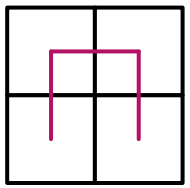
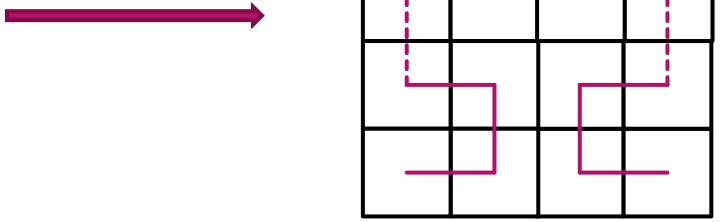
Practice7

YAO ZHAO

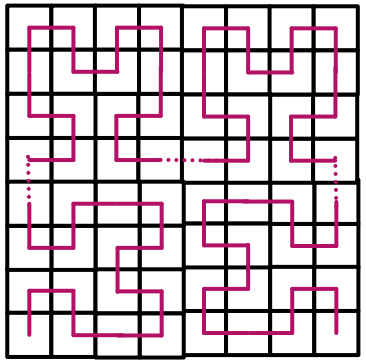
Hilbert Curve



W_1



W_2



W_3

W_2 consists of four W_1 structures with the lower-left and the lower-right ones are 90 degree rotated clockwise and counter-clockwise, respectively; the upper ones have the same structure with W_1. Connect the four structures with 3 unit lines.

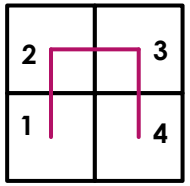
W_3 consists of four W_2 structures with the lower-left and the lower-right ones are 90 degree rotated clockwise and counter-clockwise, respectively; the upper ones have the same structure with W_2

Hilbert Curve

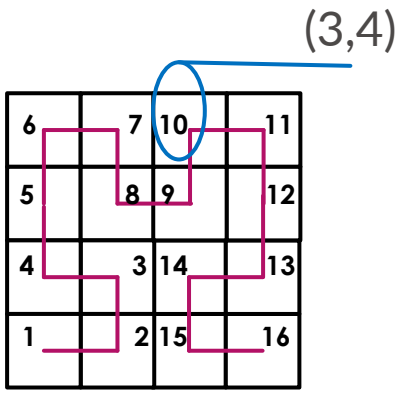
- This rule has been devised by a mathematical philosopher David Hilbert (1862 – 1943), and the resulting curve is usually called a Hilbert Curve named after him. He once talked about a space filling method using this kind of curve to fill up a square with 2^k sides.

For each vertex p on the Hilbert curve,
we define the the coordinates of p to be the location of the square of p in the squares matrix,
and we define the serial number of p to be the vertices count on the curve from the beginning to p.

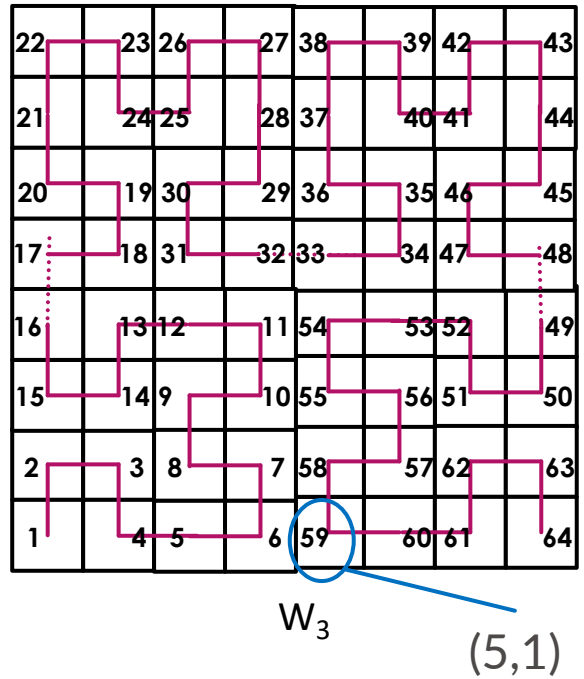
For example, when the coordinates of p is (3,4) and the order is 2, the number of p is 10;
when the coordinates of p is (5, 1) and the order is 3, the number of p is 59.



W_1



W_2

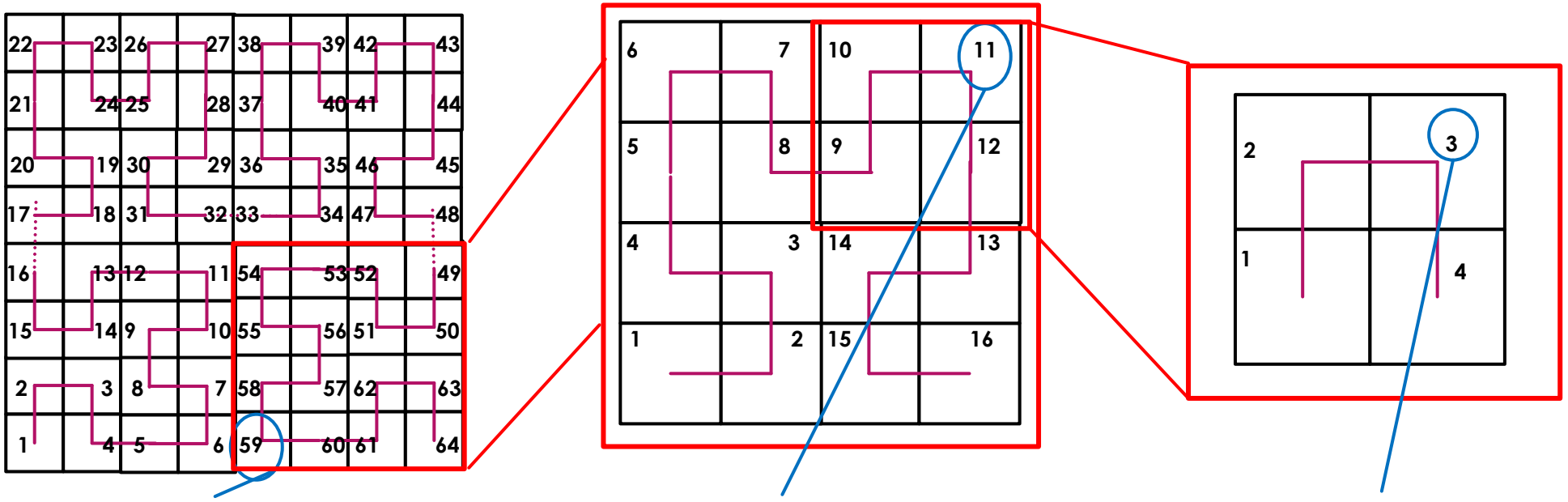


W_3

Given the order of the Hilbert curve and the coordinates of p, can you figure out the number of p?

Problem analysis

W_n consists of four W_{n-1} structures



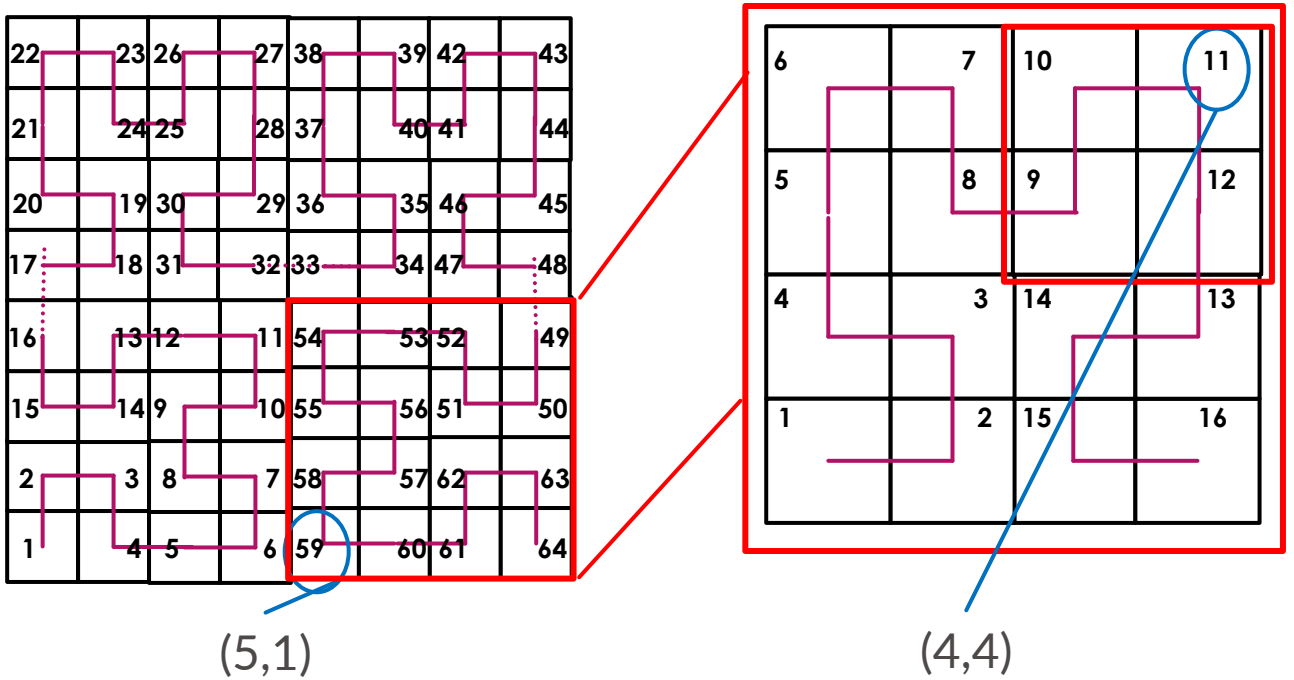
(5,1) (4,4) (2,2)

W_3 —Divide→ W_2 —Divide→ W_1

W_1 → 3

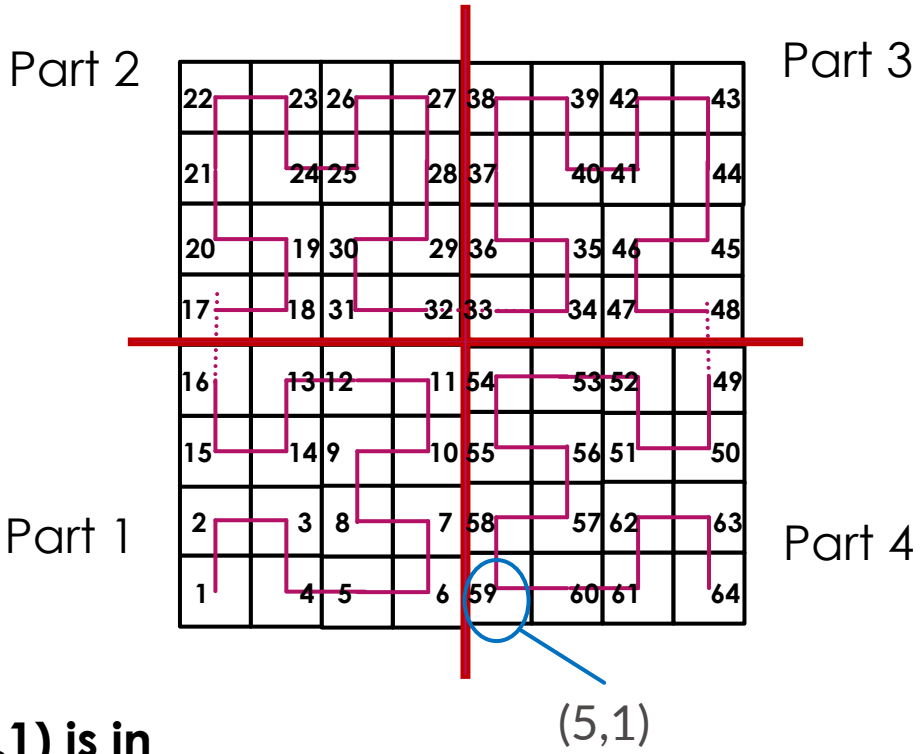
3 —Conquer→ 3+4*2=11 —Conquer→ 11+16*3=59

Problem analysis



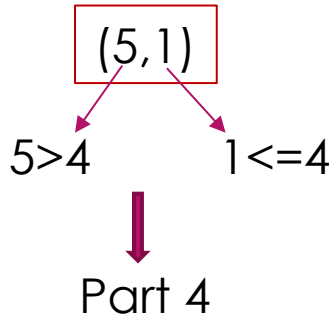
How to map (5,1) to (4,4)?

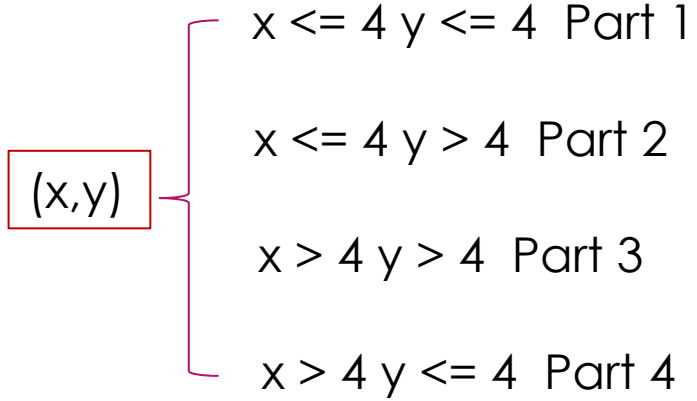
Step 1: Divide the W_3 to 4 parts



Step 2: Find which part (5,1) is in

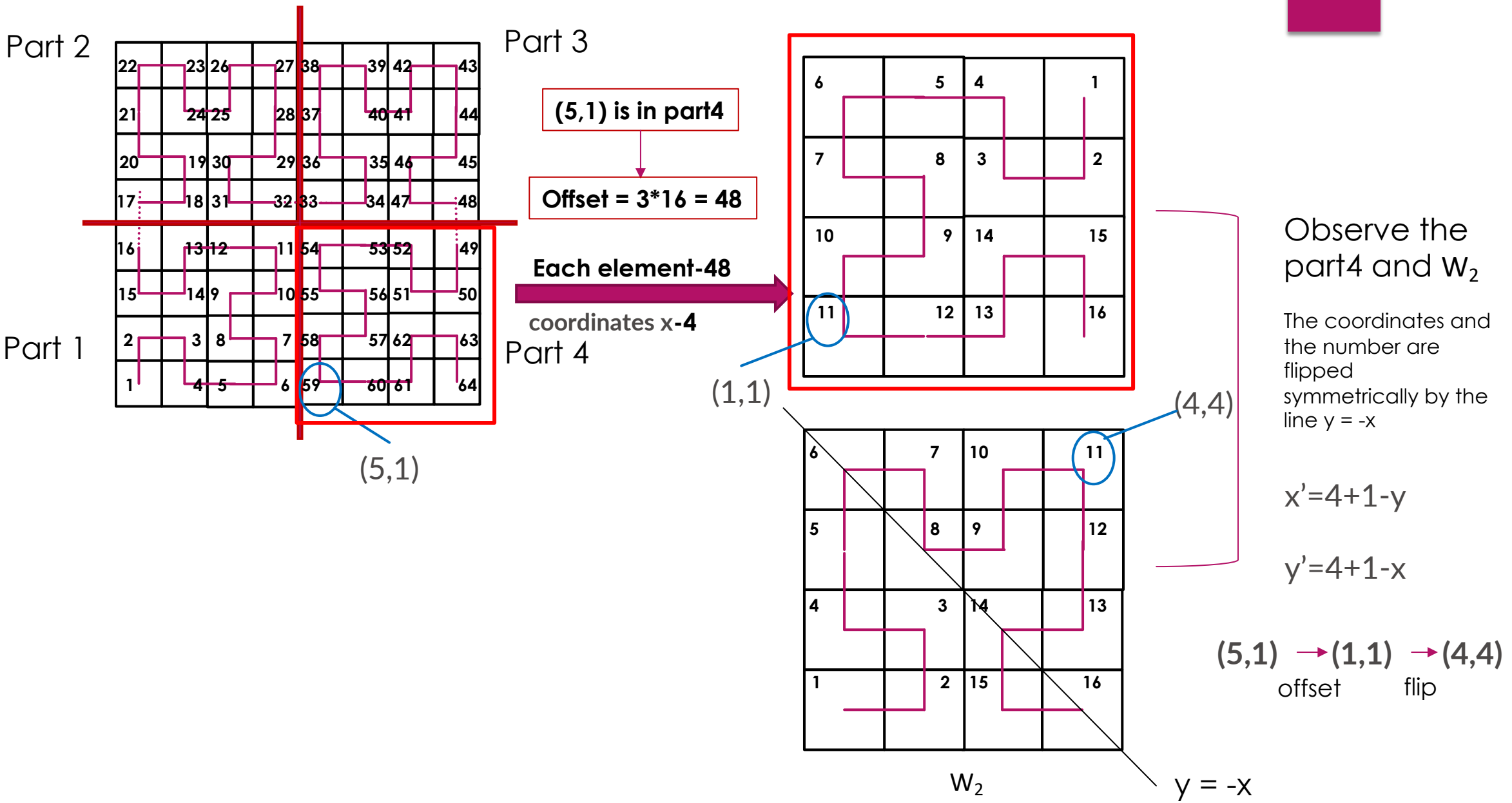
W_3 is a 8*8 square matrix





Step 3: get offset value according part number

Part 4 case



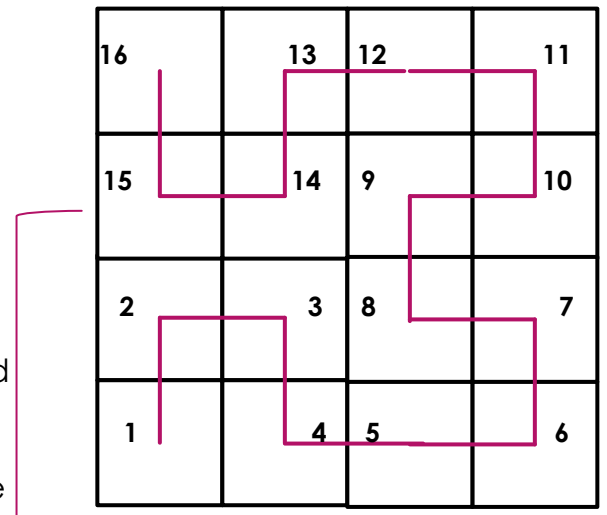
Step 3: Part 1 case

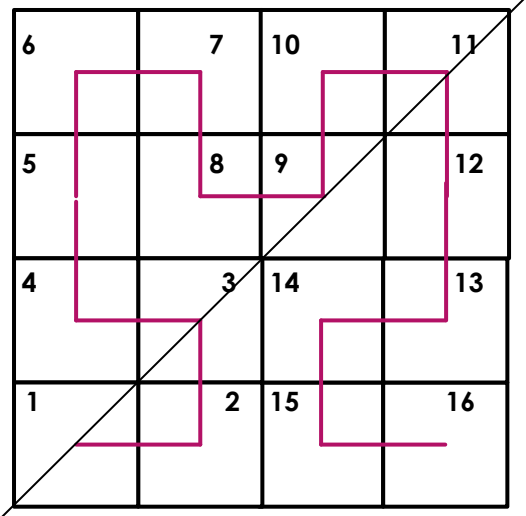
Observe the part1 and W_2

The coordinates and the number are flipped symmetrically by the line y = x

$x'=y$

$y'=x$





W_2

y = x

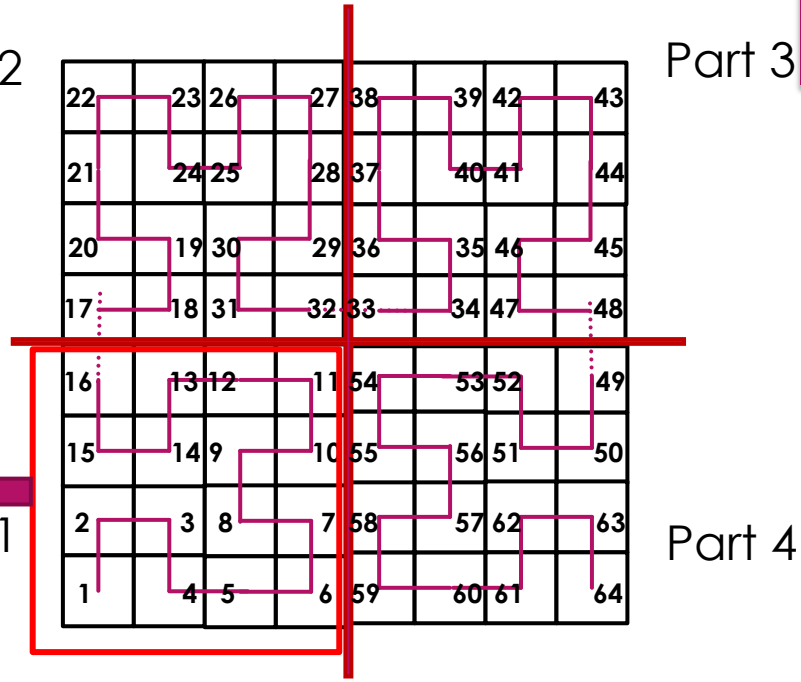
Offset = 0

Part 1

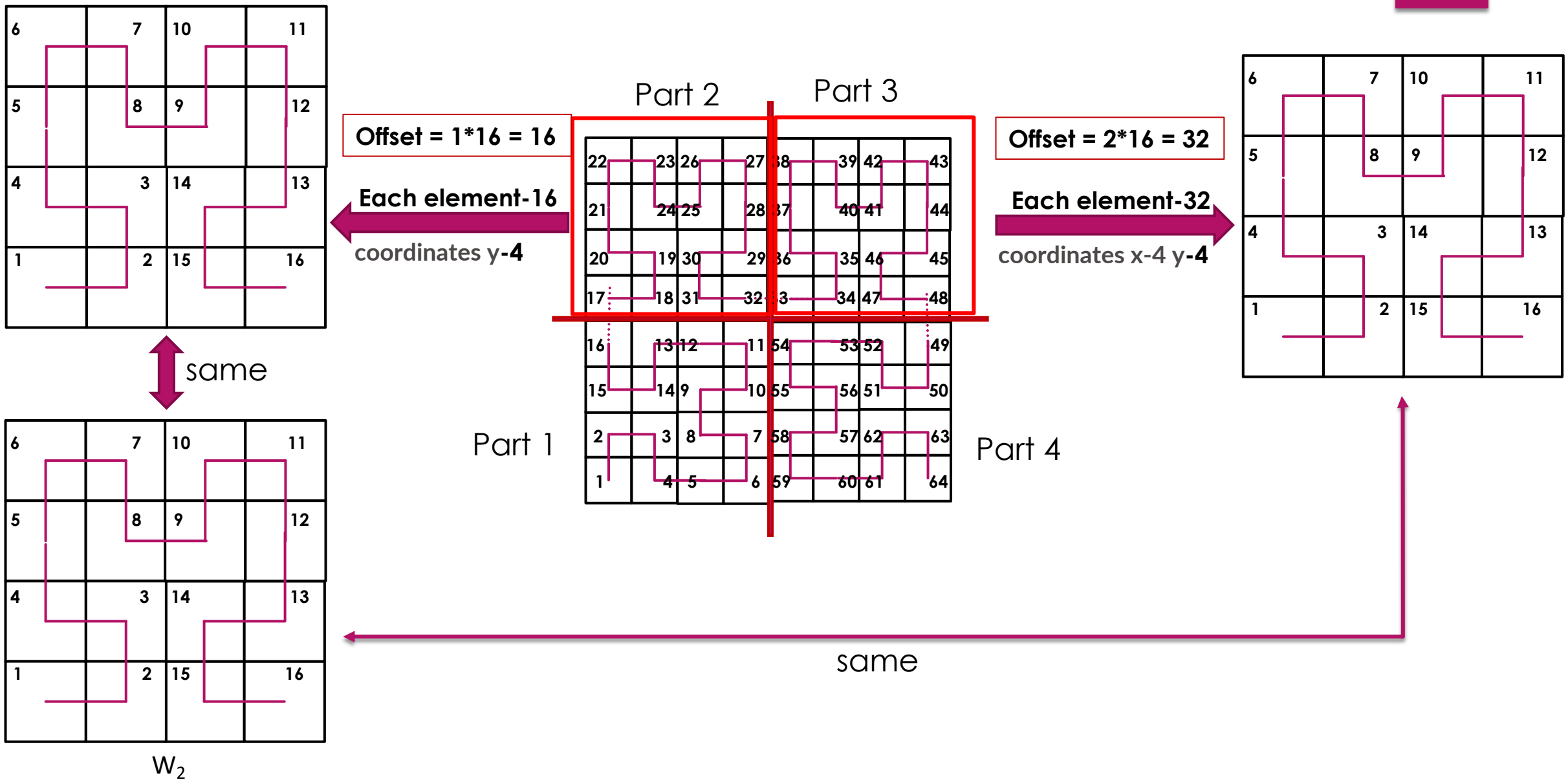
Part 2

Part 3

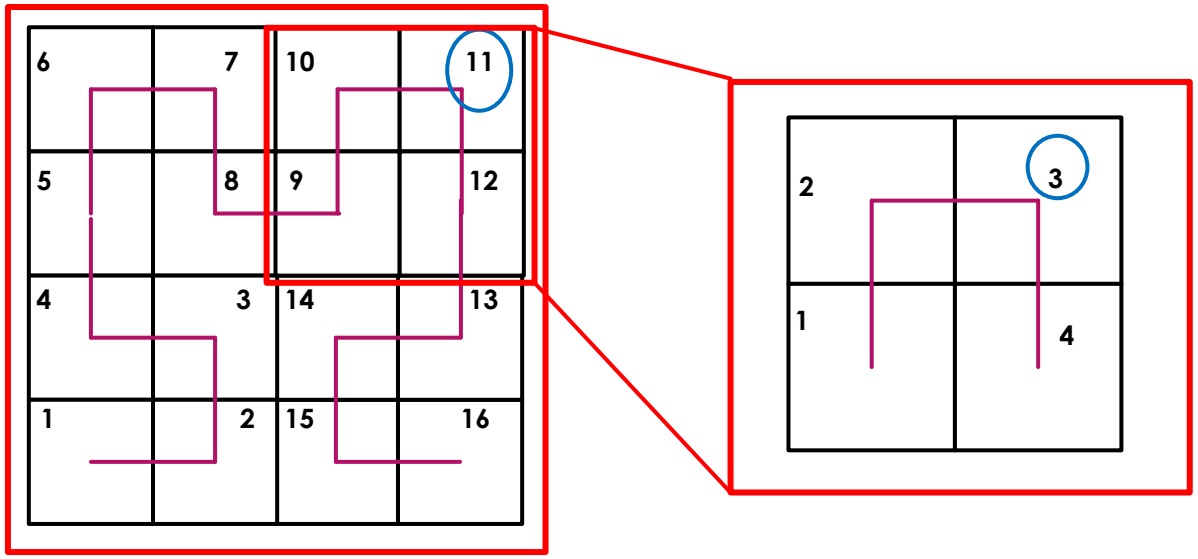
Part 4



Step 3: Part 2&3 cases



Further analysis: W_2



How to map (4,4) to (2,2)?

Step 1: Divide the W_2 to 4 parts

Step 2: Find which part (4,4) is in

(x,y)

- x <= 2 y <= 2 Part 1
- x <= 2 y > 2 Part 2
- x > 2 y > 2 Part 3
- x > 2 y <= 2 Part 4

Step 3: get offset value according part number

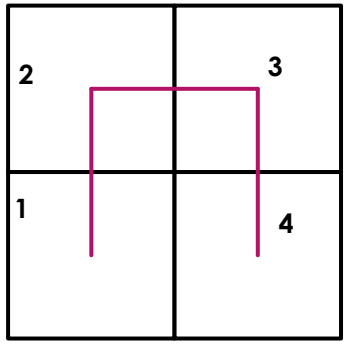
Part 3 **Offset = 2*4 = 8**

Each element-8

coordinates x-2 y-2

(4,4) → (2,2)

Base code: W_1



coordinates	number
(1,1)	1
(1,2)	2
(2,2)	3
(2,1)	4

Please **write pseudocode** for the problem.

The practice will be checked in this lab class or the next lab class(before **Apr.27**) by teachers or SAs.

This practice will contribute **1 mark** to your overall grade. Late submissions within 2 weeks after the deadline (before May.11)will incur a 20% penalty, meaning that you can only get 80% of the score.