1995 ISL G1

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Problem

Let A, B, C, D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y. The line XY meets BC at Z. Let P be a point on the line XY other than Z. The line CP intersects the circle with diameter AC at C and M, and the line BP intersects the circle with diameter BD at B and N. Prove that the lines AM, DN, XY are concurrent.

Solution

It remains to prove that quadrilateral $AMND$ so that we can use the radical axis theorem and finish the problem. Notice that quadrilateral $MBCN$ is cyclic. Now we do an angle chase.

$$\begin{aligned}\angle MND &= \angle MNB + 90^\circ = \angle MCB + 90^\circ = \angle MCA + 90^\circ \\ &= 90 - \angle MAC + 90 = 180 - \angle MAC = 180 - \angle MAD. \quad \blacksquare\end{aligned}$$