Lab2 Solution

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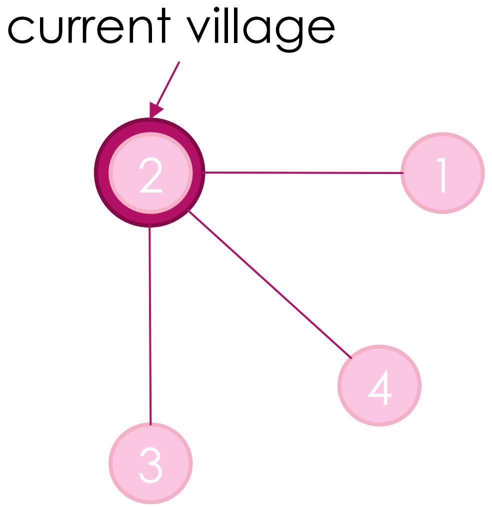
Lab2.A Pay a new year call

- XX lives in a town composed of n villages (numbered $1, 2, 3, \dots, n$ respectively), and m roads (each road connects two villages), of which the number of XX's village is $p(p \in [1, n])$. On the first day of the New Year, XX starts from his village and goes to another village to pay New Year's call through the road connected with his village. XX can also either go through the road he has passed or stay in her current village. Could you tell me the number of villages that XX might stay on the day k?

Input:

```
4 3 2 3
2 1
2 3
4 2
0
1
2
```



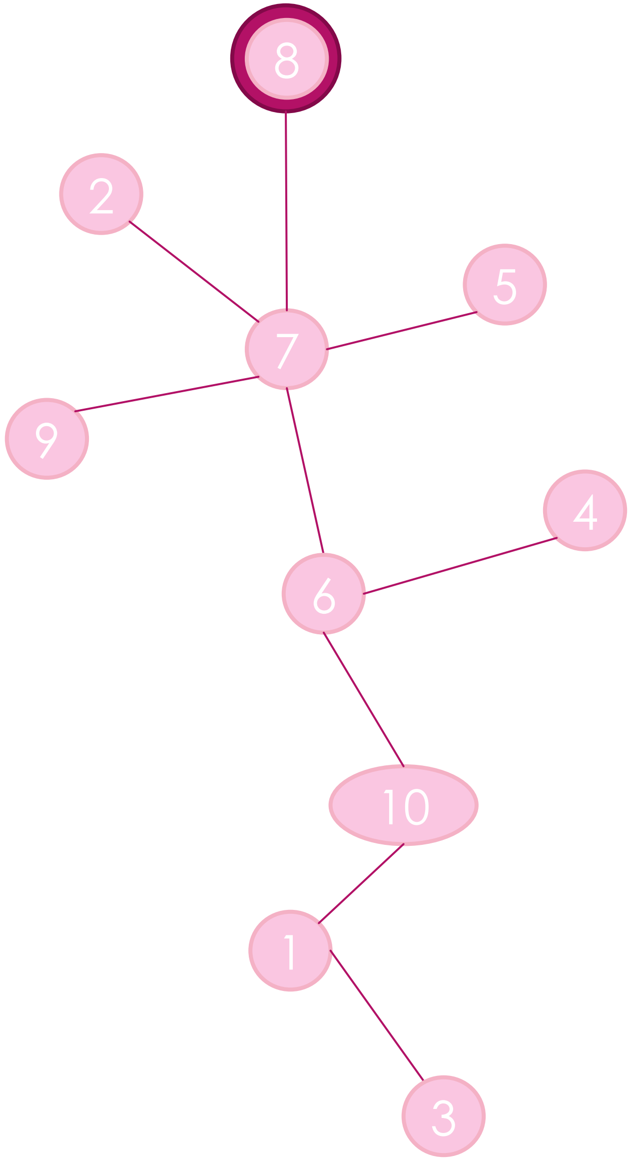
	Which villages XX may stay on	the number of villages
day 0:	2	1
day 1:	2,1,3,4	4
day 2:	2,1,3,4	4

Output:

1 4 4

Note： the range of k updated to [0, 100000]

10 9 8 20
8 7
6 7
4 6
5 7
2 7
1 10
10 6
3 1
9 7
1
3
5
8
11
10



XX can also either go through the road he has passed or stay in her current village

let Num_k is the number of villages on day k

$Num_{k+1} =$

Num_k + the number of villages with the distance from village 8 is k+1

BFS to get a table :distance->villages number

distance:	0	1	2	3	4	5
villages number :	1	1	4	2	1	1

Prefix sum

day:	0	1	2	3	4	5	…	∞
villages number :	1	2	6	8	9	10	…	10

Output：

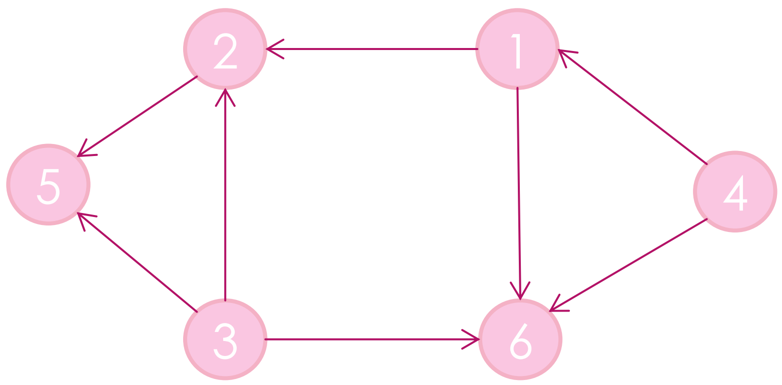
2 8 10 10 10 10

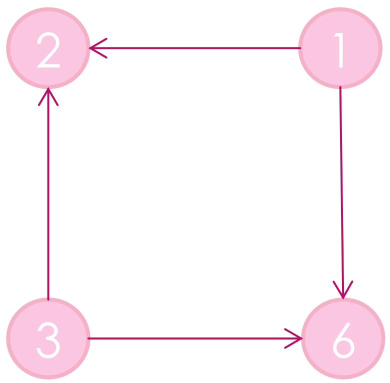
Lab2.B Simplicity Favors Regularity

- **Ihyyy** has a graph $G(V, E)$, but it's too complex. He wants to remove some vertices and edges to get a simple and regular subgraph.
- Suppose $V' = \{v_1, v_2, \dots, v_k\}$, the graph $G'(V', E')$ is called simple if k is even and $E' = \{(v_1, v_2), (v_3, v_2), (v_3, v_4) \dots, (v_{k-1}, v_k), (v_1, v_k)\}$
- To make the subgraph simple, **Ihyyy** wants to minimize $|V'|$
- However, **Ihyyy** knows nothing about graph theory, can you help him?

Input:

1
6 8
1 2
3 2
3 6
1 6
2 5
3 5
4 6
4 1

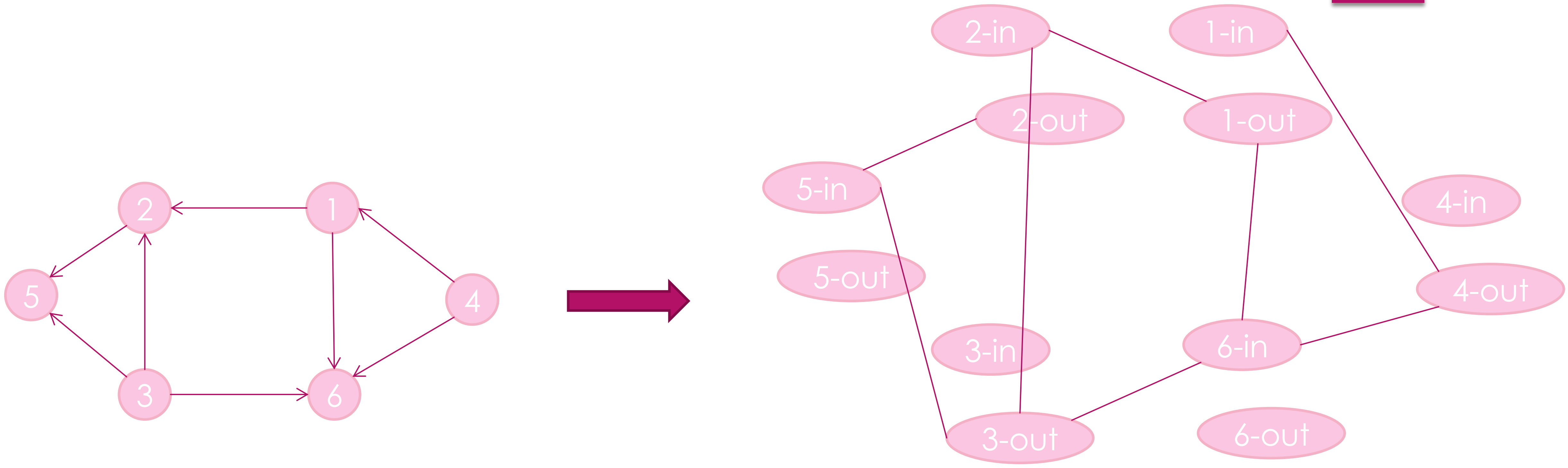




Output:

4

Given a digraph, finds the smallest “circle” so that the directions of the edges on the “circle” are staggered and the number of edges are even.



Original Graph

New Graph

each $\mathrm{v} \in G(V,E)$ $\longrightarrow$ split to $\mathrm{v-in}$ and $\mathrm{v-out}$

each $\mathrm{e} \in G(V,E)$, (v_i, v_j) $\longrightarrow$ (v_{i-out}, v_{j-in})

The new graph is an undirected graph because the node number already indicates the direction.

Original Graph:

$\{(v_1, v_2), (v_3, v_2), (v_3, v_4) \dots, (v_{k-1}, v_k), (v_1, v_k)\}$

New Graph

$\{(v_{1-out}, v_{2-in}), (v_{3-out}, v_{2-in}), (v_{3-out}, v_{4-in}) \dots, (v_{k-1-out}, v_{k-in}), (v_{1-out}, v_{k-in})\}$

The new graph is an undirected graph because the node number already indicates the direction.

$\{(v_{1-out}, v_{2-in}), (v_{2-in}, v_{3-out}), (v_{3-out}, v_{4-in}) \dots, (v_{k-1-out}, v_{k-in}), (v_{k-in}, v_{1-out})\}$

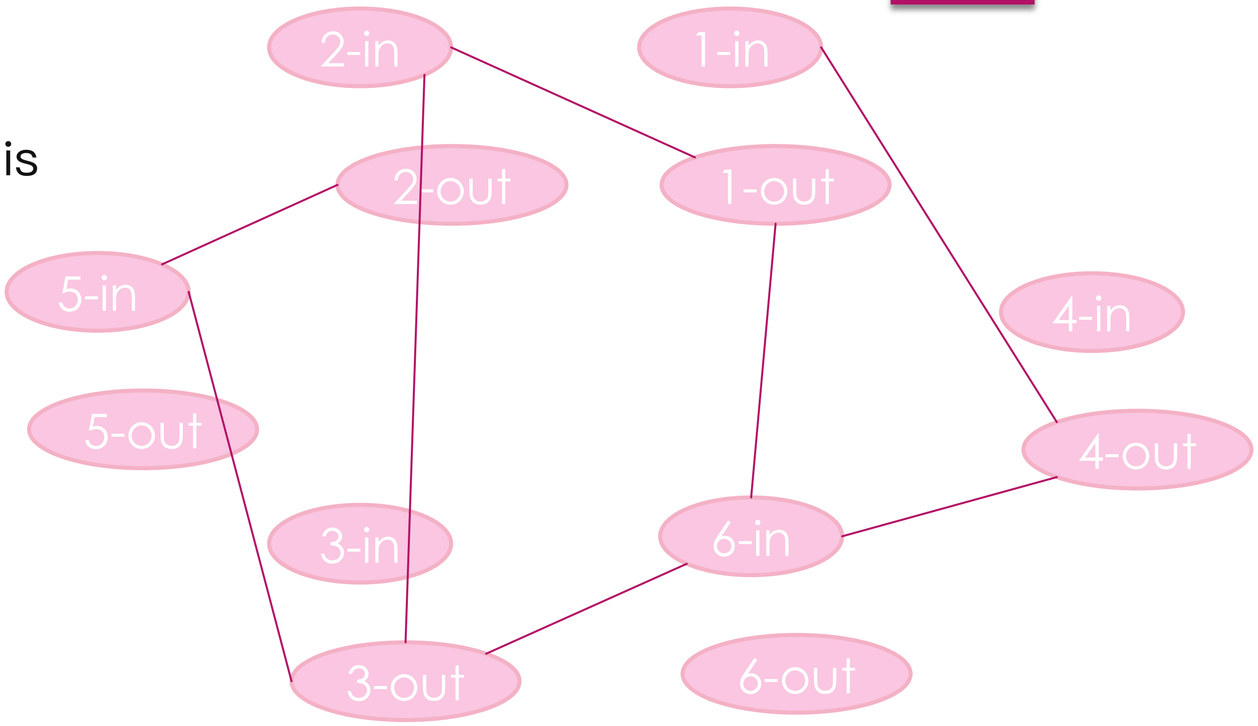
The original problem is transformed into finding the minimum circle in the new graph

How to find the minimum circle in the new graph?

Start BFS from each point

record the distance of each point

if a cross edge is found, a circle is found, which is used to update the answer.



New Graph