mean	29406.93	-	<b>29079.51</b>	<b>27820.63</b>	31927.71
	std.	294.22		187.41	65.59	0.00
X459	best	27986.71		<b>27468.29</b>	<b>28251.70</b>	28448.27
	mean	28081.82	+	<b>27986.53</b>	<b>28469.61</b>	29434.48
	std.	85.34		299.65	153.36	540.16
X573	best	56738.10		<b>55986.28</b>	57670.41	<b>56609.94</b>
	mean	57068.45	+	<b>56752.22</b>	58167.74	<b>57170.35</b>
	std.	306.68		433.90	289.61	199.81
X685	best	<b>79673.84</b>		79822.20	<b>78862.87</b>	84435.72
	mean	<b>80076.58</b>	-	80168.38	<b>79012.35</b>	84435.72
	std.	264.30		266.54	104.89	0.00
X749	best	<b>87040.68</b>		87731.50	<b>88070.85</b>	90441.60
	mean	<b>87450.01</b>	+	87807.86	<b>88381.83</b>	90441.36
	std.	265.64		63.69	214.63	0.00
X819	best	176356.85		<b>176184.24</b>	175315.97	<b>171553.21</b>
	mean	<b>176535.74</b>	-	176726.23	175540.33	<b>172355.34</b>
	std.	170.28		315.63	184.18	311.09
X916	best	<b>363784.87</b>		364092.53	363537.41	<b>353046.07</b>
	mean	<b>364557.39</b>	=	364755.93	364091.13	<b>354262.15</b>
	std.	613.26		623.46	321.96	464.68
X1001	best	83425.31		<b>83103.57</b>	<b>86064.64</b>	89611.93
	mean	83569.87	+	<b>83451.84</b>	<b>86064.64</b>	89611.93
	std.	148.98		278.26	0.00	0.00

Significant Better 6&7  
Equal 4  
Significant Worse 7&6  
Better\* 11

All algorithms use the same population size and stopping criterion for fair comparisons and SLMMAS is denoted as AS-S. In addition, the results of SLMMAS are also presented with the same parameter setting as that in the original study for analysis, which is denoted as AS-O.

\*We cannot apply the Wilcoxon rank-sum test between EACO and AS-O, for the detailed result of AS-O is not available.

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The proposed EACO outperforms SLMMAS on medium-scale instances and the largest instance.

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	std.	187.91		74.21	41.90	129.22
X351	best	29116.15		<b>28750.95</b>	<b>27749.48</b>	31927.71
	mean	29406.93	-	<b>29079.51</b>	<b>27820.63</b>	31927.71
	std.	294.22		187.41	65.59	0.00
X459	best	27986.71		<b>27468.29</b>	<b>28251.70</b>	28448.27
	mean	28081.82	+	<b>27986.53</b>	<b>28469.61</b>	29434.48
	std.	85.34		299.65	153.36	540.16
X573	best	56738.10		<b>55986.28</b>	57670.41	<b>56609.94</b>
	mean	57068.45	+	<b>56752.22</b>	58167.74	<b>57170.35</b>
	std.	306.68		433.90	289.61	199.81
X685	best	<b>79673.84</b>		79822.20	<b>78862.87</b>	84435.72
	mean	<b>80076.58</b>	-	80168.38	<b>79012.35</b>	84435.72
	std.	264.30		266.54	104.89	0.00
X749	best	<b>87040.68</b>		87731.50	<b>88070.85</b>	90441.60
	mean	<b>87450.01</b>	+	87807.86	<b>88381.83</b>	90441.36
	std.	265.64		63.69	214.63	0.00
X819	best	176356.85		<b>176184.24</b>	175315.97	<b>171553.21</b>
	mean	<b>176535.74</b>	-	176726.23	175540.33	<b>172355.34</b>
	std.	170.28		315.63	184.18	311.09
X916	best	<b>363784.87</b>		364092.53	363537.41	<b>353046.07</b>
	mean	<b>364557.39</b>	=	364755.93	364091.13	<b>354262.15</b>
	std.	613.26		623.46	321.96	464.68
X1001	best	83425.31		<b>83103.57</b>	<b>86064.64</b>	89611.93
	mean	83569.87	+	<b>83451.84</b>	<b>86064.64</b>	89611.93
	std.	148.98		278.26	0.00	0.00

AS-S has overtaken AS-O\* on 12 instances, by comparing the mean values\*\*, especially on the medium-scale and large-scale instances.

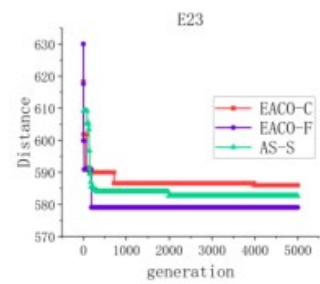
It indicates that the SLMMAS can find better solutions using more iterations rather than larger population sizes.

All algorithms use the same population size and stopping criterion for fair comparisons and SLMMAS is denoted as AS-S. In addition, the results of SLMMAS are also presented with the same parameter setting as that in the original study for analysis, which is denoted as AS-O.

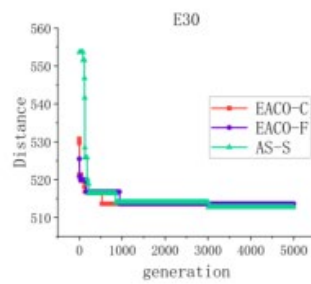
\*The algorithm setting of AS-O is that the stopping criterion is defined as fixed execution time and population size is set to  $n = |I| + 1$  for each instance.

\*\*We cannot apply the Wilcoxon rank-sum test between EACO and AS-O, for the detailed result of AS-O is not available.

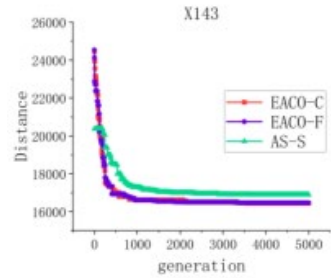




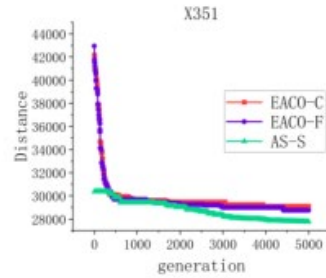
(a) E23



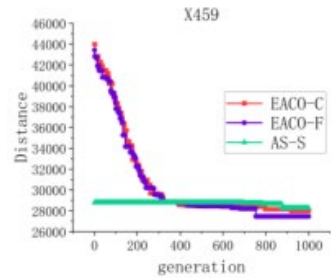
(b) E30



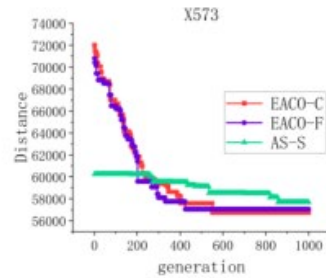
(c) X143



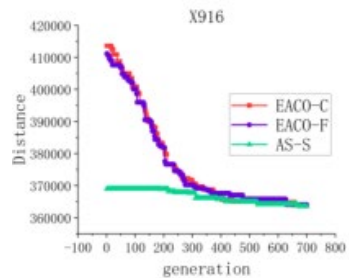
(d) X351



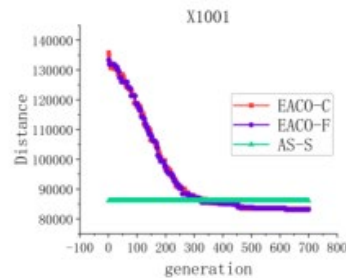
(e) X459



(f) X573



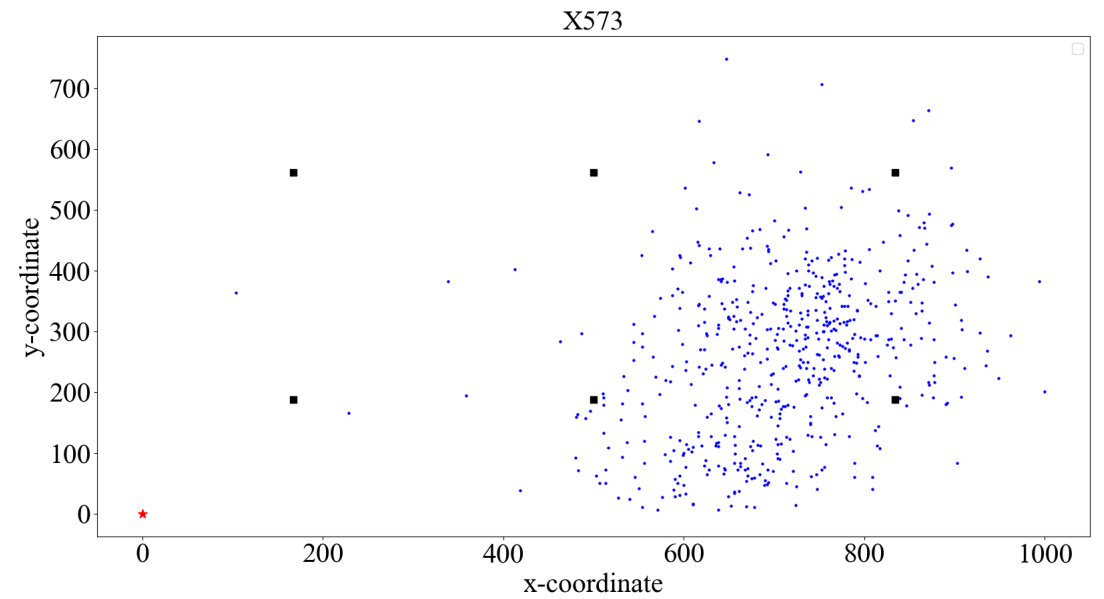
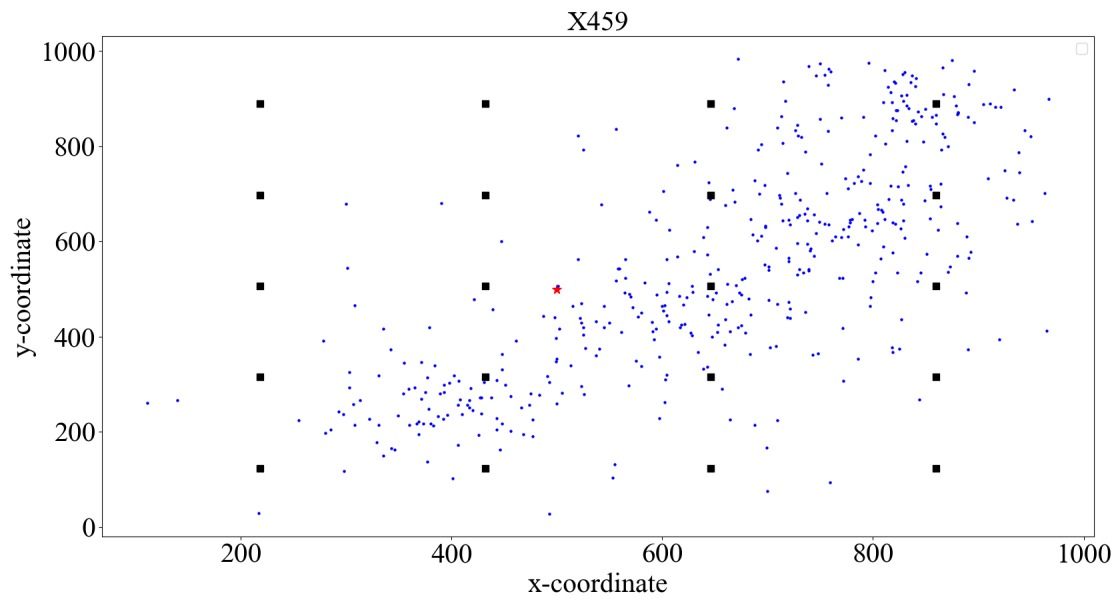
(g) X916



(h) X1001

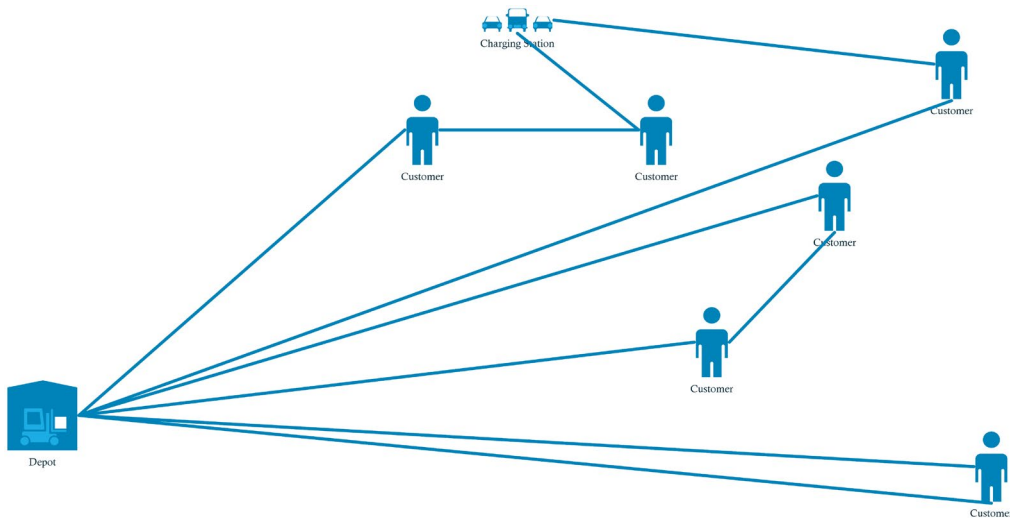
Generally, SLMMAS can construct good solutions in early stage and may stagnate quickly. **It can deal with small-scale problems, but failed on large-scale instances.** By contrast, though EACO finds worse solutions in the early stage but converges quickly and surpasses SLMMAS on most instances. **In addition, EACO presents stronger exploration ability, especially on medium-scale and large-scale instances.**

All algorithms use the same population size and stopping criterion for fair comparisons and SLMMAS is denoted as AS-S.



**EACO-F is better**

**EACO-C is better**



**The EVs run frequently between depot and customers with a long distance, which makes the search space of EACO-F extremely huge and hard to find better solutions.**



**We generally recommend**

**EACO**

**with fine route construction method**

# THANKS

2022.12

## **Efficient Extended Ant Colony Optimization for Capacitated Electric Vehicle Routing**

Bo-Cheng Lin, Xiao-Fang Liu, Yi Mei