Chapter 6 Exercises

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Tuesday, September 23, 2014

6-1

A) The value of `sum(approxMass)` is 0.9991. This is less than 1 because it is not using the analytical solution - instead, this is a numerical approximation of the beta distribution.

B) We cannot use theta on the interval from 0 to 1. We must use the midpoints. Using 0 and 1 as endpoints gives an incorrect numerical approximation. While the widths are the same as the other rectangles, the edges actually extend outside of the beta distribution, giving an over-approximation of the area. Indeed, using such a parameterization, we see a total probability estimate of 1.001, which is clearly impossible.

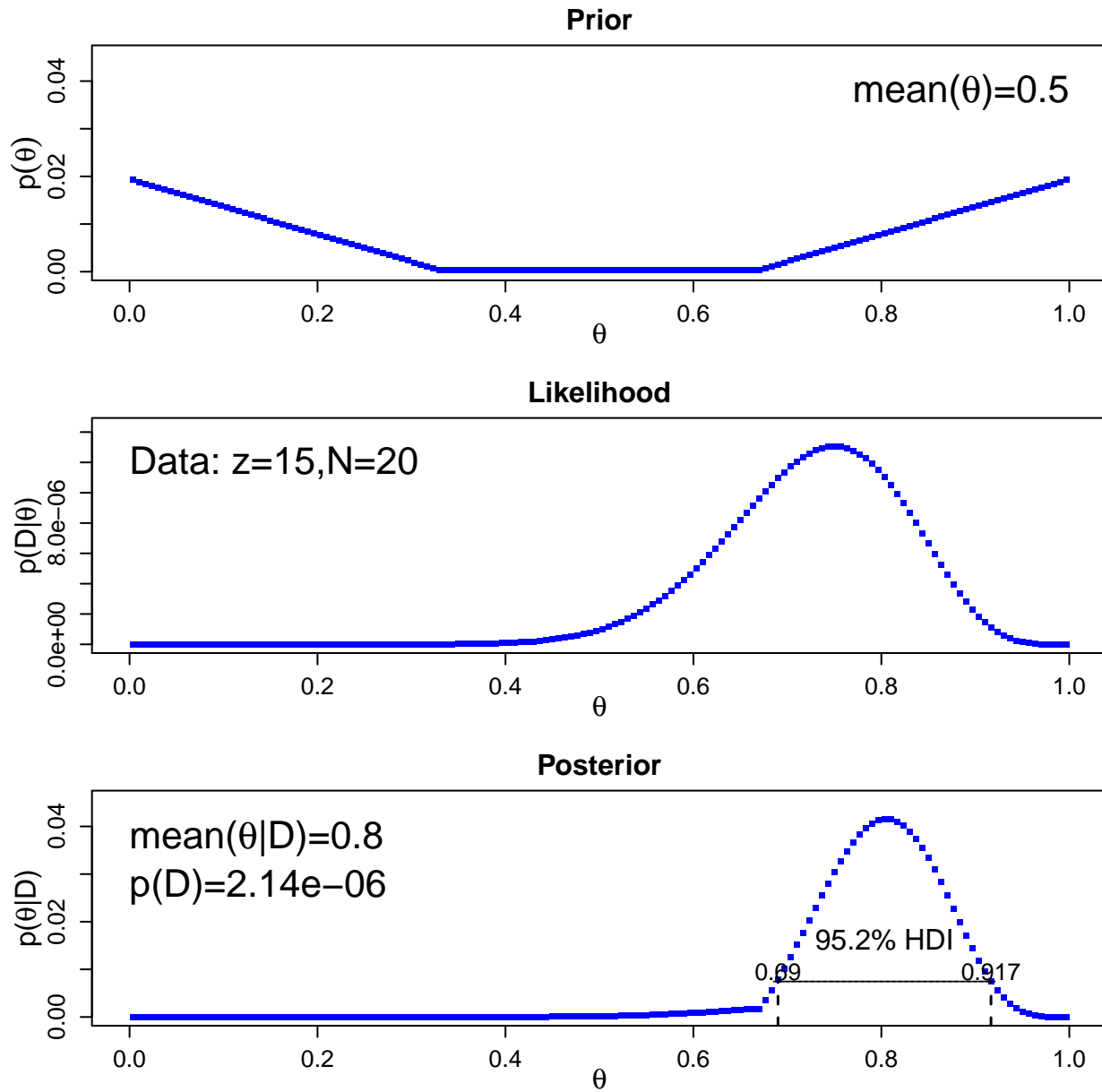**6-2)** We'll use the example from the book, for ease and convenience (it's late.)

A)

```
ptheta <- c(50:1, rep(1, 50), 1:50)
ptheta <- ptheta/sum(ptheta)

width = 1/length(ptheta)

theta = seq(from=width/2, to=1-width/2, by=width)
```
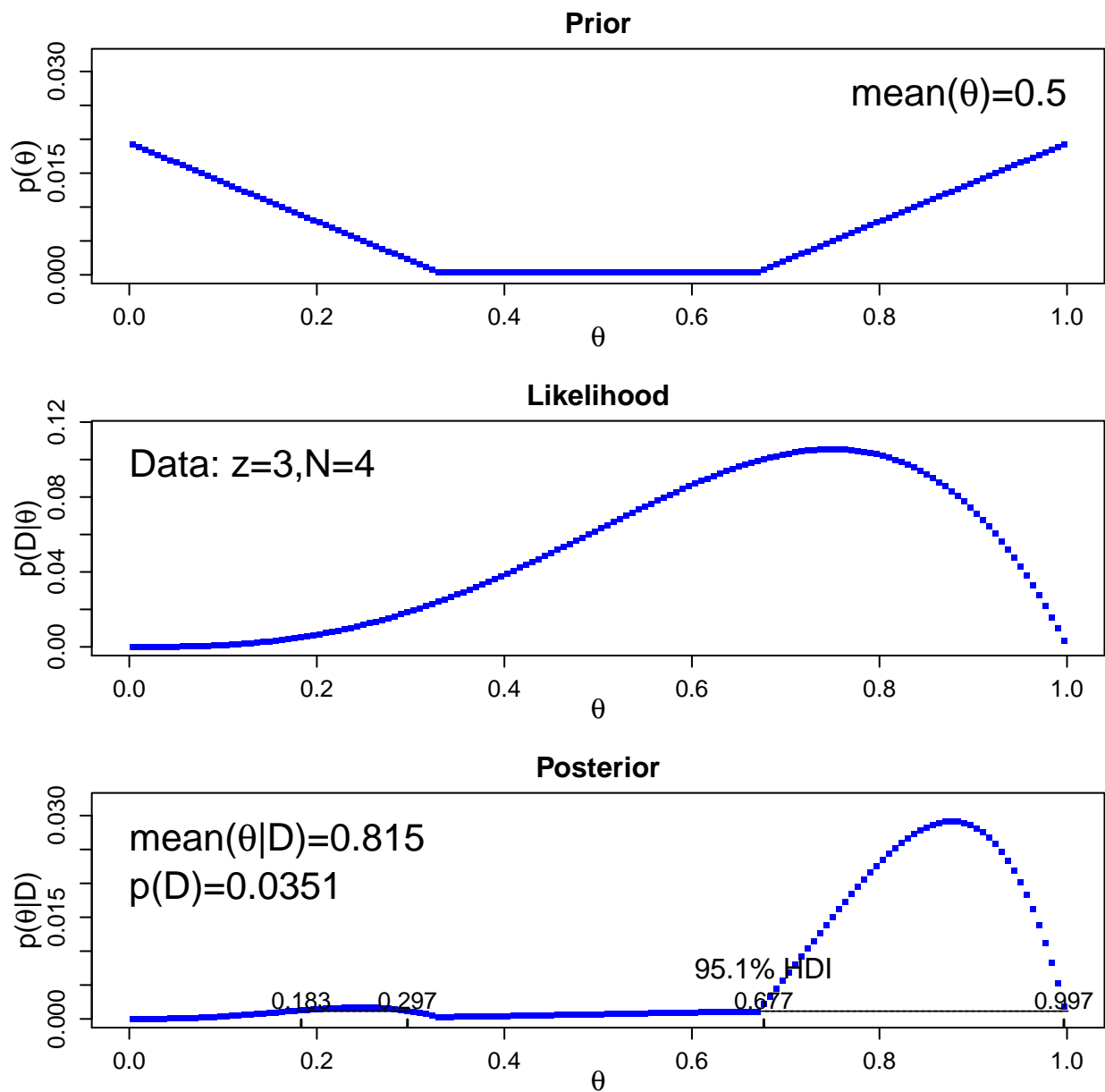
B) Suppose now that we flip a coin 20 times and get 15 heads. What are our new posterior beliefs? We use `BernGrid.r` to find out. After typing it out for several minutes, I decided it was worth copy and pasting instead from the official website.

Prior

mean(θ)=0.5
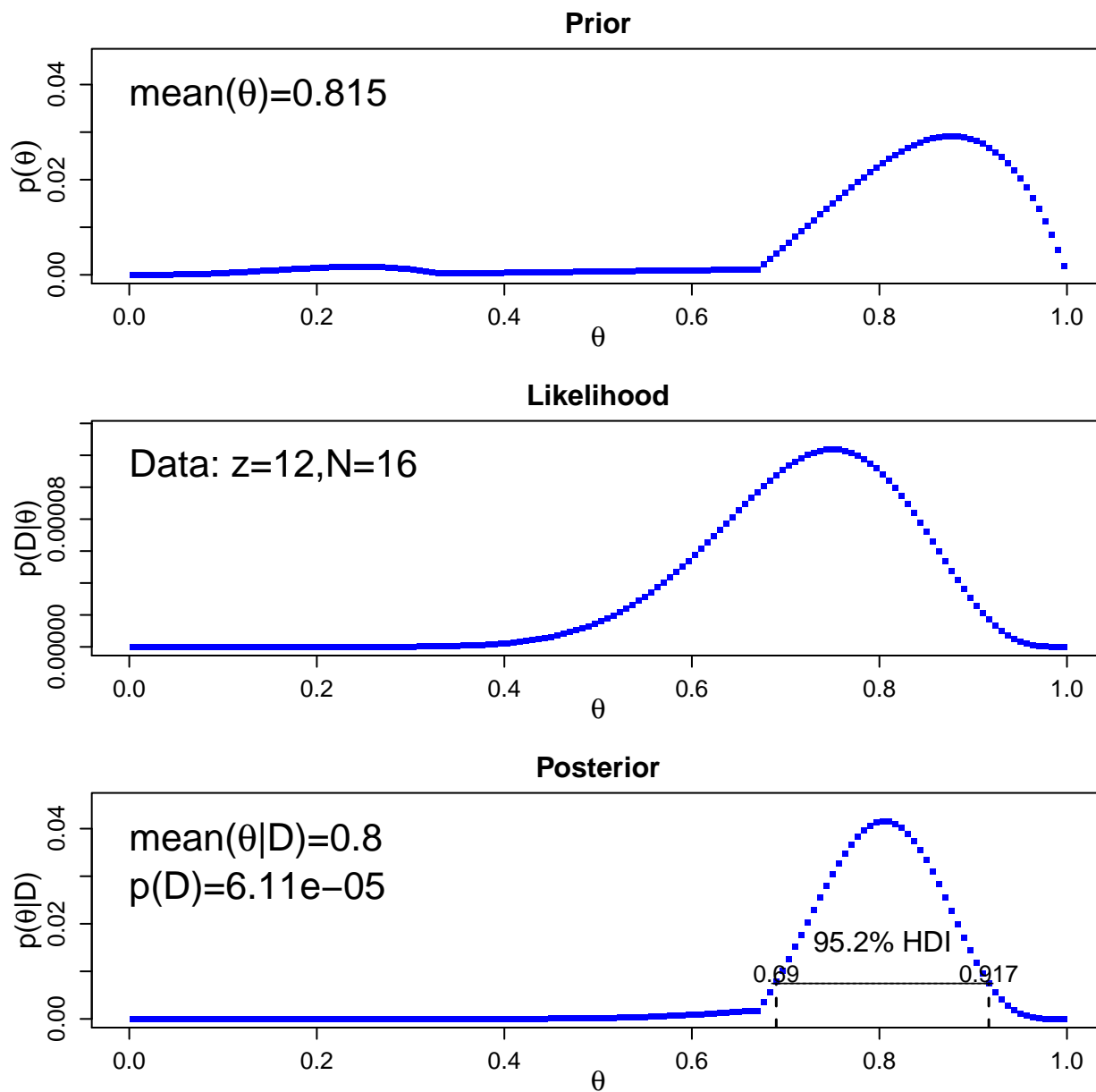
p(θ) vs θ

Likelihood

Data: z=15,N=20

p(D|θ) vs θ

Posterior

mean(θ|D)=0.8
p(D)=2.14e−06

95.2% HDI
0.69   0.917

p(θ|D) vs θ

6-3

A) This time, we use the same prior as before. However, we flip a coin 4 times and get 3 heads.

Prior

mean(θ)=0.5

Likelihood

Data: z=3,N=4

Posterior

mean(θ|D)=0.815
p(D)=0.0351

95.1% HDI

0.183   0.297   0.677   0.997

B) Now, let's do some updating, with 16 flips and 12 heads. Using the previous posterior distribution as our prior, we see a different result.
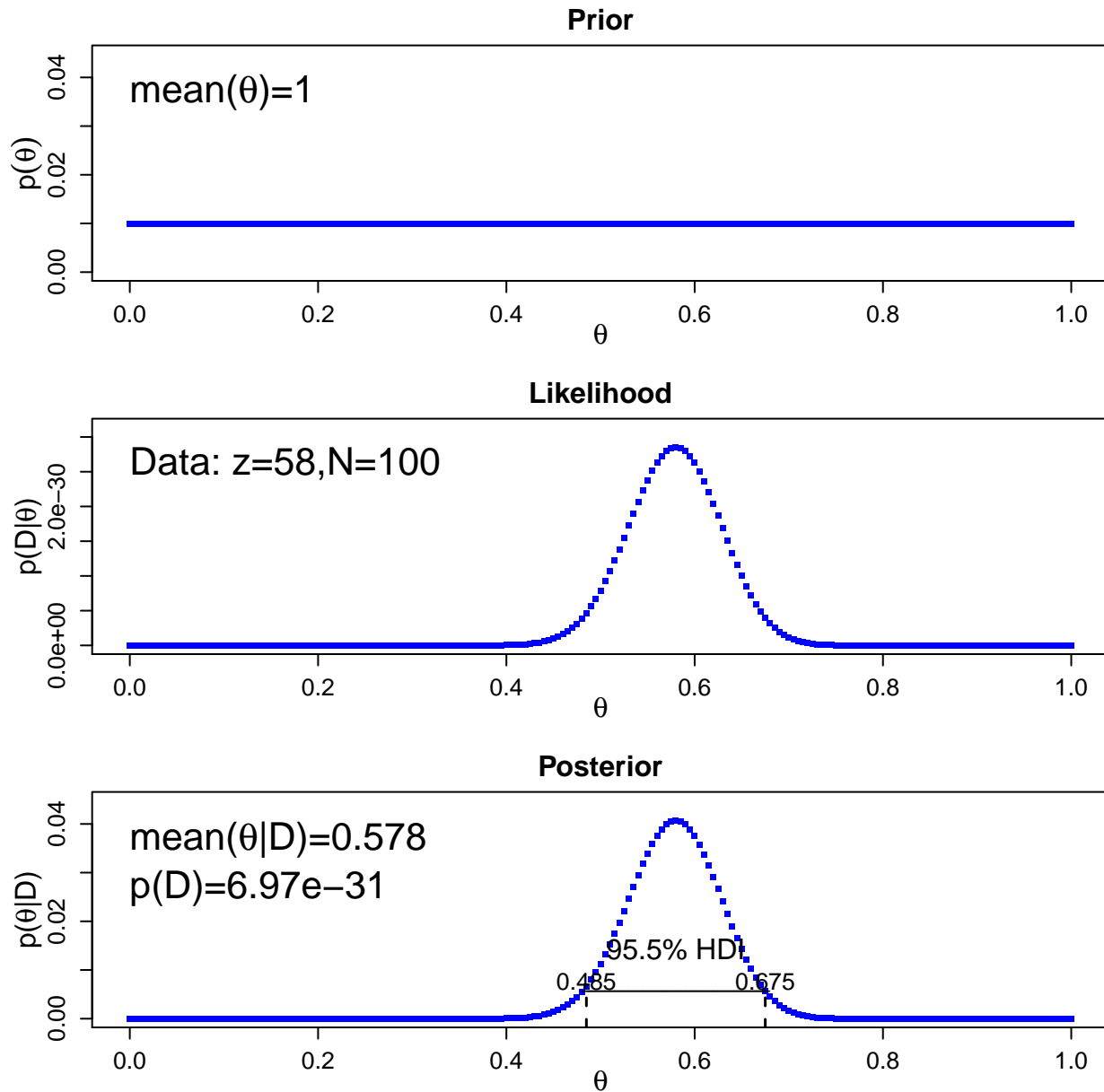
Prior

mean(θ)=0.815

p(θ) 0.04 0.02 0.00

θ
0.0 0.2 0.4 0.6 0.8 1.0

Likelihood

Data: z=12,N=16

p(D|θ)

θ
0.0 0.2 0.4 0.6 0.8 1.0

Posterior

mean(θ|D)=0.8
p(D)=6.11e−05

95.2% HDI
0.69   0.917

p(θ|D) 0.04 0.02 0.00

θ
0.0 0.2 0.4 0.6 0.8 1.0

6-4

A) Our prior is a standard uniform distribution.

```
theta <- seq(0,1, by=0.005)
ptheta <- dunif(theta)/sum(theta)
data <- c(rep(1,58), rep(0,42))

ptheta <- BernGrid(theta, ptheta, data)
```
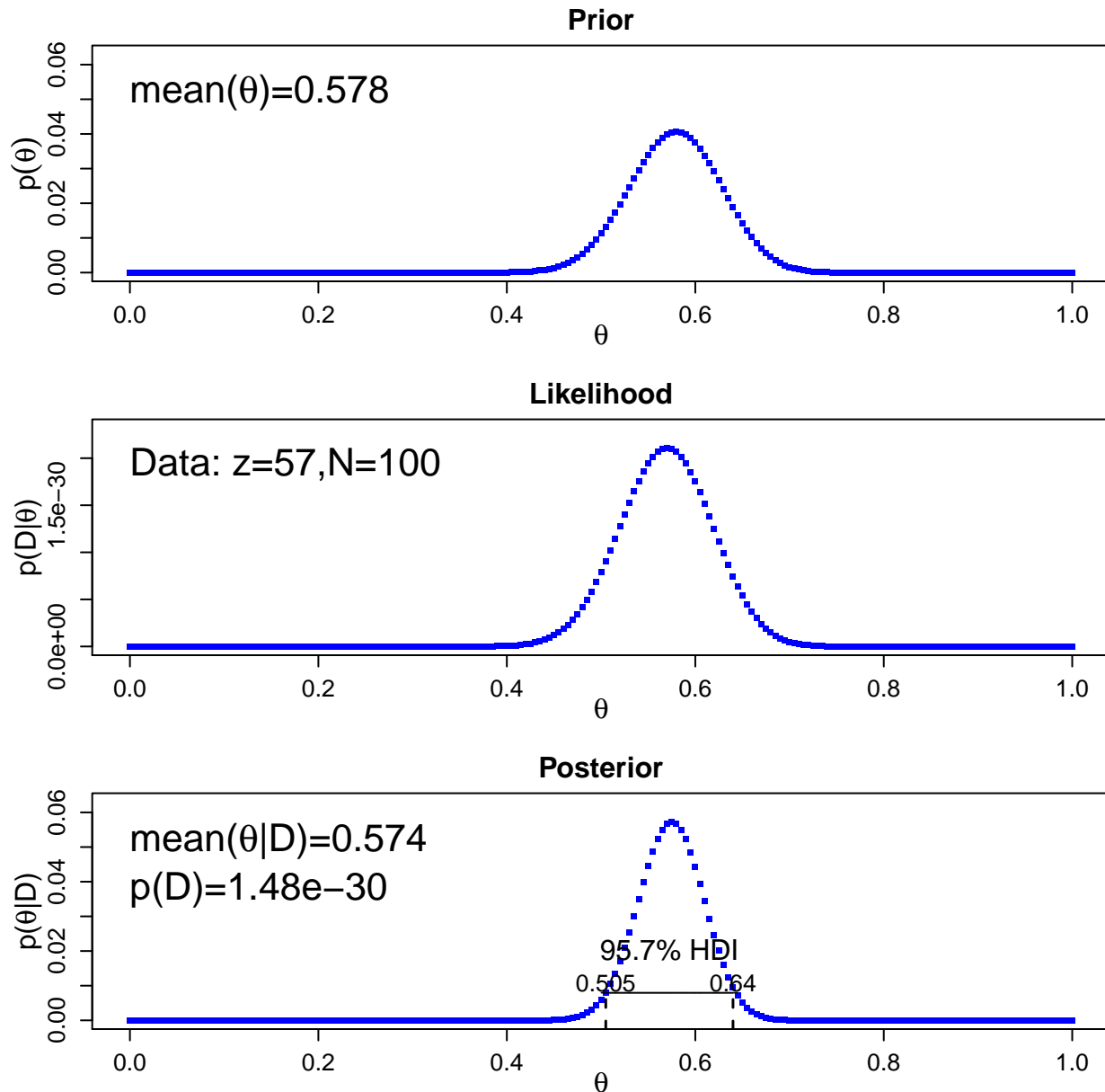
Prior

mean(θ)=1

Likelihood

Data: z=58,N=100

Posterior

mean(θ|D)=0.578
p(D)=6.97e−31

95.5% HDI

0.485   0.675

This means that the 95% HDI is $[0.48, 0.67]$.

B) Based on this information, yes, it's somewhat plausible to believe that the population is divided equally. The HDI contains 50%, so the notion that $\theta = 0.5$ isn't far outside the realm of possibility.

C) However, once we take another follow up poll, we get another 100 people. Using the results of our previous poll as our prior distribution gives us very different results.

Prior

mean(θ)=0.578

Likelihood

Data: z=57,N=100

Posterior

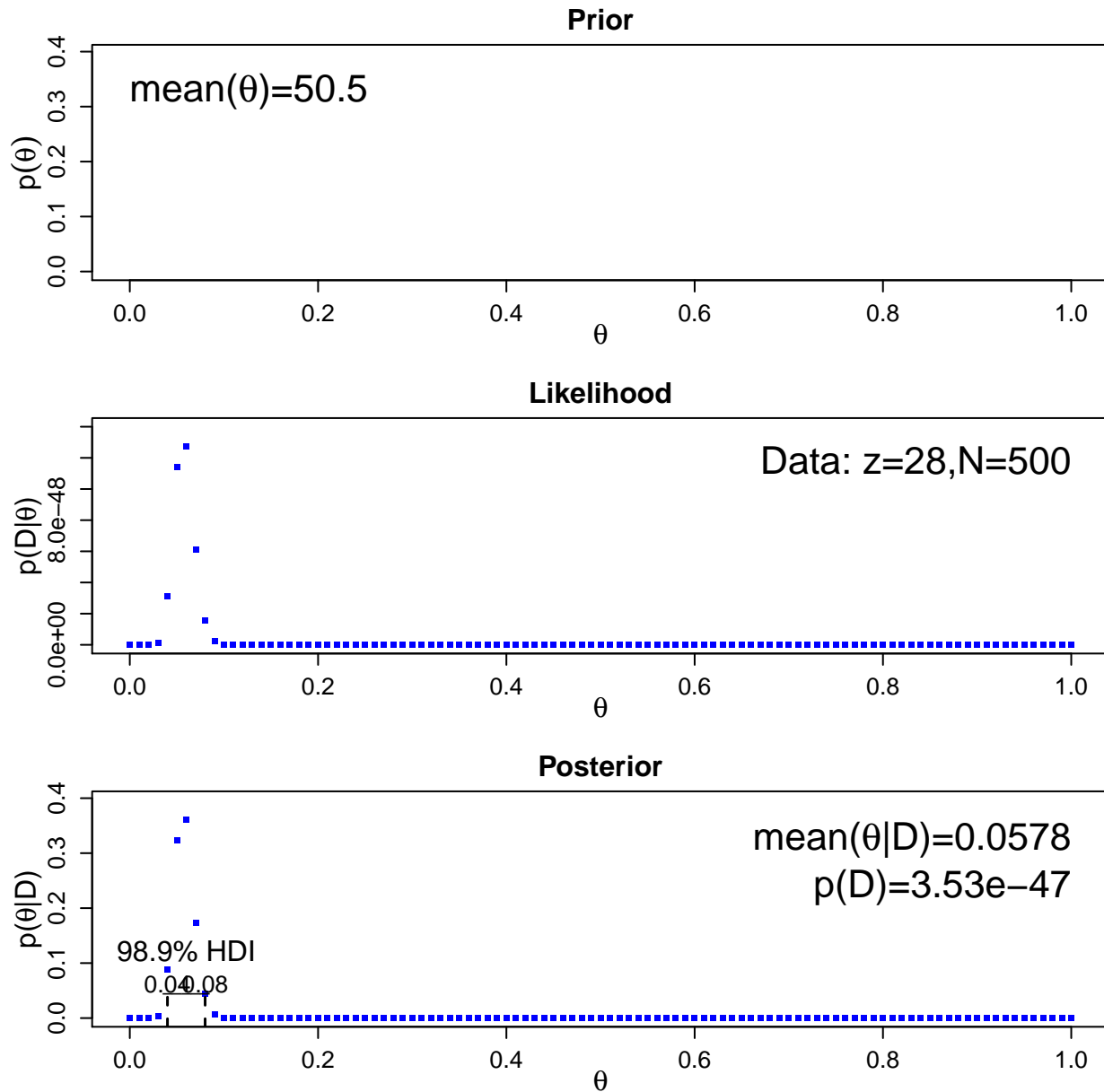mean(θ|D)=0.574
p(D)=1.48e−30

95.7% HDI
0.505   0.64

Now, $\theta = 0.5$ is outside our highest density interval, and it doesn't really make sense to believe that candidate A is preferred over candidate B.

6-5

A) If you are trying to convince the CEO that less than 10% of widgets are defective, this depends on your prior distribution. No previous data is available, so realistically, there are three options for the analysis.

- Option 1: Assume a uniform prior. This is supposed to be noninformative, though of course, this is not the case in practice. Instead, this prior assumes that all values are equally likely, which will in fact be the most conservative estimate - it will take more evidence to overcome this prior when compared to option 2...

- Option 2: Assume a prior beta distribution with mean = 10, and make the overall strength of this prior relatively weak. This is somewhat akin to hypothesis testing in the frequentist paradigm, but the biggest problem is assigning your strength of belief to this prior.

We will choose option 1, since it is the most conservative.

Prior

mean(θ)=50.5

Likelihood

Data: z=28,N=500

Posterior

mean(θ|D)=0.0578
p(D)=3.53e−47

98.9% HDI
0.04 0.08

Given this data, it would be right to tell the CEO that the probability of a defect is less than 10%. In fact, there is about a 97% chance that $\theta \in [0.04, 0.08]$.

6-6

Suppose we think a coin is biased towards tails such that $P(\theta) = \theta^2$. Then, suppose we flip a coin and see 2 heads and 2 tails. Our new posterior distribution is below.

Prior

Likelihood

Posterior
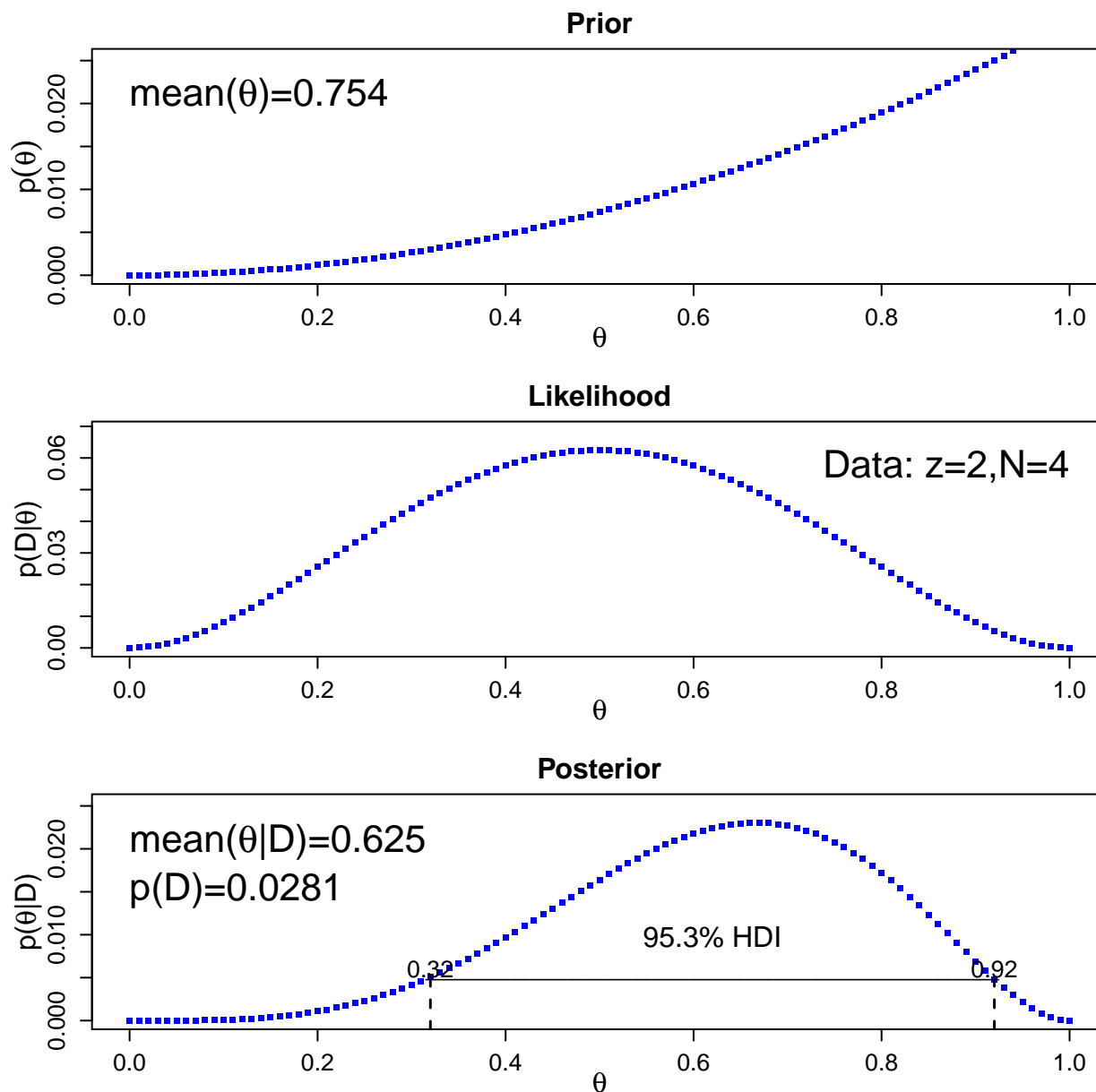
6-8 We're going to skip to 6-8.

Suppose we have a study with 50 couples. After taking an experimental drug, 30 of them have boys, and 20 of them have girls.

A) An appropriate prior distribution would be to assume that the drug does not work at all. Thus, any evidence we find to the contrary must overcome the prior. However, we must also assume a certain level of certainty. For this, we will use the sample size of the experiment we ran, and say that our prior belief is that $\theta \sim \beta(25, 25)$.
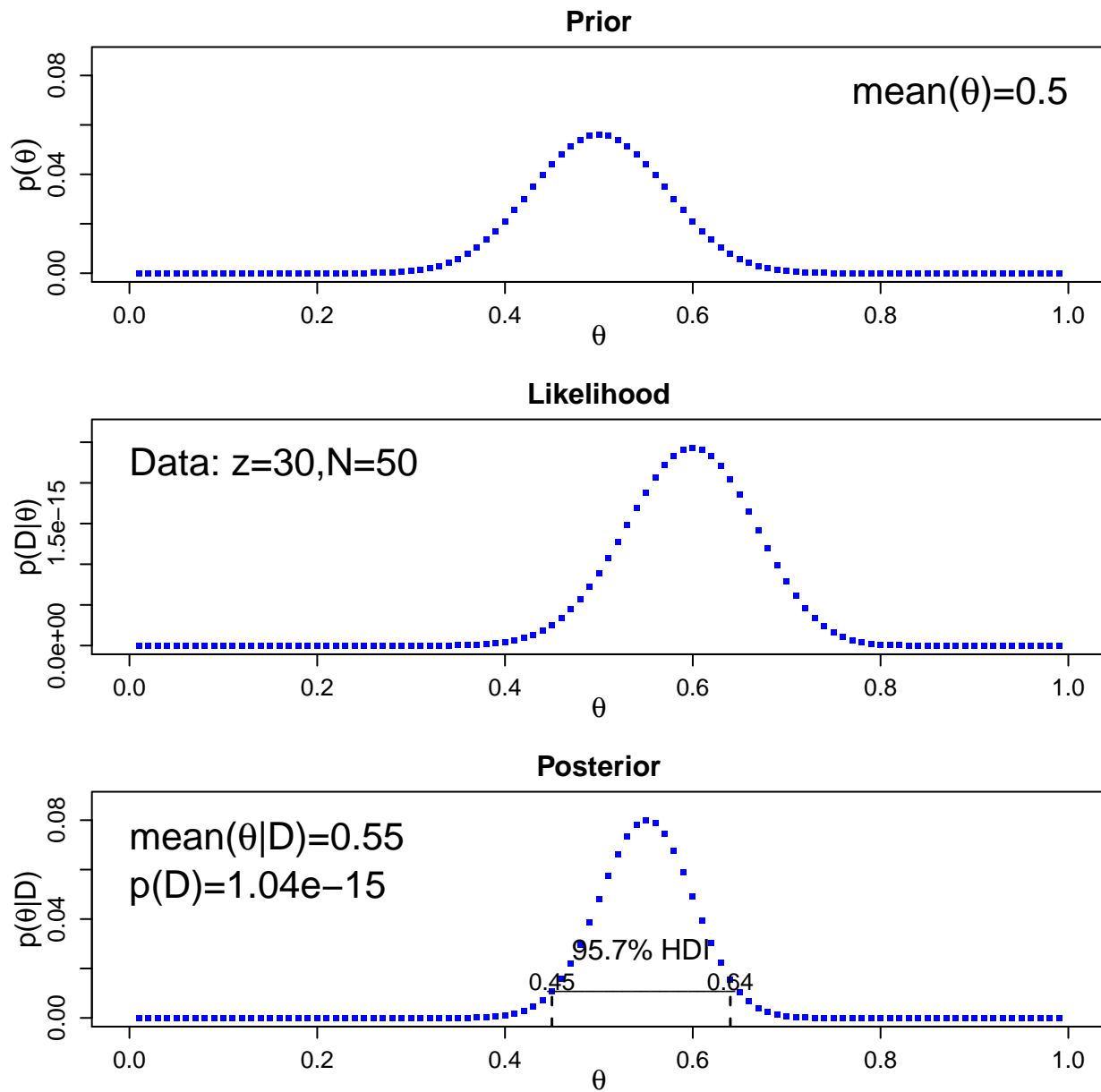
B) Using our `BernGrid` function, we define a prior and posterior in this way.

```
intervals  <- 100
binwidth <- 1/intervals

theta <- seq(binwidth,1 - binwidth, by=0.01)
ptheta <- dbeta(theta, 25, 25)*binwidth
```

```
data <- c(rep(1, 30), rep(0, 20))

post <- BernGrid(theta, ptheta, data)
```


Prior

mean(θ)=0.5

Likelihood

Data: z=30,N=50

Posterior

mean(θ|D)=0.55
p(D)=1.04e−15

95.7% HDI
0.45   0.64

Given these results, it is certainly credible to assume that a couple who takes the drug has a 50% chance of conceiving a boy. In fact, it is credible to assume any probability $\in [0.45, 0.64]$