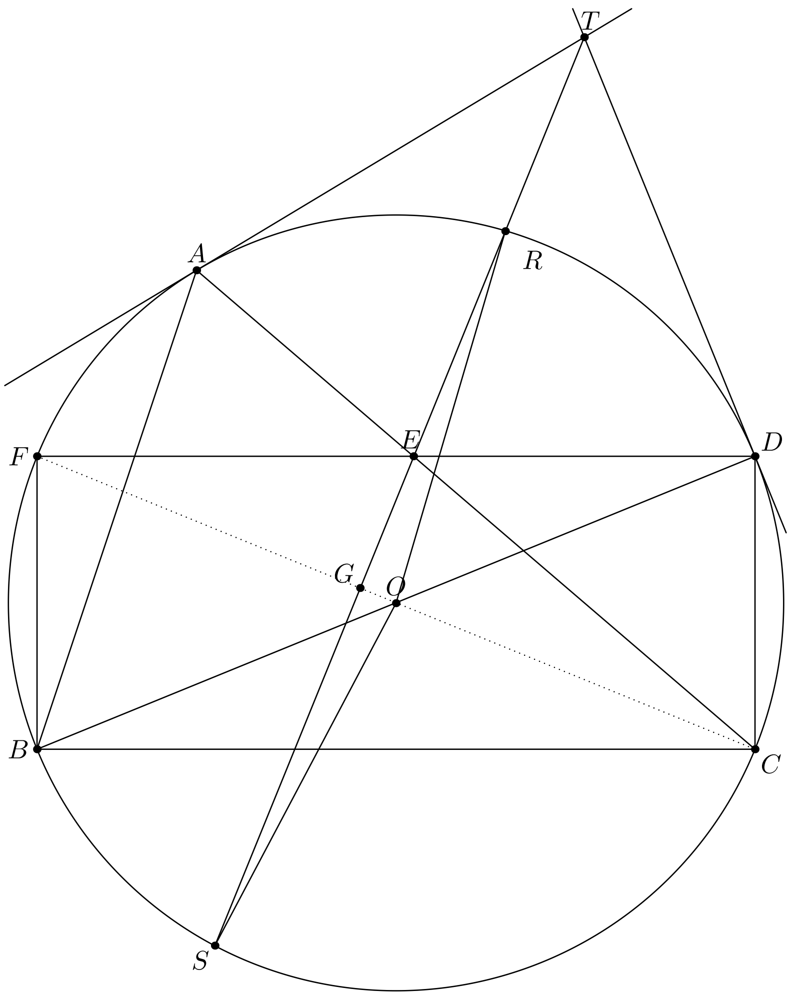
# 2020 Centroamerican Shortlist G2

LIN LIU

September 18, 2021



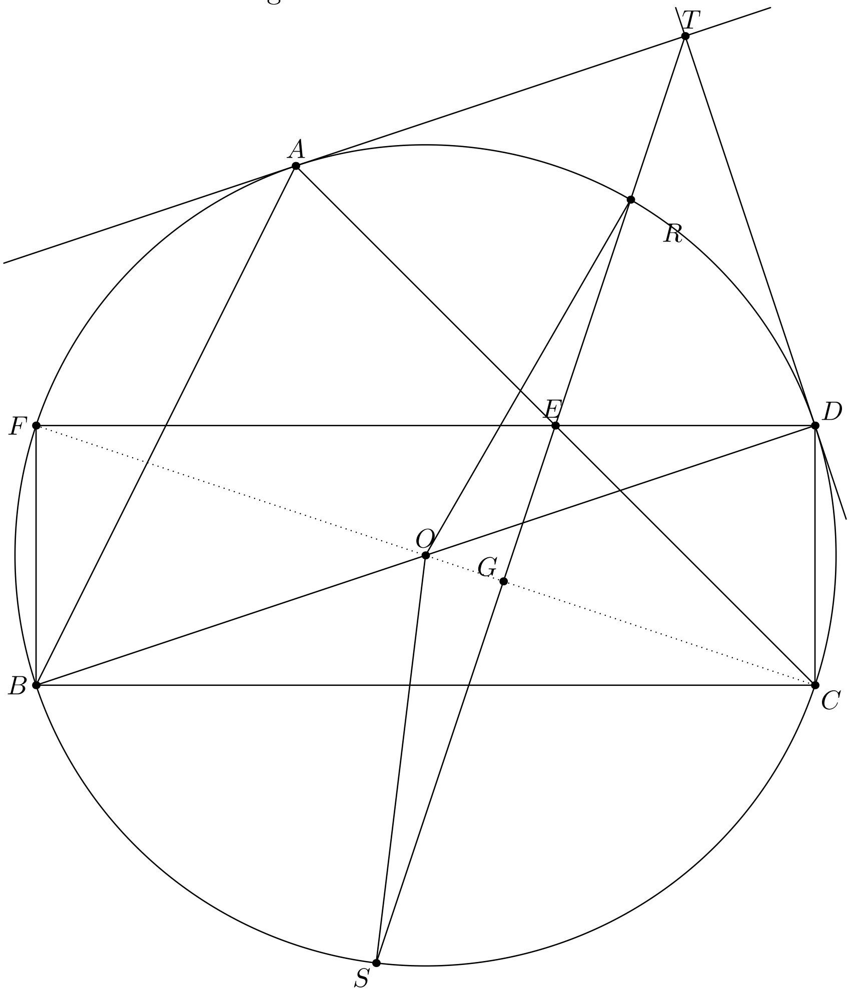

Let $G = \overline{OC} \cap \overline{TS}$.
**Claim:** Quadrilateral $FBCD$ is a rectangle.

*Proof.* Because $BD$ is a radius of $\Gamma$, we have that $\angle BFD = \angle DCB = 90°$. Now denote $\angle DBC = \angle CFD = \alpha$. This means that $\angle BFC = \angle FBD = 90 - \alpha$ which means that $\angle FBC = \angle FDC = 90°$. Since all four angles are 90 we can conclude that it is a rectangle. □

**Claim:** $F, G, O, C$ are collinear.

*Proof.* Notice the rectangle $FBCD$ and that $FC$ is a diameter of $\Gamma$. Because $O$ is the intersection of the diagonals of the rectangle, we are done. □

**Case 1:** $G$ lies on segment $OC$




Because $\angle OGS = \angle OGR = 90^\circ$ we have $\triangle OGS \cong \triangle OGR$. Thus we have

$$\angle SGO = \angle RGO \implies \angle FOS = \angle FOR.$$

**Case 2:** $G$ does not lie on segment $OC$.
Use diagram 1 for reference.
Again, we have

$$\angle OGS = \angle OGR = 90^\circ \implies \triangle GOS \cong \triangle GOR \implies \angle GOS = \angle GOR.$$