2016 JMO P4

Lin Liu

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Problem

Find, with proof, the least integer N such that if any 2016 elements are removed from the set $\{1, 2, \ldots, N\}$, one can still find 2016 distinct numbers among the remaining elements with sum N.

Solution

We claim that the minimum working number is $2017 + 2018 + \cdots + 4032 = 6049 \cdot 1008$. Notice that any $n < 6049 \cdot 1008$ would not work because of size reasons. Then notice that if we form 3024 pairs

$$(1, 6048), (2, 6047), \ldots, (3024, 3025)$$

and if we remove any 2016 numbers, then we would have at least $3024 - 2016 = 1008$ pairs left and thus we would achieve a sum of $1008 \cdot 6049$ so we are done.