VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

Evolutionary Computation Research Group

Ant Colony Optimization

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2024.1.26

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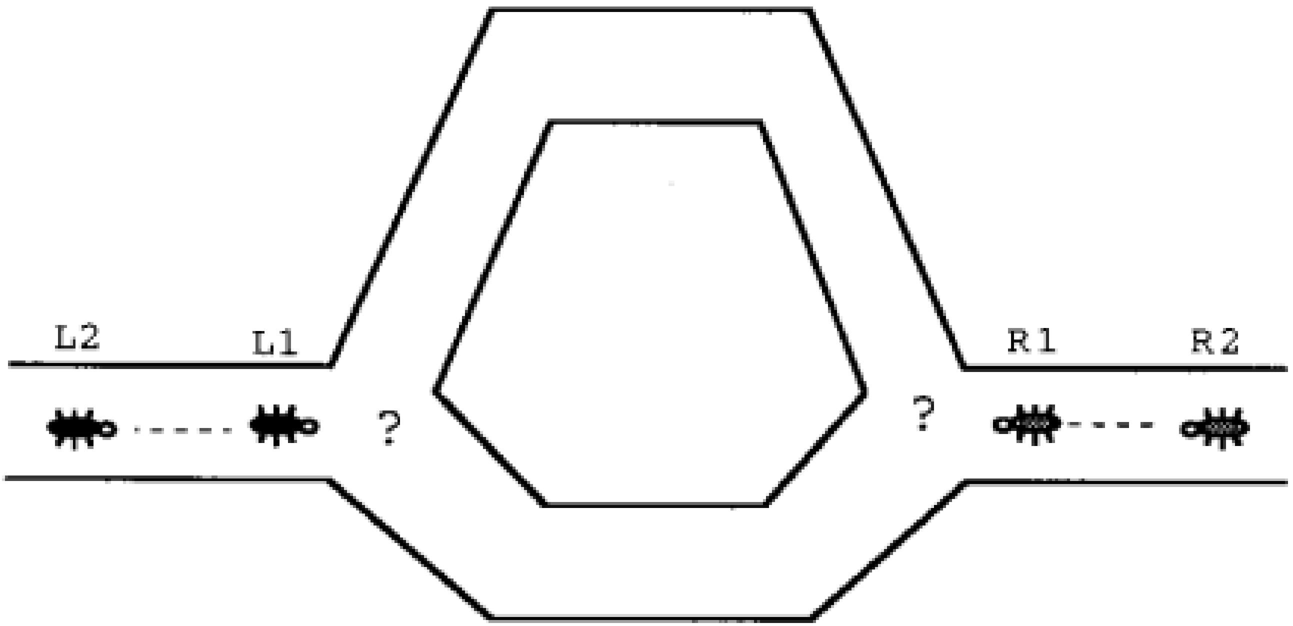
PART I

What is ACO?

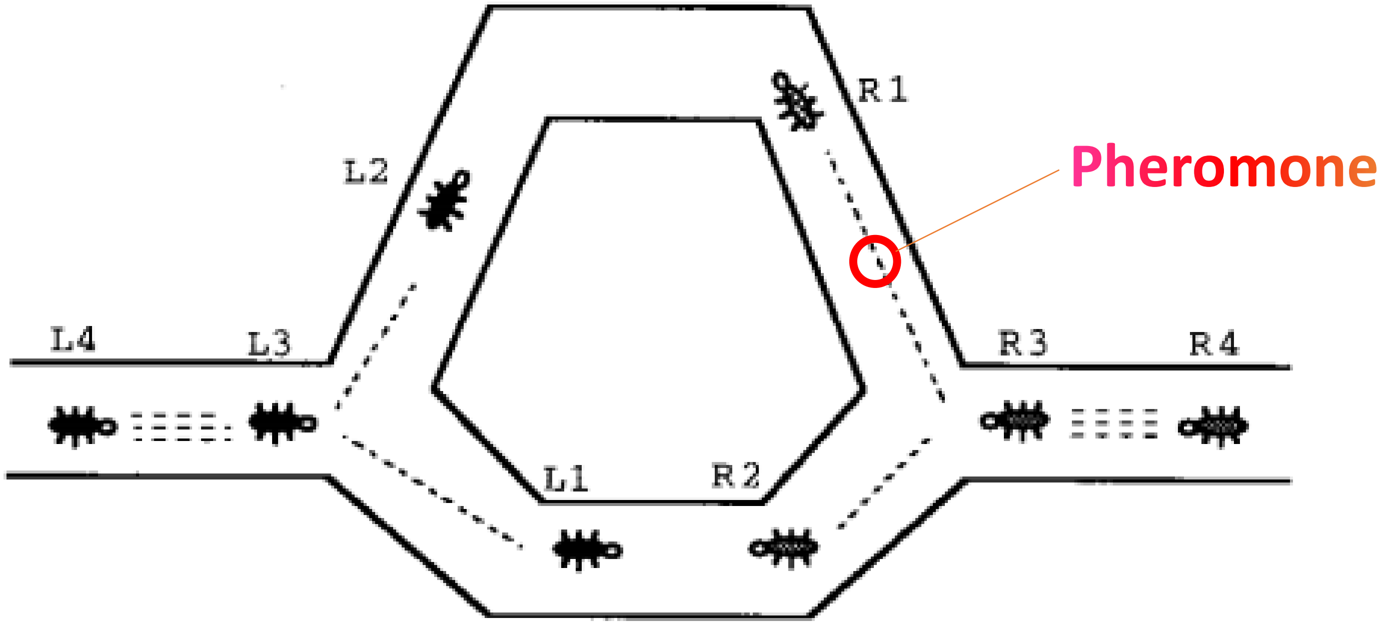
Ants arrive at a decision point.
How real ants find a shortest path?



(a)

[1] M. Dorigo and L. M. Gambardella, “Ant colony system: a cooperative learning approach to the traveling salesman problem,” IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53–66, Apr. 1997, doi: 10.1109/4235.585892.

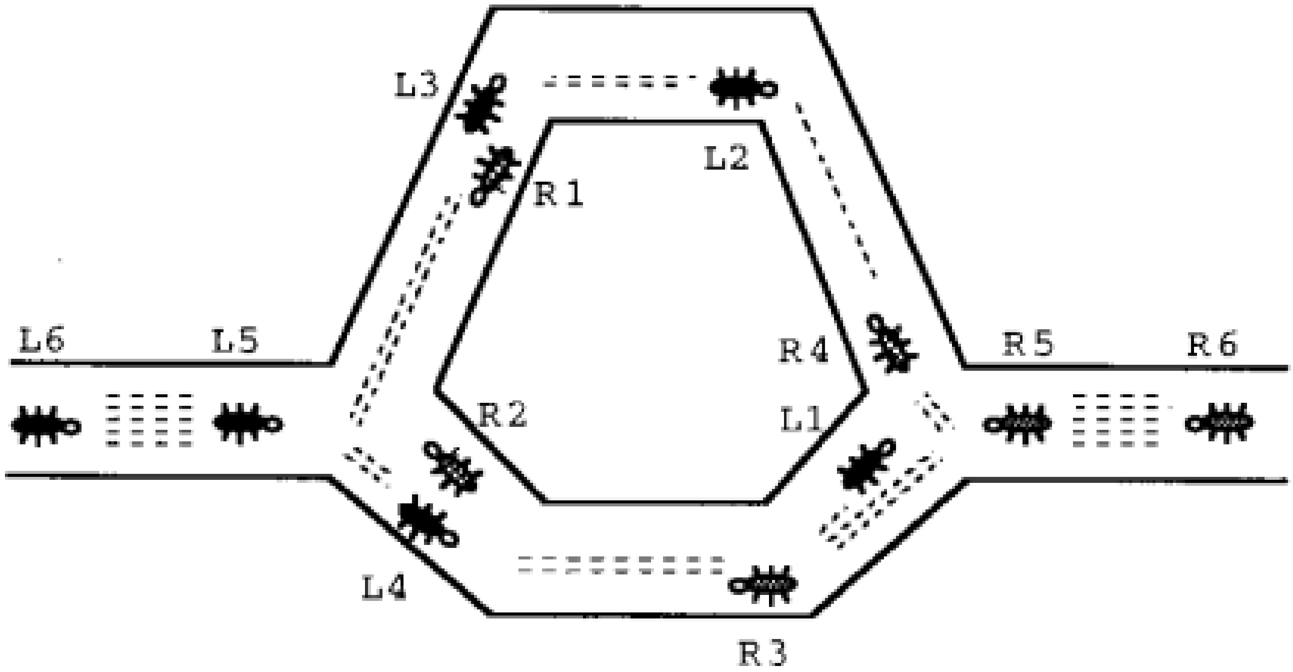
Some ants choose upper path, and some the lower path, randomly.



(b)

[1] M. Dorigo and L. M. Gambardella, “Ant colony system: a cooperative learning approach to the traveling salesman problem,” IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53–66, Apr. 1997, doi: 10.1109/4235.585892.

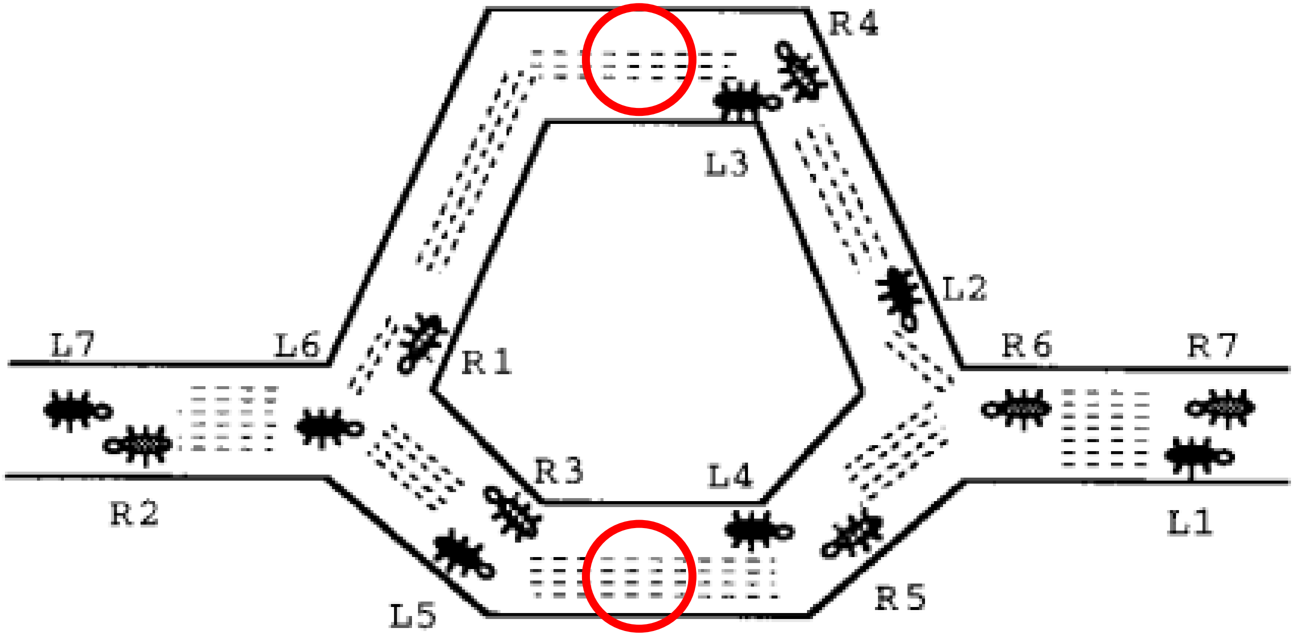
The ants move at same speed, so the ants choose shorter path reach opposite point faster.



(c)

[1] M. Dorigo and L. M. Gambardella, “Ant colony system: a cooperative learning approach to the traveling salesman problem,” IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53–66, Apr. 1997, doi: 10.1109/4235.585892.

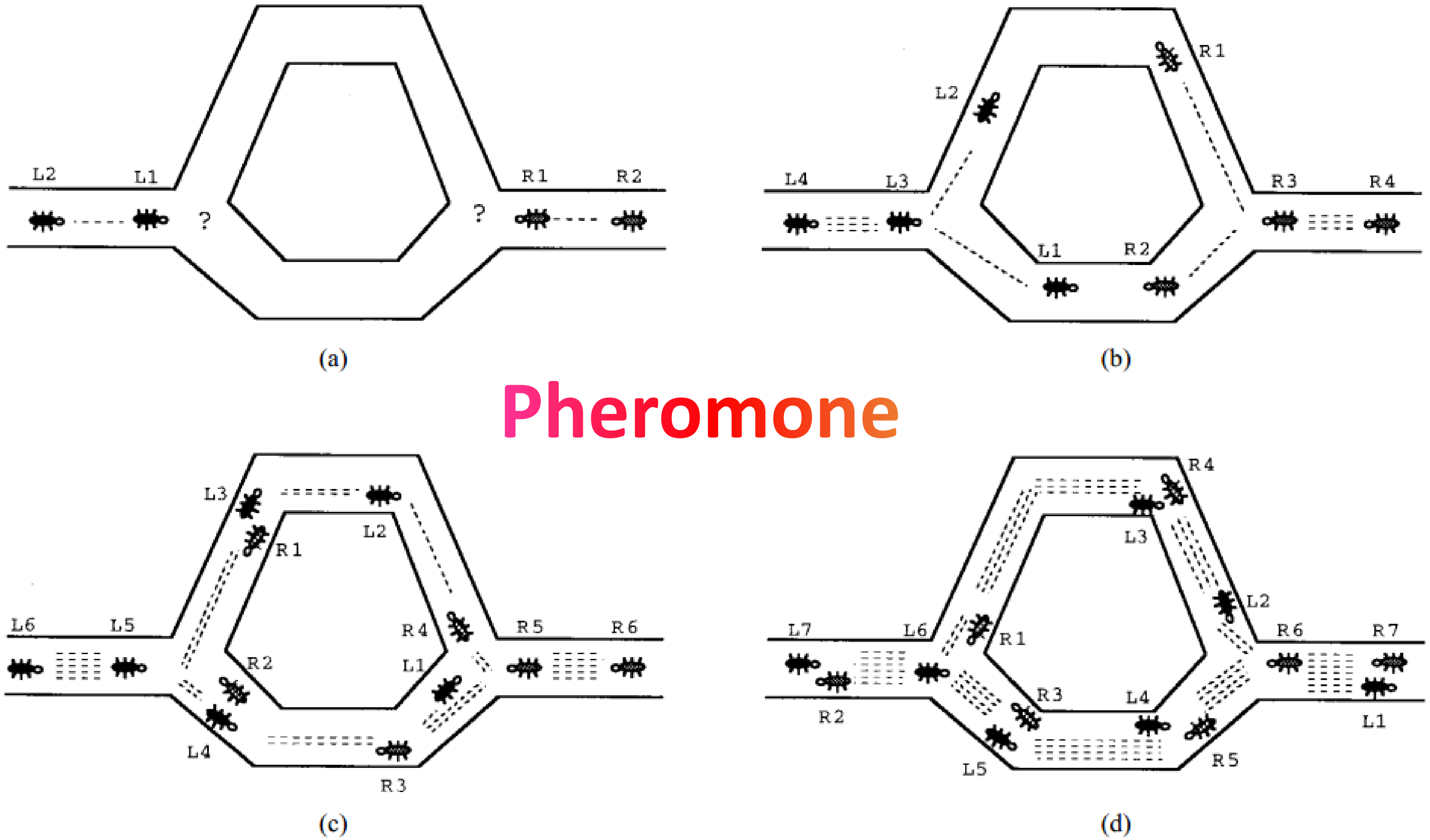
Pheromone accumulates at a higher rate on shorter path.



(d)

[1] M. Dorigo and L. M. Gambardella, “Ant colony system: a cooperative learning approach to the traveling salesman problem,” IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53–66, Apr. 1997, doi: 10.1109/4235.585892.

This is how ants find shorter path.

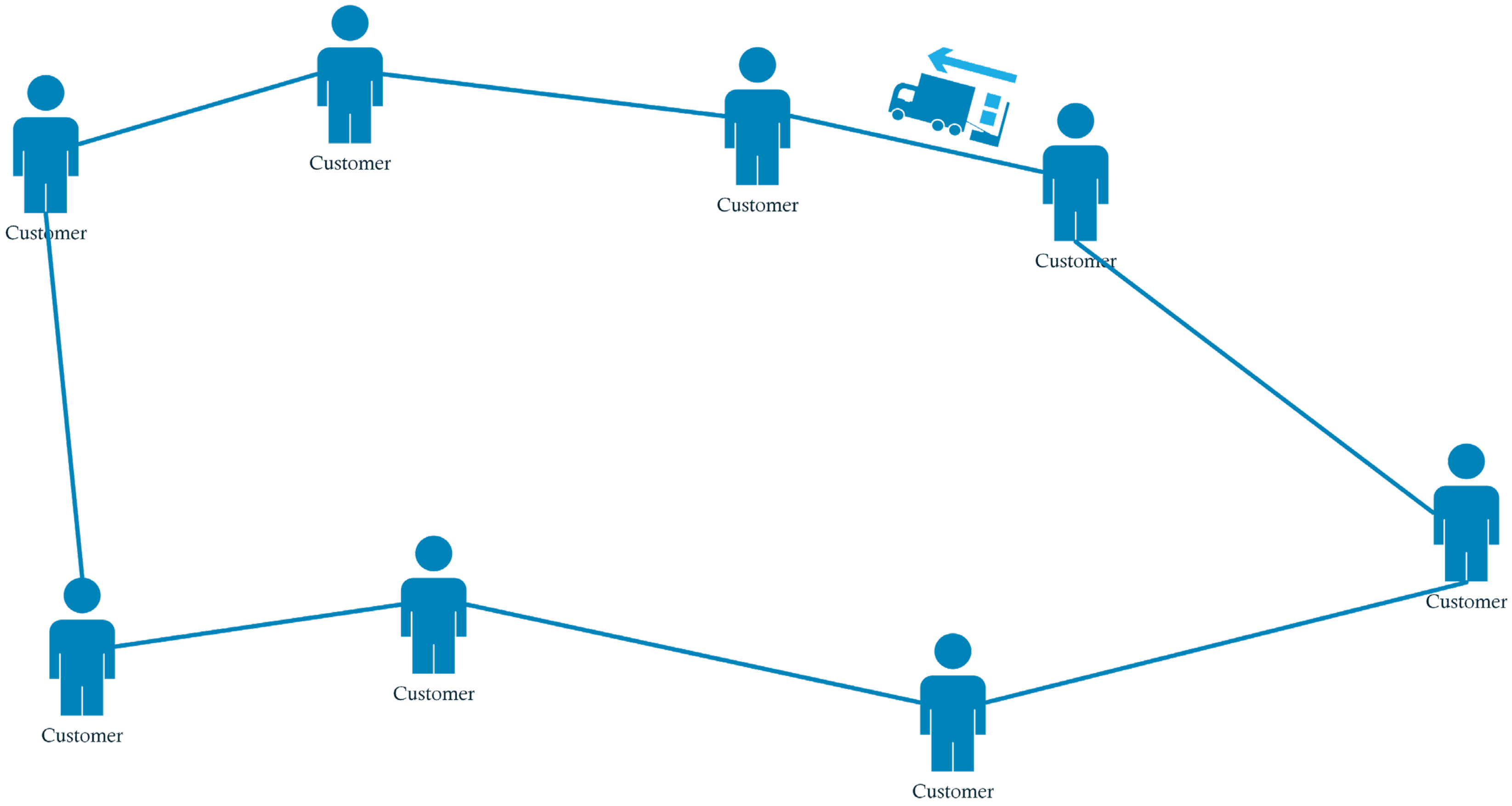


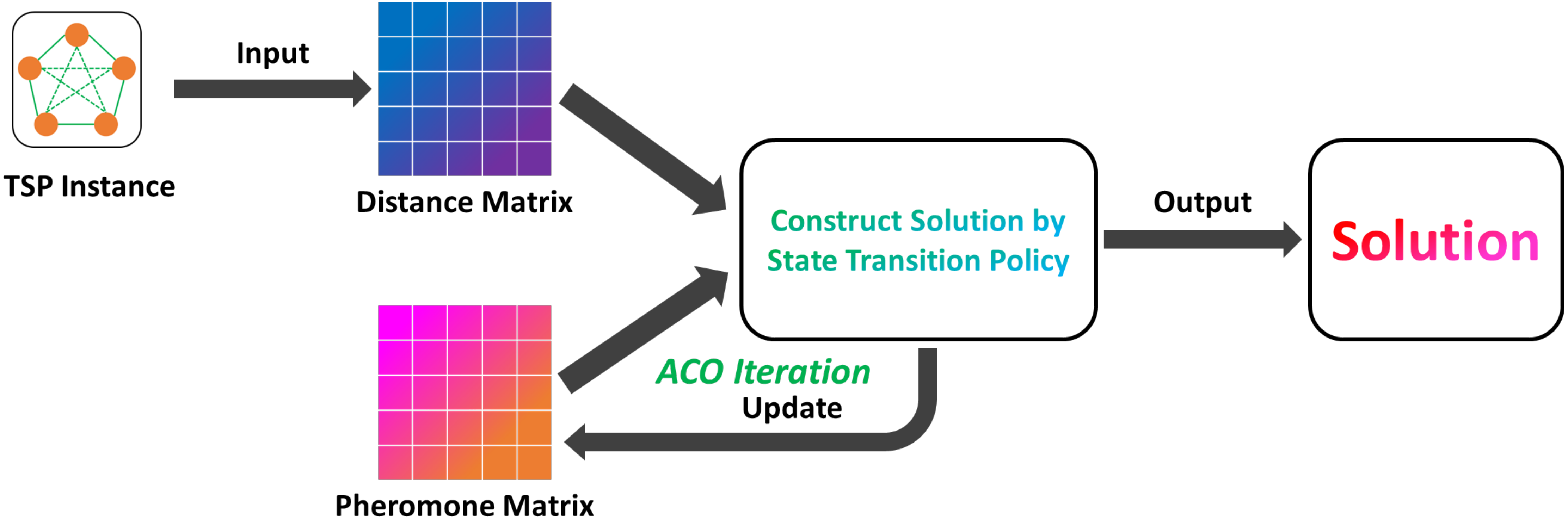
[1] M. Dorigo and L. M. Gambardella, “Ant colony system: a cooperative learning approach to the traveling salesman problem,” IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53–66, Apr. 1997, doi: 10.1109/4235.585892.

How ants find shortest path on more complex graph?

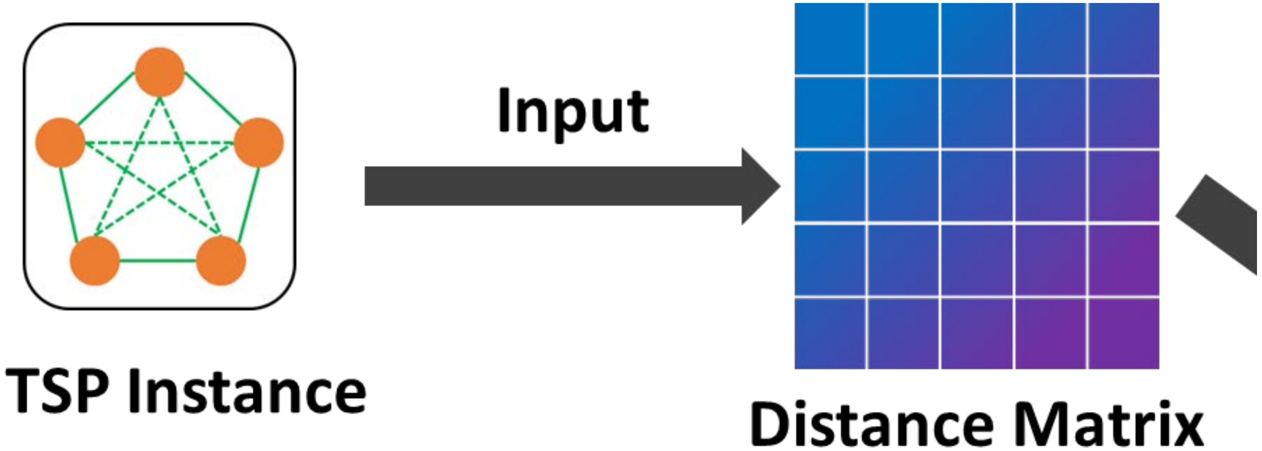
Travelling Salesman Problem

TSP

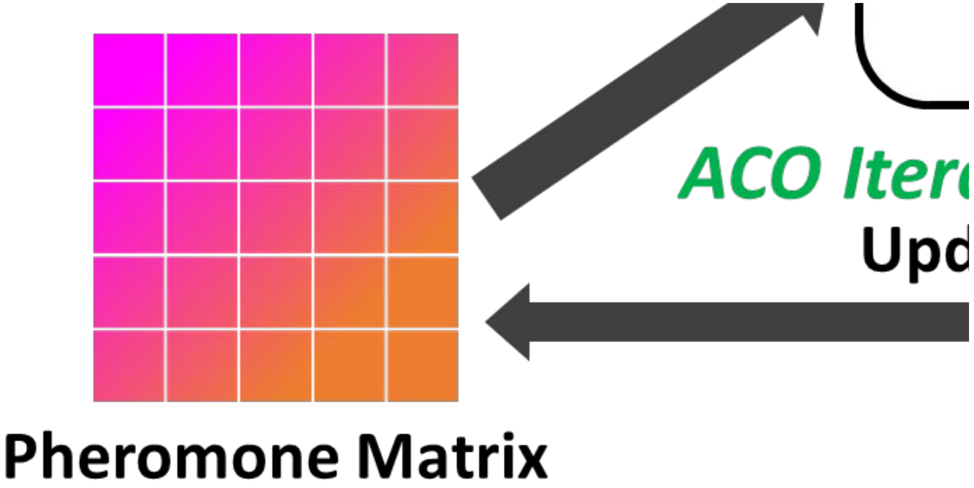



Input
TSP Instance
Distance Matrix
Construct Solution by
State Transition Policy
Output
Solution
ACO Iteration
Update
Pheromone Matrix

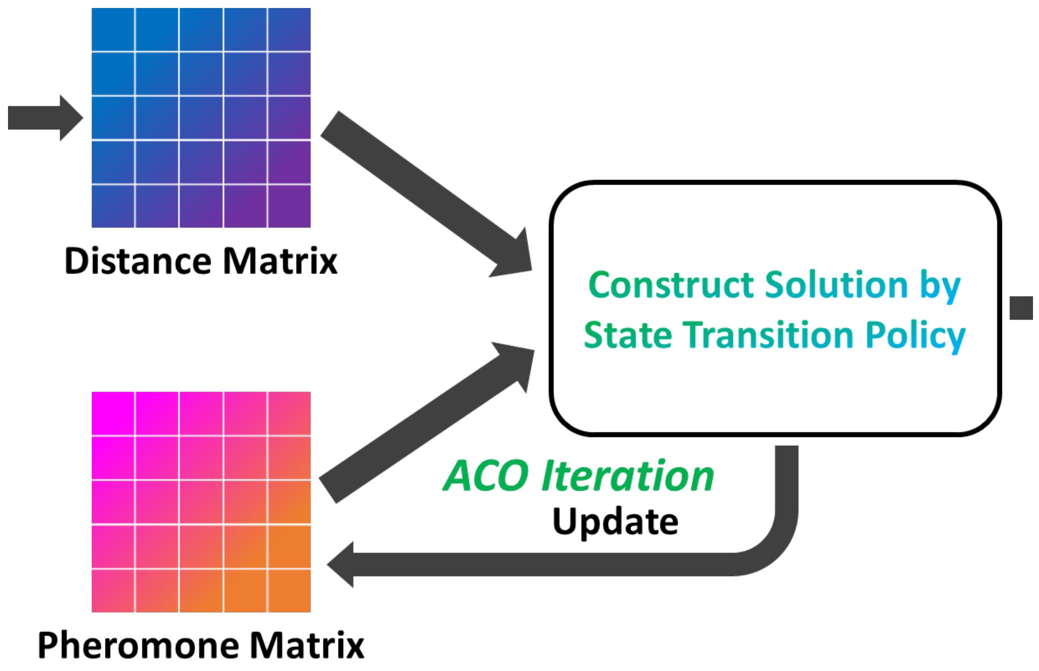
A graph contains n nodes



A $n \times n$ matrix records the distance d_{ij} between nodes



A $n \times n$ matrix records the pheromone φ_{ij} between nodes



The core component in ACO

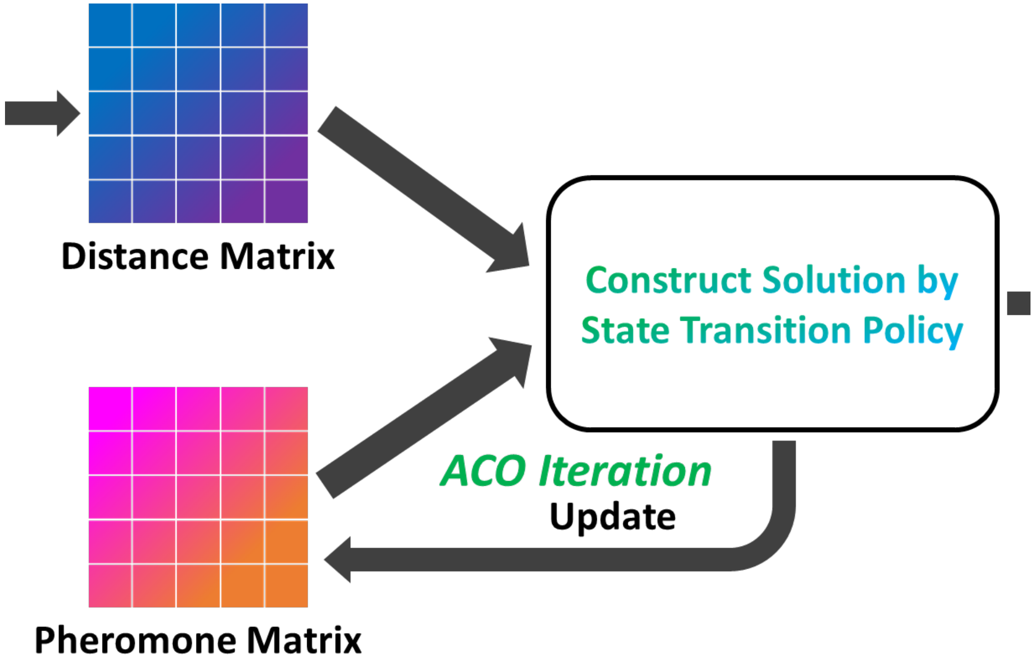
Algorithm 4 ACO

Input: instance

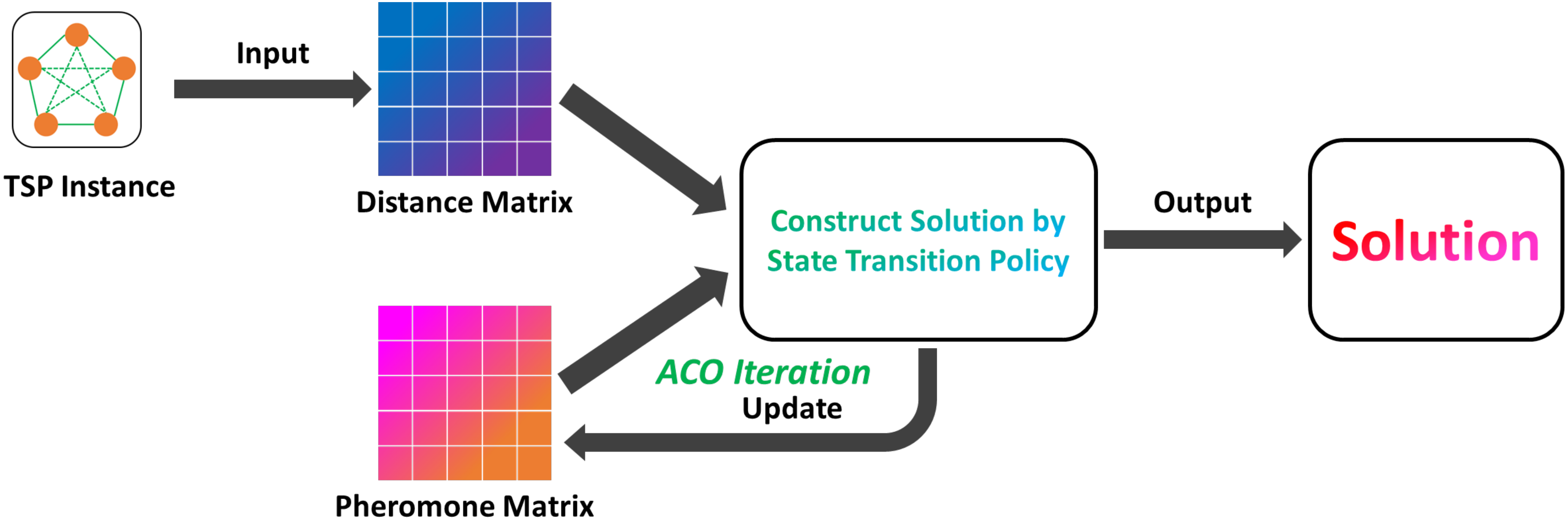
Output: path

```
load instance;
initialize pheromone matrix;
for i = 0 to max_iteration
   initialize solutions;
   for n = 0 to num_ants
      Randomly choose a node as the start
      while candidate_nodes is not empty
         next_node = State_Transition_Policy(pheromone, distance);
         append next_node to solution;
      end while
      n++;
   end for
   record best_solution;
   update pheromone matrix;
   i++;
end for
record best_solution;
return best_solution;
```

```
for n = 0 to num_ants
    Randomly choose a node as the start
    while candidate_nodes is not empty
        next_node = State_Transition_Policy(pheromone, distance);
        append next_node to solution;
    end while
    n++;
end for
```



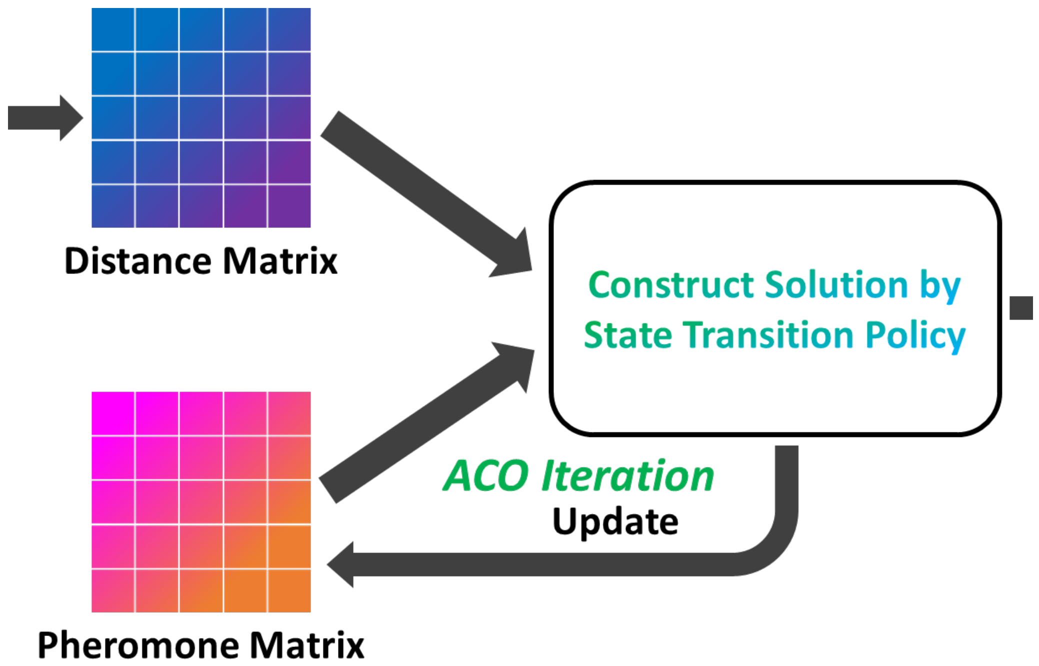
$$p_{ij} = \frac{\varphi_{ij}^{\alpha} / d_{ij}^{\beta}}{\sum_{l \in I_s} \varphi_{il}^{\alpha} / d_{il}^{\beta}}$$



TSP Instance
Input
Distance Matrix
Construct Solution by
State Transition Policy
Output
Solution
ACO Iteration
Update
Pheromone Matrix

PART II

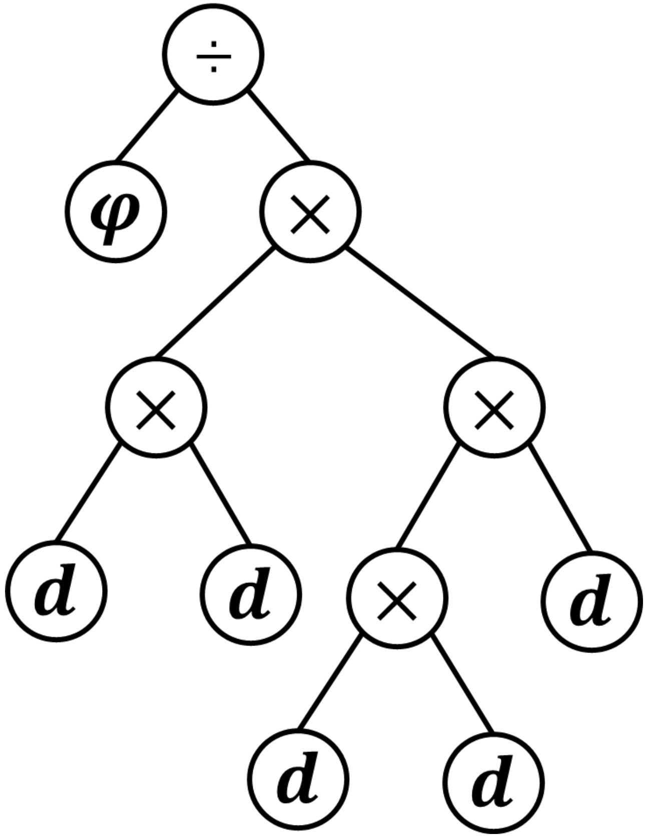
Automated Design of ACO by GP

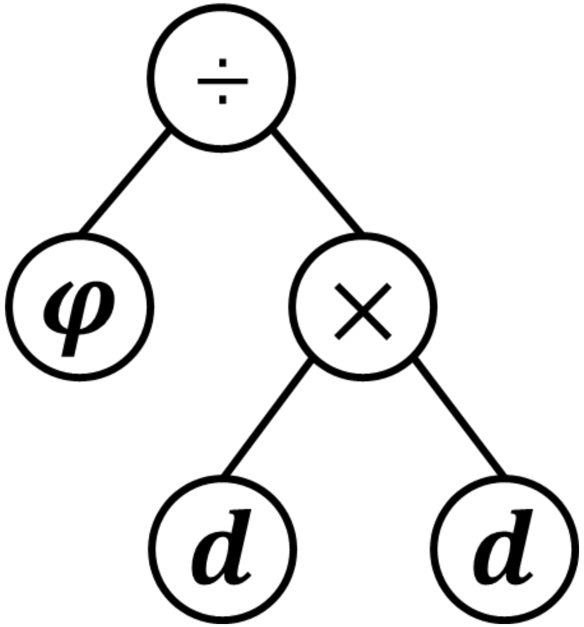


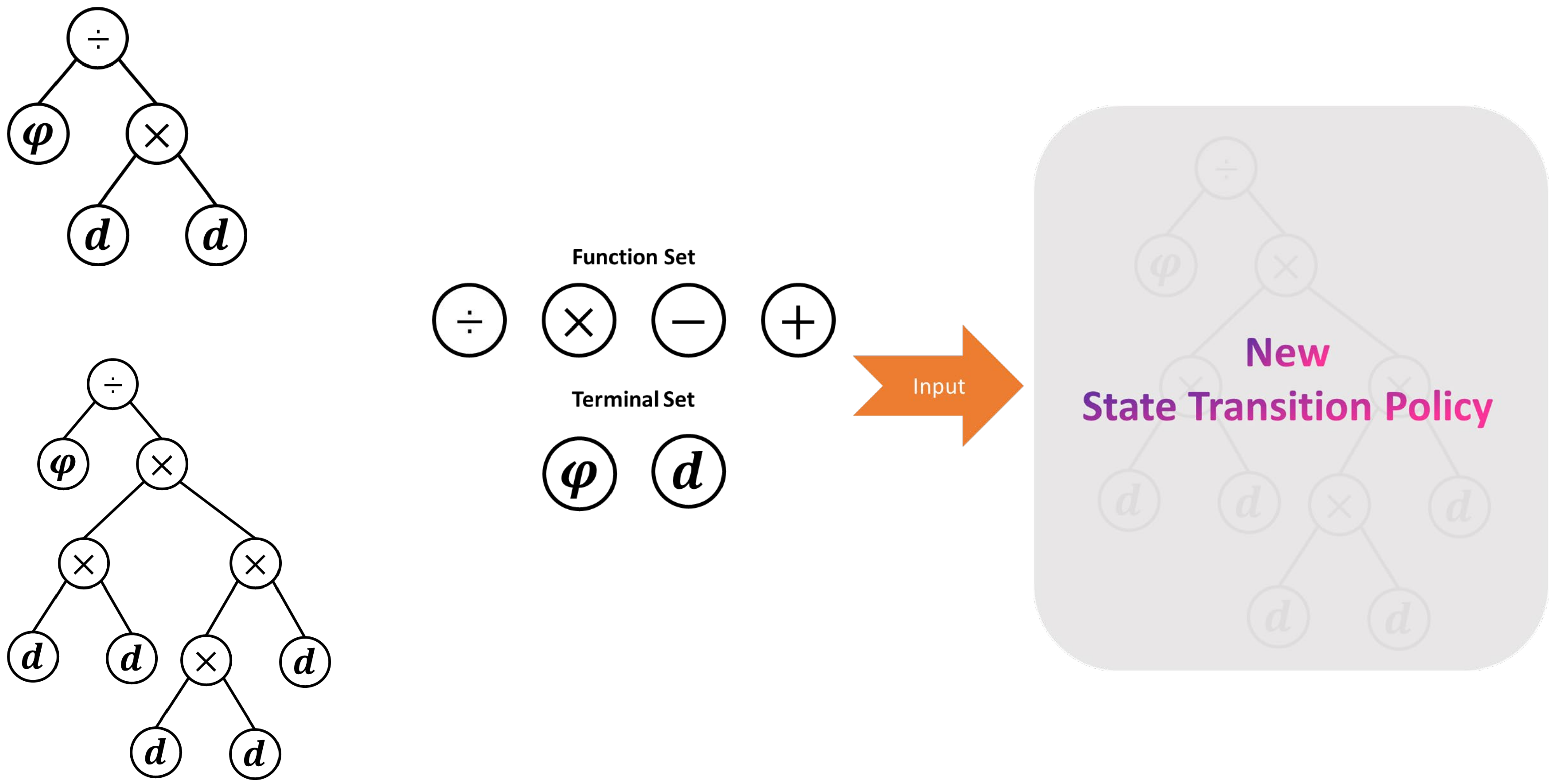
$$p_{ij} = \frac{\boxed{\varphi_{ij}^{\alpha} / d_{ij}^{\beta}}}{\sum_{l \in I_s} \varphi_{il}^{\alpha} / d_{il}^{\beta}}$$

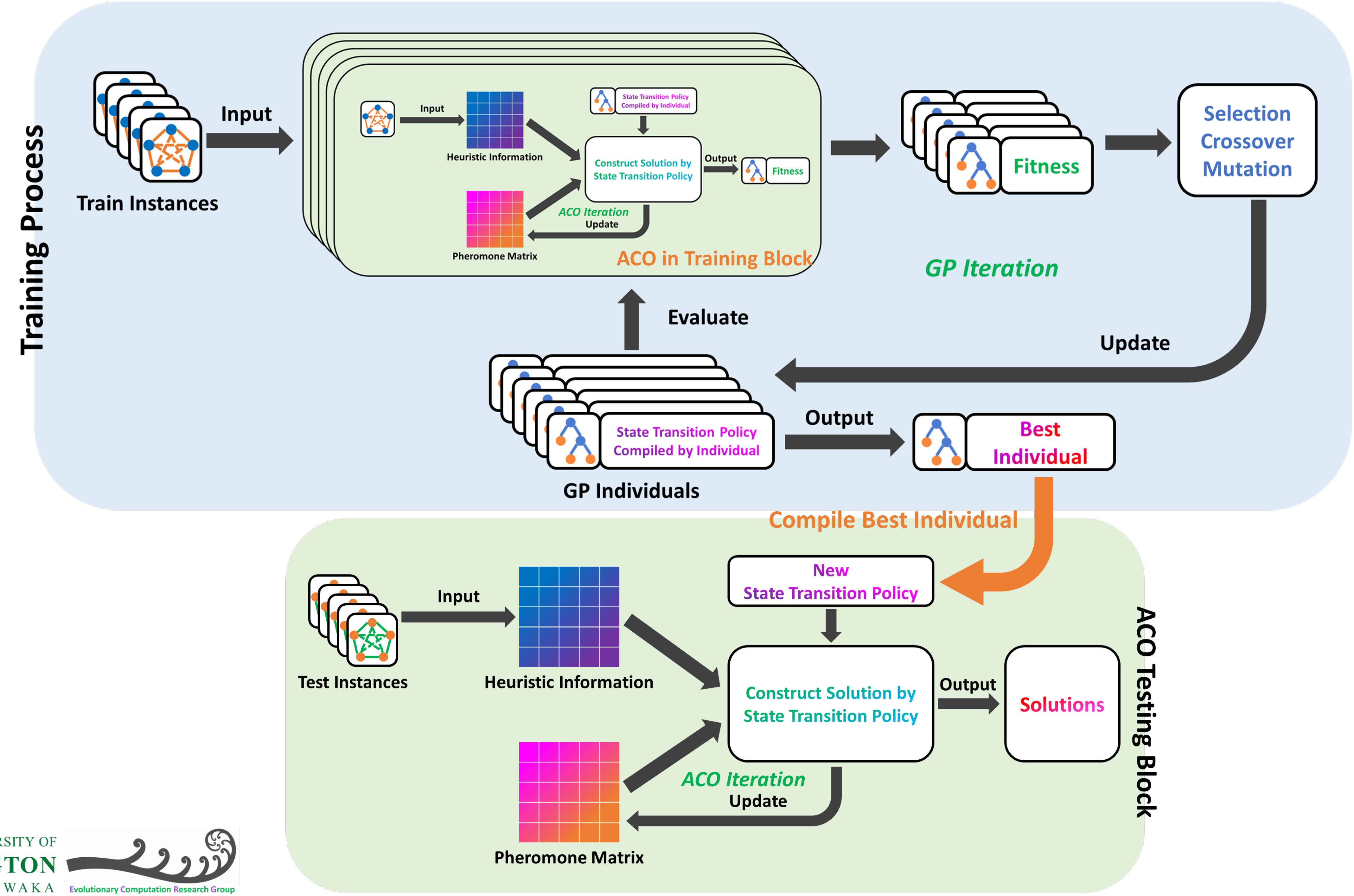
The core component in ACO

$$p_{ij} = \frac{\boxed{\varphi_{ij}^{\alpha} / d_{ij}^{\beta}}}{\sum_{l \in I_s} \varphi_{il}^{\alpha} / d_{il}^{\beta}}$$






Function Set
Terminal Set
Input
New
State Transition Policy



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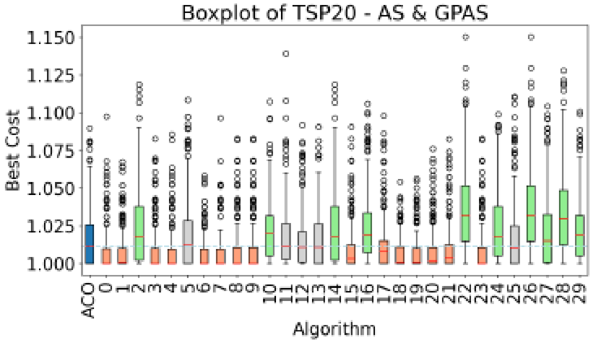
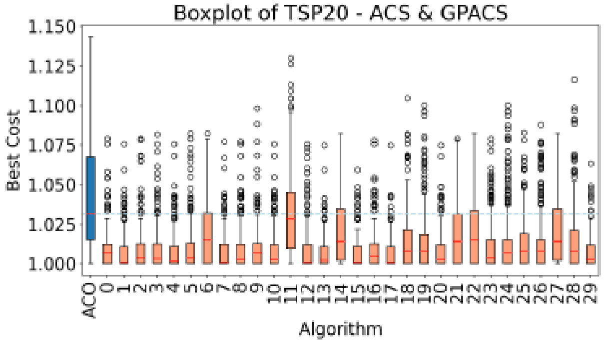
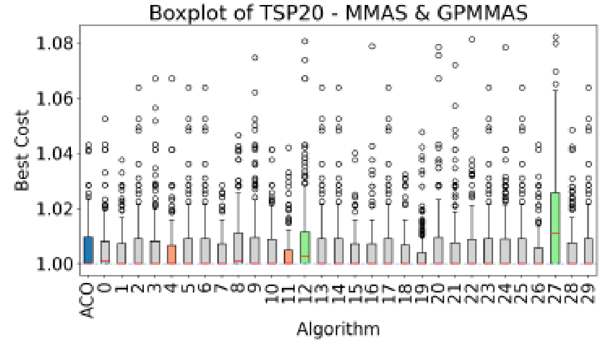
TSP		AS			ACS			MMAS		
		baseline	**GP**		**baseline**	**GP**		**baseline**	**GP**	
20	mean	1.01581	1.01600	=	1.04235	**1.01226**	+	1.00504	1.00547	=
	std	0.01744	0.02131		0.03516	0.01726		0.00763	0.00947	
	min	1	1		1	1		1	1	
	max	1.08989	1.15052		1.14352	1.13015		1.04311	1.08232	
20-100	mean	1.06770	**1.06158**	+	1.10647	**1.05448**	+	**1.04658**	1.04895	-
	std	0.02786	0.02914		0.03386	0.02430		0.02422	0.02917	
	min	1	1		1.02539	1		1	1	
	max	1.14753	1.18756		1.19654	1.15790		1.11980	1.21483	
100	mean	1.07363	**1.06846**	+	1.09604	**1.05931**	+	1.06005	**1.04381**	+
	std	0.02670	0.02941		0.03193	0.02037		0.02445	0.01964	
	min	1.01843	1		1.02711	1.00375		1.00571	1	
	max	1.15084	1.20156		1.16721	1.15470		1.12775	1.13291	



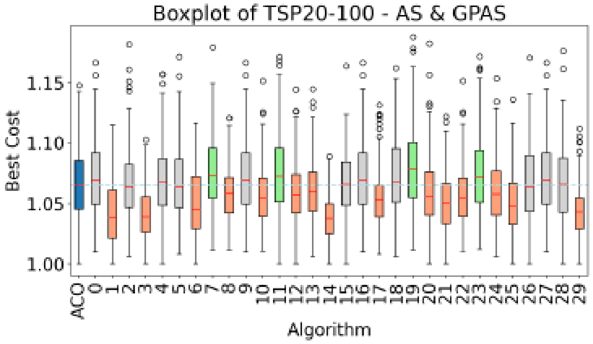
(a) TSP20 - AS



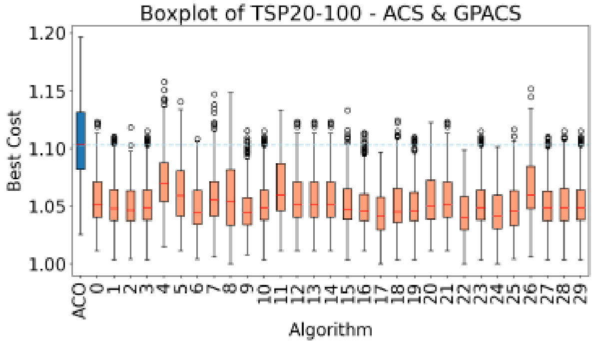
(b) TSP20 - ACS



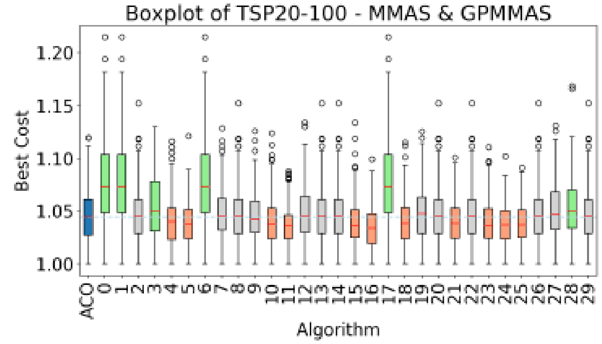
(c) TSP20 - MMAS



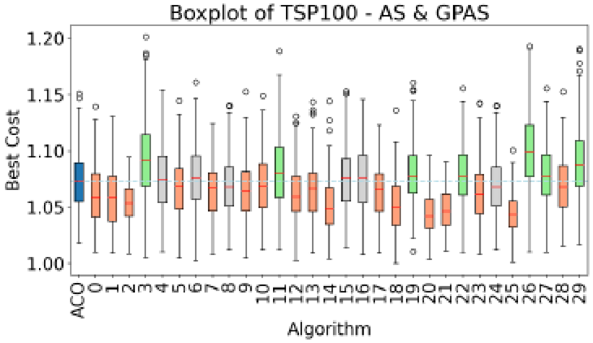
(d) TSP20-100 - AS



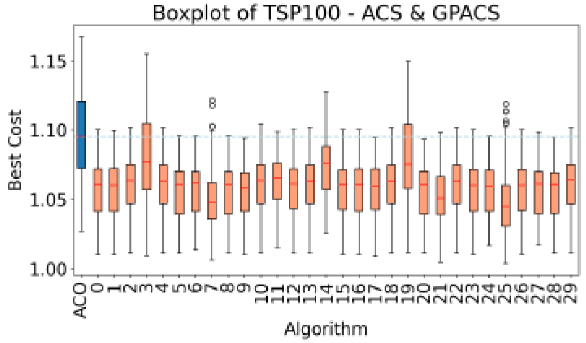
(e) TSP20-100 - ACS



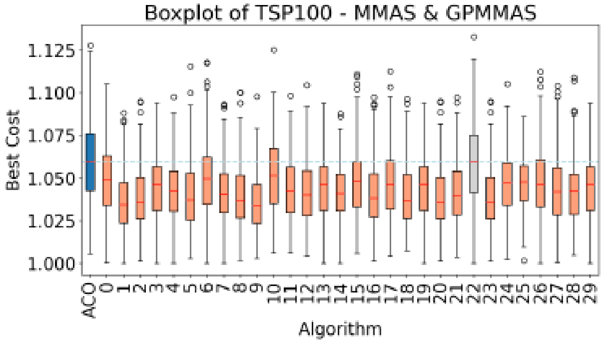
(f) TSP20-100 - MMAS



(g) TSP100 - AS

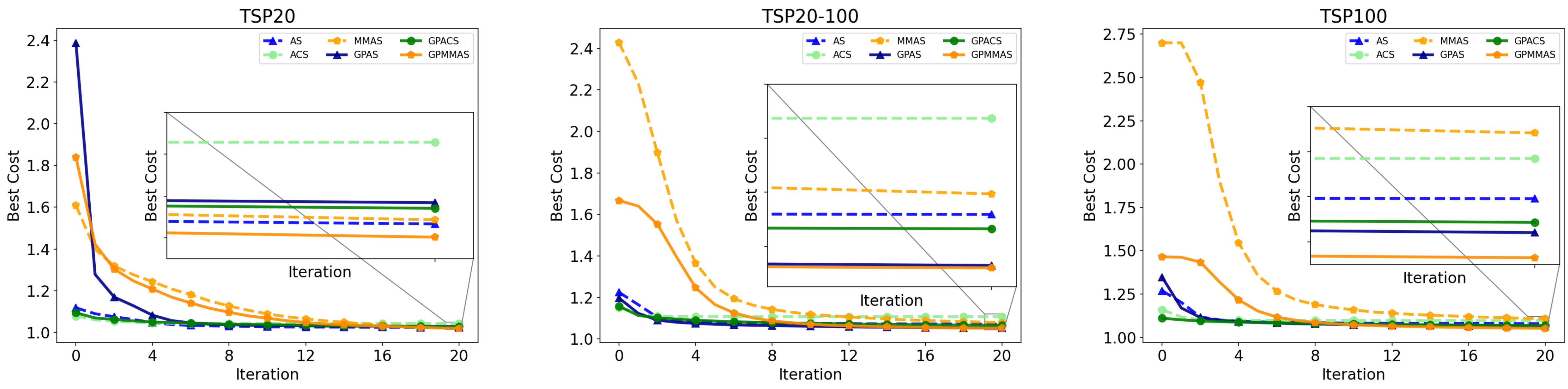


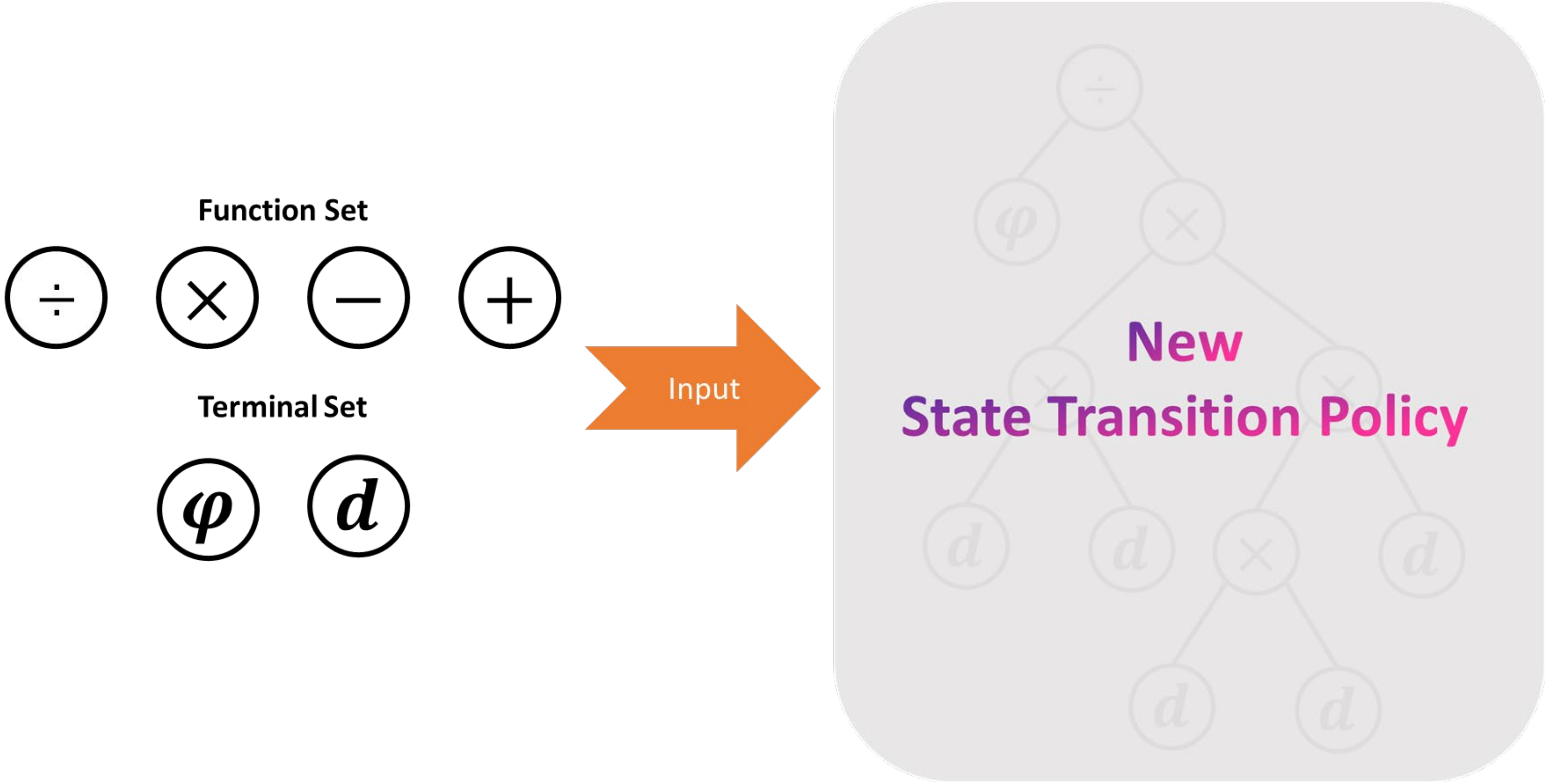
(h) TSP100 - ACS



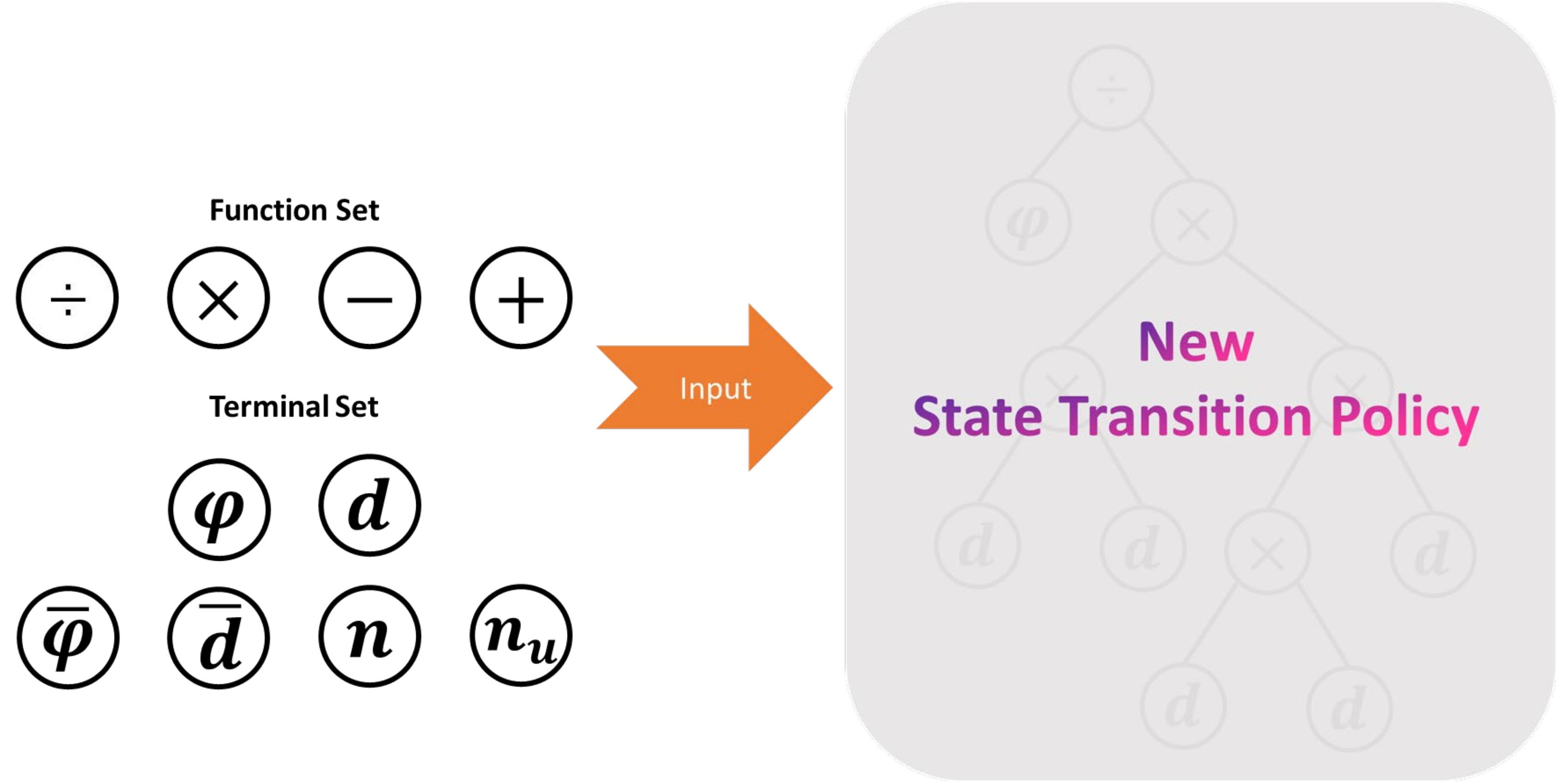
(i) TSP100 - MMAS

Algorithm	TSP20	TSP20-100	TSP100
GP-AS	$\frac{\varphi}{(d-2\varphi)d^2} - \varphi$	$\frac{\varphi}{(\varphi+d^4)d^5}$	$\frac{2\varphi}{(2\varphi+d^2)(2\varphi+d^4)d^3}$
GP-ACS	$\varphi d^3 - \frac{d^3}{\varphi} + 1$	$\frac{(d^3-1)\varphi}{(\varphi-d)d^4} - d$	$\frac{2\varphi}{(\varphi+d)^2 d} - 2d^2 + 1$
GP-MMAS	$-\varphi d - \frac{2d^2}{\varphi+d}$	$-\frac{2\varphi}{(\varphi+\varphi d+2d^2)d}$	$\frac{\varphi+\varphi(\varphi+d)d^2}{(\varphi+d)^2 d^4}$





Why not add more terminals?



average pheromone to the candidate node from the current node

average distance to the candidate node from the current node

total number of nodes

number of candidate nodes

TSP		AS					ACS					MMAS				
		baseline	GP	xGP			baseline	GP	xGP			baseline	GP	xGP		
20	mean	1.01581	1.01600	**1.01054**	+	+	1.04235	**1.01226**	1.01413	+	-	**1.00504**	**1.00547**	1.00610	-	-
	std	0.01744	0.02131	0.01766			0.03516	0.01726	0.01999			0.00763	0.00947	0.01048		
	min	1	1	1			1	1	1			1	1	1		
	max	1.08989	1.15052	1.16435			1.14352	1.13015	1.13878			1.04311	1.08232	1.10493		
20-100	mean	1.06770	1.06158	**1.05509**	+	+	1.10647	1.05448	**1.05101**	+	+	**1.04658**	1.04895	1.04814	=	-
	std	0.02786	0.02914	0.03057			0.03386	0.02430	0.02582			0.02422	0.02917	0.02910		
	min	1	1	0.99317			1.02539	1	**0.98033**			1	1	0.98722		
	max	1.14753	1.18756	1.22094			1.19654	1.15790	1.16051			1.11980	1.21483	1.27336		
100	mean	1.07363	1.06846	**1.06610**	+	+	1.09604	1.05931	**1.05575**	+	+	1.06005	**1.04381**	1.04666	+	-
	std	0.02670	0.02941	0.03652			0.03193	0.02037	0.02410			0.02445	0.01964	0.02477		
	min	1.01843	1	0.99699			1.02711	1.00375	**0.98780**			1.00571	1	0.99496		
	max	1.15084	1.20156	1.27354			1.16721	1.15470	1.14783			1.12775	1.13291	1.24038		

xGP-AS	xGP-ACS	xGP-MMAS
$\frac{2\bar{d}\bar{\varphi}^2+n_u\varphi\bar{\varphi}d}{d^6}$	$n_u n\left(\frac{\bar{d}}{d}\right)^4\left(\varphi+\frac{\bar{d}}{d}\right)$	$\frac{\varphi\bar{d}}{(\varphi+d^2)^2\bar{\varphi}d^3}$

PART III

Future work

1. Design more powerful features as terminals
2. learn GP by Graph Neural Networks (GNNs)

...

Shot by Yuan Tian

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Thank you for your listening!

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