Implementation of an Equal-area Gridding Method for Global-scale Image Archiving

Jeong Chang Seong

Abstract
Constructing global-scale image databases is a challenging task because of large data volume and various map distortions. The raster pixel data structure on a projected flat surface has been used frequently; however, research shows that it may duplicate or lose original features. For a more accurate image-database construction method, this research investigated an equal-area global gridding method. An extended header structure was developed that allows facet indexing using geographic latitude and longitude. Three algorithms were developed in a C/C++ programming environment that performs new layer creation, accessing facet values, and importing image data in other projections. The output of this research can be considered for archiving global-scale satellite imagery and for publishing global-scale thematic raster datasets such as land cover and population density.

Introduction
Designing an accurate and effective global-scale data storing method is a challenging task because of large data volumes and various distortions. Traditionally, many remote sensing and geographic information systems (GIS) have used the raster pixel structure on projected surfaces. Appropriate map projection selection, therefore, has been a major part of global raster database construction. Previous research (Mulcahy, 1999; Seong, 1999; Mulcahy, 2001; Seong and Usery, 2001; Usery and Seong, 2001; Kimerling, 2002; Seong, *et al.*, 2002; Seong, 2003), however, indicates that raster database construction with conventional map projections at the global-scale causes the pixel value duplication and loss problems. This research investigates an implementation of a global gridding method for removing the pixel value duplication and loss problems at a global scale.

Various approaches to partitioning the globe have been proposed (Bailey, 1956; Dutton, 1999; Kimerling *et al.*, 1999; Sahr *et al.*, 2003). One method is to use the meridians and parallels on a flat 2-dimentional surface, resulting in an equally placed, rectangular meridian/parallel grid (i.e., Plate Carrée projection). Even though this method produces different sizes of grid cells as latitude changes, it has been used frequently (Tobler *et al.*, 1995; Brown *et al.*, 1999), but area calculation is difficult, and data are voluminous since unnecessarily high-resolution grid cells are created at high latitude regions. Another method is to use irregular-size grid cells. For example, the SeaWiFS Level-3 data products use irregular-size grid cells on the sinusoidal projection (Campbell *et al.*, 1995). Methods based on polyhedral subdivision have also been proposed. These include the Icosahedral Snyder Equal Area (ISEA)–based grids proposed by Kimerling *et al.* (1999), and the small circle subdivision method proposed by Song *et al.* (2002). Ottoson and Hauska (2002) proposed equal-area and irregular-shape facets on global ellipsoids. They proposed an equal-area quadtree structure for archiving global data. A Voronoi polygon network was also proposed by Lukatela (1987) in order to partition the globe based on computational geodesy in which direction cosines are used instead of the conventional latitude and longitude.

Even though these methods have many advantages over the raster pixel data structure on projected surfaces, a more efficient method still needs to be investigated. This research investigates an implementation of a global gridding method for building global image databases by using equal-area facets and an anchor table which will remove the pixel duplication and loss problems.

Global Grid Using Equal-area Facets on an Ellipsoid
A global grid using meridians and parallels can be designed in many ways (Sahr *et al.*, 2003). Figure 1 shows three different methods of making equal-area facets with four example facets in the solid black color. Each facet in the figure represents 10,000 km^2 on the Earth. Figure 1a shows a method that fixes the latitude difference and longitude changes. This approach produces a raster-like. grid structure; however, vertical distances change slightly on an ellipsoid when latitude differences are fixed. For example, one degree distances along a meridian changes as latitudes change. Figure 1b shows another method that fixes the longitude difference and latitude changes. Ottoson and Hauska (2002) proposed this method with a quadtree data structure. This approach produces vertically long and narrow facets in high latitude regions. Fixing either the latitude or longitude difference, therefore, may not give the best solution if someone wants to produce a square-like grid with a fixed metric distance along a meridian or a parallel. Figure 1c shows another method that fixes the metric distances along meridians and distance changes along parallels. This approach produces the most square-like, equal-area facets. However, it still creates ellipsoidal triangles at the two Poles.

This research investigated the last approach, shown in Figure 1c, that is, to fix the metric distance along a meridian. The following procedures were used for creating equal-area facets with the area of d^2 meters:

Department of Geography, Northern Michigan University, 1401 Presque Isle Ave., Marquette, Michigan 49855 (jseong@nmu.edu).

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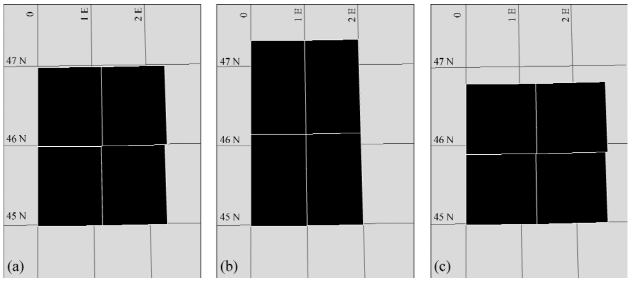

Figure 1. Three different methods of gridding the earth with equal-area facets. Latitudes are geocentric. The facet area is 10,000 km^2.

- Setting up the origin: the North Pole and the 180^0 E/W meridian were used as the origin of global equal-area gridding.
- Creating parallel strips of which facet distances are d.
- Calculating longitude difference ($\Delta\lambda$).

The center coordinates were used as the nominal location of each facet. Figure 2 shows an example of parallel strips, facets, facet distance d, and longitude difference $\Delta\lambda$. The following two sections explain how to calculate the bottom and top latitudes of each parallel strip, and how to calculate the longitude difference that makes equal-area facets.

Calculation of Latitudes, β_1 and β_2, of which Distance is d.

Supposing the latitude and longitude of the first point (β_1, λ_1), the geodetic azimuth (α_{1-2}) from the point, and the distance (d) between two points, the second point (β_2, λ_2) can be calculated using the direct formula (Vincenty, 1975). Even though the formula has a limitation in calculating the antipode of a point on an ellipsoid, it calculated the second point very precisely in less than six iterations. With the direct formula, the latitudes of parallel strips were calculated with the North Pole as the origin point, the distance from the origin to a parallel strip, and the 180° azimuth along the 180° E/W meridian.

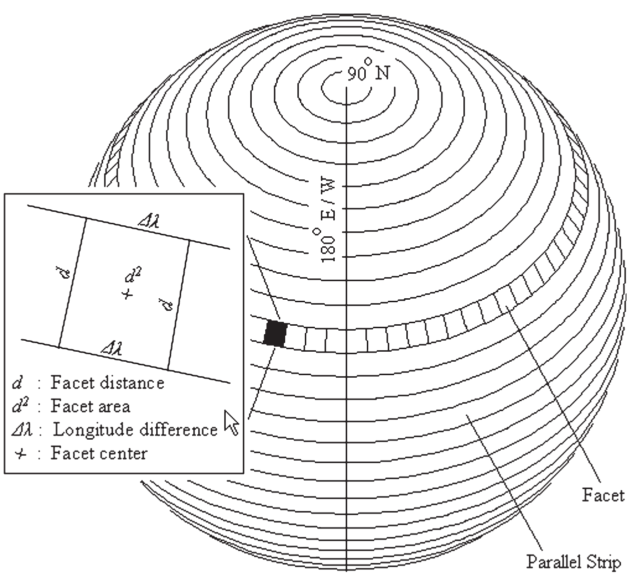

Figure 2. Parallel strips and facets.

Calculation of Longitude Difference

After calculating the latitudes of parallel strips along the 180° E/W meridian from the North Pole, the longitude difference ($\Delta\lambda$) that satisfies the equal-area condition was calculated in each parallel strip. Supposing a is the semi major axis, and f is the flattening of an ellipsoid, the facet area S, bounded by two meridians and two parallels, can be calculated using the following equation (Marcialis, unpublished data, 2003):

$$S = \int_{\lambda_1}^{\lambda_2}\int_{\beta_1}^{\beta_2} r^2 \cos\beta \, d\beta \, d\lambda \tag{1}$$

where, $r = a(1 - f\sin^2\beta - \frac{3}{8}f^2\sin^2 2\beta)$ to the order of f^2.

The longitude difference can, therefore, be calculated as follows:

$$\begin{aligned}\Delta\lambda &= \frac{S}{\int_{\beta_1}^{\beta_2} r^2 \cos\beta \, d\beta} \\ &= S/\Bigg[a^2\Bigg[\sin\beta - \frac{2f}{3}\sin^3\beta + \frac{f^2}{5}\sin^5\beta - \frac{12}{5}f^2\sin^5\beta \\ &\quad + \frac{3}{14}f^2\sin^7\beta - \frac{1}{12}f^2\sin^9\beta + \frac{12}{7}f^3\sin^7\beta\,\frac{24}{9}f^3\sin^9\beta \\ &\quad + \frac{12}{11}f^3\sin^{11}\beta + \frac{9}{20}f^4\sin^5\beta - \frac{9}{14}f^4\sin^7\beta \\ &\quad + \frac{1}{64}f^4\sin^9\beta\Bigg]_{\beta_1}^{\beta^2}\Bigg]\end{aligned} \tag{2}$$

The longitude differences change as latitude changes. As latitudes become higher, the longitude difference increases in order to satisfy the equal-area condition. The above equations should be used carefully because they are using geocentric latitudes. The geocentric latitude is the angle between the equatorial plane and a vector connecting the point of interest and the origin of the coordinate system (Escobal, 1965; JPL, 2002). The latitudes that are conventionally used in global scale referencing are not geocentric, but geographic. Geographic latitudes are defined as the angle between a line perpendicular to the surface of the ellipsoid at the given point and the plane of the equator (Snyder, 1987). Therefore, non-geocentric latitudes need to be converted with the equation, $\beta_g = \tan^{-1}[(1 - e^2)\tan\beta]$, where β_g is a geocentric latitude, β is a geographic latitude and e is the eccentricity of an ellipsoid, defined as $e^2 = 2f - f^2$.

A Grid Data Structure With An Extended Header

Design of An Extended Header File with An Anchor Table

Currently, many raster data formats are composed of header information and actual data. The header usually includes information such as pixel size, data type, corner coordinates, number of bands, projection, units, columns and rows, and byte order. In this conventional structure, geographic referencing is accomplished by referencing rows and columns. If we know the reference coordinates of a cell (usually the upper left corner cell), row and column, and the cell size, the conversion between image coordinates and reference coordinates can be performed easily. This approach can be-implemented on a planar surface; however, it is difficult to apply the approach to the spherical gridding method described in the former section. As an intuitive approach to building a grid data structure, this research investigated the use of an anchor table, shown in Table 1, in addition to existing header information.

Table 1. An Example of Anchor Table with 1,000 m Facet on the WGS84 ellipsoid.

Latitudes in radians	Longitude differences in radians	Number of facets in each parallel strip	Number of cumulative facets
1.570718	2.020475	3	0
1.570562	0.673492	9	3
1.570406	0.404095	15	12
1.570249	0.288639	21	27
1.570093	0.224497	27	48
1.569937	0.183680	34	75
1.569781	0.155421	40	109
1.569624	0.134698	46	149
1.569468	0.118851	52	195
1.569312	0.106341	59	247
1.569156	0.096213	65	306
1.568999	0.087847	71	371
1.568843	0.080819	77	442
1.568687	0.074832	83	519
1.568531	0.069672	90	602
:	:	:	:

The anchor table was designed as an N-by-4 array. N is the number of parallel strips along the 180° E/W meridian. The first column was used for the latitude at the center of each facet. The second column was used for the longitude difference between the left and right meridians of a facet at the parallel strip. The total number of facets in each parallel strip, M, was assigned to the third column. The fourth column was used for storing the number of cumulative facets, T, from the origin. The latitude and longitude were stored in the anchor table because the latitude difference and the longitude difference change continuously on an ellipsoid.

Even though Vincenty's formula can calculate the latitude of parallel strips, it is not efficient for calculating the N values, the number of parallel strips, along the 180° E/W meridian, because all latitudes of parallel strips should be calculated and counted. An easy way is to use an equation that calculates the meridian perimeter of an ellipsoid and to divide the perimeter by the facet distance, d. Figure 2 shows a facet and facet distance. The meridian perimeter, P, can be very accurately approximated by the following Ramanujan's formula (Barnard *et al.*, 2001):

$$P \approx \pi[3(a + b) - \sqrt{(3a + b)(a + 3b)}] \tag{3}$$

where, a is the major axis of the earth and b is the minor axis. From the equation, the meridian perimeter is 40,007,862.916 meters with the WGS84 spheroid (a = 6,378,137.000 meters, and b = 6,356,752.314 meters). The number of parallel strips, N, along a meridian can be calculated as follows:

$$N = P/(2d) \tag{4}$$

where, d is a facet distance. If d in each parallel strip is 1,000 meters, there will be 20,004 parallel strips between the North and South Poles. If d is 30 meters, there will be 666,798 parallel strips.

The total number of facets in each parallel strip, M, can be calculated as follows by using the $\Delta\lambda$:

$$M = 2\pi/\Delta\lambda \tag{5}$$

Values in the anchor table easily become very large because facets cover the entire globe. The maximum value of the fourth column of the anchor table will be at least 510,065,622, if 1,000 meter parallel strips are used, because the surface area of the globe is 510,065,621,710,996 m^2 ($S = [2\pi a^2 (1 + (1 - e^2) \operatorname{atanh}(e)/e)]$, where $e^2 = 1 - b^2/a^2$). As the facet distance d decreases, the total number of facets increases dramatically. For example, if a 30 meter facet distance is used, there will be at least 566,739,579,679 facets. This requires an integer number that is represented by more than 32-bits, such as 64-bit numbers, for direct memory addressing.

Anchor table size becomes [N total size of columns]. When 64-bit double precision types are used, latitude and longitude in radians can be represented very accurately because the 64-bit double precision, usually, has the precision of 19 digits. With a 1,000 meter facet distance, the anchor table size will be about 640 KB (20004 × 4 × 8 bytes), and with 30-meter facets it will be 2.1 MB (666798 × 4 × 8 bytes).

Finally, facet values were written in a one-dimensional array. Considering the surface area of the globe, approximately 0.5, 2, 8, and 566 GB are required for 1,000 m, 500 m, 250 m, and 30 m facets, respectively, when the 8-bit quantization is used for representing facet values without any data compression. Because 0.5 and 2 GB of memory can be addressed directly in a conventional 32-bit workstation environment without complicated programming techniques, this research tested the equal-area global gridding method with 1,000 m facets with a 32-bit laptop computer.

Coordinate Conversions Between Latitude/Longitude and Facet Location

The number of parallel strips and the number of facets in each strip can be used for calculating latitude and longitude just as rows and columns are used in a regular raster format. In this research, rows and columns will be used to denote parallel strips and facets in each strip, respectively, even though they are not exactly the same as in traditional raster format.

Finding Latitude and Longitude When A Facet Location is Given

This is an essential procedure for further analyses using the proposed equal-area global grid data structure. This is particularly important for importing datasets in other projections. The latitude and longitude of the i^{th} grid cell in a data set can be calculated by using the following algorithm:

- Search the fourth column of the anchor table that shows the total number of facets, T, in order to find the row that satisfies [$T_{Row} < i \leq T_{Row} + 1$]. A binary search algorithm will increase the speed of finding the line. In cases where 30-meter facets are used, the total number of facets becomes about 666,798 between the North and South Poles. Because [$2^{19} < 666798 < 2^{20}$], twenty or less comparisons give the row value.
- Once the row is found, the column in the row can be calculated by, [Column = i – AnchorTable [Row][3] – 1], assuming the column index starts not from one, but from zero.
- Finally, latitude = AnchorTable [Row][0], and
- Longitude = AnchorTable [Row][1] · (Column + 1/2) at the middle of each facet.

Using the above algorithm, we can design a data importing algorithm as follows:

- Find the latitude and longitude of the i^{th} facet.
- Resample a facet value of the latitude and longitude from the dataset to be imported.
- Update the value of the i^{th} facet.

Finding A Facet When Latitude and Longitude Are Given

Another important algorithm that a data structure should support is the finding of a facet when latitude and longitude are given. This allows the update of facet values. The i^{th} grid cell in the proposed data structure can be found using the following algorithm:

- Find the row using the binary search in the first field of the anchor table.
- Column = ceil ((longitude + 180)/AnchorTable[Row][1]). 180 is added to the longitude because the 180° E/W is used for the origin of parallel strips.
- i = AnchorTable[Row][3] + Column.

Implementations

The equal-area global grid method was implemented with the DJGPP C/C++ compiler (Hagerty *et al.*, 2001). The DJGPP compiler supports a complete 32-bit memory addressing, so that a large memory such as 2 GB can be allocated in the command prompt mode. This is sufficient to handle 1,000 m facets with the 8-bit or 32-bit facet value representation.

Three main algorithms were coded and tested. The first algorithm is to create a new layer covering the entire globe with a facet distance. It takes five parameters: layer name, data type, initial value, semi major axis, and semi minor axis. Three files are created as the output: header, anchor table, and data. When 1,000 m facet size (510,616,264 total facets) was used on the WGS 84 ellipsoid with the unsigned character representation, it took 123 seconds to create a new layer with a laptop (CPU-Intel Celeron 1.07 GHz RAM, 1 GB. Hard Disk, IBM Travelstar™, 20GB, 4200 RPM). Interestingly, it took 121 seconds to create a new layer without making the anchor table. Therefore, there were only two seconds of difference between the proposed structure and the traditional raster structure.

The second algorithm is to retrieve a facet using latitude and longitude. This algorithm is important for retrieving and updating facet values, and it was tested with 1000 m facets and the WGS84 ellipsoid on the same laptop. It took 55 seconds to read the data file into memory. When the facet access time using latitude and longitude was modeled with randomly-generated 100-million latitude/longitude pairs, it took 106 seconds. A direct access method using the row and column index values without the anchor table, however, took 10 seconds. Therefore, the proposed system showed 10.6 times slower than a direct access.

The third algorithm is to import existing datasets in a different map projection. As an example, an algorithm was developed to import the 1-km Global Land Cover Characteristics (GLCC) dataset in the Goode interrupted homolosine projection (Eidenshink and Faundeen, 1994; Steinwand, 1994; Brown *et al.*, 1999). The projection algorithm coded in C by Steinwand (1994) was used with slight modifications. The GLCC dataset is using the unsigned 8-bit representation in 17,347 rows and 40,031 columns, making a 694,417,757 byte file. In the test computer, it took 70 seconds to read the 694 megabyte input file into memory, and 668 seconds to re-project and write data. The total output file size was 511,256,413 bytes including header (53 bytes), anchor (640,096 bytes), and data (510,616,264 bytes). Therefore, the proposed data structure used about 26 percent less storage than the original. When the number of facets in each category was compared, the root mean weighted square error (RMWSE = $\sqrt{\Sigma(f_i \cdot e_i^2)/\Sigma f_i}$, where f_i is the frequency of category i of the original data, and e_i is the percent difference between the original and new datasets in each category) was 0.13 percent.

Discussion and Conclusions

The proposed data structure and gridding method shows various advantages over traditional global data archiving methods. First, the proposed gridding method produces the most square-like facet shapes so that remotely sensed imagery can be stored with the least shape distortion. Second, the proposed system uses ellipsoids so that any image data can be archived very accurately on any ellipsoid. Third, projection change can be performed easily by using the algorithms suggested in this research. Fourth, the data structure is simple and straight-forward, so that further analysis algorithms can be developed easily. Fifth, 64-bit processing systems would allow global-scale image database construction in high resolution. Sixth, area calculation from the proposed database is accurate. Seventh, the proposed system allows data compression in a serial way, so that the run-length encoding and other data compression methods can be implemented directly. Last, while data storage increases as a second-order function of the facet distance, the anchor table increases as a first-order function of the facet distance.

Currently, the proposed system may have some limitations. First, the construction of a subset database using the same data structure is not available because the proposed database assumes a complete coverage of the globe. Second, the last facet in each parallel strip does not exactly represent actual size, because the last facet of a parallel strip is determined by rounding the remaining longitude difference. For example, if the $\Delta\lambda$ in a parallel strip is 25.0 degrees, there will be 14 facets and 0.4 facet is ignored because 360/25 = 14.40. If $\Delta\lambda$ is 26 degrees, there will be 14 facets too because 360/26 = 13.85. In this case, the last facet represents a smaller area than the other facets in the same strip. The same problem appears at the bottom-most parallel strip, so that the bottom strip may not cover the South Pole. Third, the proposed method requires a map projection in order to display data. Unlike most databases, the proposed method uses a non-planar coordinate system. Therefore, a map projection must be applied for displaying data, which is not efficient for displaying imagery fast.

In conclusion, an implementation of a direct global-gridding method using meridians and parallels was investigated for global-scale image database construction because the raster data structure on a projected surface may bring pixel value loss and duplication problems. The global-gridding method used in this research provides very square-like shapes covering the earth with equal-area facets. An anchor table was designed as an extended header for effective referencing between data and latitude/longitude on any ellipsoids. The proposed data structure took 106 seconds for accessing facet values with randomly generated 100-million latitude/longitude coordinates with a test machine. Even though the proposed method took more time than traditional row/column accessing methods, it provides a possibility of representing global imagery and raster data more accurately with the most square-like facets because of the removal of pixel value loss and duplication problems due to map projections. The output of this research may be considered for archiving global-scale satellite imagery and for publishing global-scale thematic raster datasets such as land cover and population density.

References

Bailey, H.P., 1956. Two grid systems that divide the entire surface of the Earth into quadrilaterals of equal area, *Transactions of the American Geophysical Union*, 37:628–635.

Barnard, R. W., K. Pearce, and L. Schovanec, 2001. Inequalities for the perimeter of an ellipse, *Journal of Mathematical Analysis and Applications*, 260:295–306.

Brown, J. F., T. R. Loveland, D. O. Ohlen, and Z. Zhu, 1999. The Global Land-Cover Characteristics Database: The Users' Perspective, *Photogrammetric Engineering & Remote Sensing*, 65(9):1069–1074.

Campbell, J. W., J. M. Blaisdell, and M. Darzi, 1995. *Level-3 SeaWiFS data products: Spatial and temporal binning algorithms*, NASA Technical Memorandum 104566, GSFC, Vol. 32, Appendix A, Jan 13.

Dutton, G.H. 1999. *A Hierarchical Coordinate System for Geoprocessing and Cartography*, Berlin: Springer-Verlag, 230 p.

Eidenshink, J. C., and J. L. Faundeen, 1994. The 1 km AVHRR global land data set-first stages in implementation, *International Journal of Remote Sensing*, 15(17):3443–3462.

Escobal, P.R., 1965. *Methods of Orbit Determination*, Krieger Publishing Company, Malabar, Florida, USA, p. 479.

Hagerty, D., M. Weisshaus, E. Zaretskii, D. Hagerty, D. Hagerty, M. Weisshauss, and E. Zaretskii, 2001. *GNU Software for MS-Windows and MS-DOS*, Free Software Foundation, 150 p.

JPL (Jet Propulsion Laboratory), 2002. *Planetary Data System Standards Reference*. JPL D-7669, Part-2. JPL, California. URL: *http://pds.jpl.nasa.gov/stdref/*, (last date accessed: 22 February 2005).

Kimerling, A. J., K. Sahr, D. White, and L. Song, 1999. Comparing geometrical properties of global grids, *Cartography and Geographic Information Science*, 26(4):271–287.

Kimerling, A. J., 2002. Predicting data loss and duplication when resampling from equal-angle grids, *Cartography and Geographic Information Science*, 29(2):111–126.

Lukatela, H., 1987. Hipparchus Geopositioning Model: an Overview, *Proceedings of the AUTO-CARTO 8*, Baltimore, March, URL: *http://www.geodyssey.com/papers/hlauto8.html* (last date accessed: 22 February 2005).

Marcialis, R. L., 2003. Calculation of Cell Areas on an Oblate Planet, URL: *http://www.lpl.arizona.edu/~umpire/work/spheroid_area.html* (last date accessed: 22 February 2005).

Mulcahy, K. A., 1999. *Spatial Data Sets and Map Projections: An Analysis of Distortion*. Ph. D. Dissertation, Graduate School and University Center, City University of New York, New York, DAI-A 60/01, 210 p.

Mulcahy, K. A., 2001. Two new metrics for evaluation of pixel-based change in data sets of global extent due to projection transformation, *Cartographica*, 37(2):1–11.

Ottoson, P., and H. Hauska, 2002. Ellipsoidal quadtrees for indexing of global geographical data, *International Journal of Remote Sensing* 16(3):213–226.

Sahr, K., White, D., and Kimerling, A.J., 2003. Geodesic discrete global grid systems, *Cartography and Geographic Information Science*, 30(2):121–134.

Seong, J. C., 1999. *Multitemporal integrated global GIS databases and land cover dynamics: Asia, 1982–1994*, Ph.D. Dissertation. Department of Geography, University of Georgia, Athens, Georgia, 208 p.

Seong, J. C., 2003. Modeling the accuracy of image data reprojection, *International Journal of Remote Sensing*, 24(11):2309–2321.

Seong, J. C., K. A., Mulcahy, and E. L. Usery, 2002. The sinusoidal projection: a new importance in relation to global image data, *The Professional Geographer*, 54(2):218–225.

Seong, J. C., and E. L. Usery, 2001. Modeling the raster representation accuracy using a scale factor model, *Photogrammetric Engineering & Remote Sensing*, 67(10):1185–1191.

Snyder, J. P., 1987. *Map Projections-A Working Manual*, U. S. Geological Survey Professional Paper 1395, Washington, D. C., U. S. Government Printing Office.

Song, L., A. J. Kimerling, and K. Sahr, 2002. *Developing an Equal Area Global Grid by Small Circle Subdivision*, in: Discrete Global Grids M. Goodchild and A. J. Kimerling (Editors), Santa Barbara, California, National Center for Geographic Information & Analysis, URL: http://www.ncgia.ucsb.edu/globalgrids-book/song-kimmerling-sahr/, A Web Book (last date accessed: 22 February 2005).

Steinwand, D.R., 1994. Mapping raster imagery to the Interrupted Goode Homolosine projection, *International Journal of Remote Sensing*, 15(17):3463–3472.

Tobler, W., Deichmann, U., Gottsegen, J., and Maloy, K., 1995. *The global demography project. National Center for Geographic Information and Analysis*, Technical Report TR-95–6.

Usery, E. L., and J. C. Seong, 2001. All equal-area map projections are created equal, but some are more equal than others, *Cartography and Geographic Information Systems*, 28(3):183–193.

Vincenty, T., 1975. Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations, *Survey Review* XXII, 176 p.

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