

# 2011 USAJMO P1

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## Problem

Find, with proof, all positive integers  $n$  for which  $2^n + 12^n + 2011^n$  is a perfect square.

**Solution**

We claim that the only possible  $n$  is when  $n = 1$ . It is easy to see that this does satisfy the condition. Now assume  $n > 1$ . We have

$$2011^n \equiv (-1)^n \pmod{4}.$$

This means that  $n$  must be even. We also have

$$2^n + 1^n \equiv (-1)^n + 1^n \pmod{3}.$$

This means that  $n$  must be odd, a contradiction. Thus we are done.