Линдеманн Никита, 874

$A = 14$, $B = 16$

№2

$$\int_1^2 \left(t\,\dot{x}^2(t) - x(t)\right)dt \longrightarrow \min$$

$$x(1) = 0, \quad x(2) = 1$$

$$L = t\dot{x}^2 - x$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0 \Rightarrow -1 - \frac{d}{dt}(2t\dot{x}) = 0$$

$$1 + 2\dot{x} + 2t\ddot{x} = 0$$

$$x(t) = C_1 \log t + C_2 - \frac{t}{2}$$

Из $x(1) = 0$ и $x(2) = 1 \Rightarrow$

$$\hat{x} = \frac{\log(2)\cdot(1 - t) + 3\log t}{\log 4}$$

Пусть $\eta \in C^1[1,2]$ и $\eta(1) = \eta(2) = 0$

$$J(\hat{x}+\eta) - J(\hat{x}) = \int_1^2 \left(t(\dot{\hat{x}}+\dot{\eta})^2 - (\hat{x}+\eta)\right)dt -$$

$$-\int_1^2 \left(t\dot{\hat{x}}^2 - \hat{x}\right)dt = \int_1^2 \left(t(2\dot{\hat{x}}\dot{\eta}+\dot{\eta}^2) - \eta\right)dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt + \int_1^2 t\,2\dot{\hat{x}}\dot{\eta}\,dt - \int_1^2 \eta\,dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt + \int_1^2 2t\dot{\hat{x}}\,d\eta - \int_1^2 \eta\,dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt + \overset{0}{\cancel{2t\dot{\hat{x}}\eta\Big|_0^1}} - \int_1^2 \eta\,d(2t\dot{\hat{x}}) - \int_1^2 \eta\,dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt - \int_1^2 \eta(2\dot{\hat{x}} + 2t\ddot{\hat{x}})\,dt - \int_1^2 \eta\,dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt - \int_1^2 \eta\,\overset{0}{\cancel{(2t\ddot{\hat{x}} + 2\dot{\hat{x}} + 1)}}\,dt =$$

$$= \int_1^2 t\dot{\eta}^2\,dt > 0 \Rightarrow \hat{x} \text{ дает min}$$

№1

$$J = \int_0^1 (\dot{x}_1\dot{x}_2 + 14x_1x_2)\,dt + x_1(0)x_2(1) \to extr$$

$$L = \dot{x}_1\dot{x}_2 + 14x_1x_2, \quad l = x_1(0)x_2(1)$$

№ 6

$$\dot{x}(t) = W x(t) + V u(t)$$

$$z = D x(t)$$

$$\underset{n\times n}{W} = \begin{pmatrix} 1 & 0 & 4 \\ 4 & 0 & 6 \\ 0 & 6 & 0 \end{pmatrix}, \quad \underset{n\times p}{V} = \begin{pmatrix} 4 & 1 \\ 6 & 2 \\ 0 & 1 \end{pmatrix}, \quad \underset{q\times n}{D} = \begin{pmatrix} 4 & 1 & 6 \end{pmatrix}$$

$n = 3, \quad p = 2, \quad q = 1$

1) $Q = (V \ \ WV \ \ W^2 V \ \dots \ W^{n-1} V)$ – матрица управляемости

$\Rightarrow$ управляемость зависит только от W и V

2) Наблюдаемость определяется матрицей

$Q_D = (D \ \ DW \ \ DW^2 \dots DW^{n-1})^T \Rightarrow$

наблюдаемость зависит только от D и W

3) Система будет вполне наблюдаемой $\Leftrightarrow \operatorname{rg} Q_D = n$

$$DW = (8 \ \ 36 \ \ 22)$$

$$DW^2 = DW \cdot W = (152 \ \ 132 \ \ 248)$$

$$\operatorname{rg} Q_D = \operatorname{rg} \begin{pmatrix} 4 & 1 & 6 \\ 8 & 36 & 22 \\ 152 & 132 & 248 \end{pmatrix} = 3 = n \Rightarrow$$ Система вполне наблюдаема

№3

$$B(x)=\int_0^4 (\dot{x}^2+x^2)\,dt+16x^2(4)\longrightarrow \min$$

$$L=\dot{x}^2+x^2, \quad l=16x^2(4)$$

$$\frac{\partial L}{\partial x}-\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}=0$$

$$2x-\frac{d}{dt}(2\dot{x})=0$$

$$x-\ddot{x}=0$$

$$x(t)=C_1e^{t}+C_2e^{-t}$$

$$\begin{cases} L'_{\dot{x}}(0)=l'_{x(0)} \\ L'_{\dot{x}}(4)=-l'_{x(4)} \end{cases} \Rightarrow \begin{cases} 2\dot{x}(0)=0 \\ 2\dot{x}(4)=32x(4) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x}(0)=0 \\ \dot{x}(4)=16x(4) \end{cases}$$

$$\dot{x}(t)=C_1e^{t}-C_2e^{-t}$$

$$0=C_1-C_2 \Rightarrow C_1=C_2=C$$

$$Ce^4-Ce^{-4}=16Ce^4+16Ce^{-4}$$

$$C=0 \Rightarrow \hat{x}(t)=0.$$

№4

$$\int_0^{\pi} \dot{x}^2(t)\,dt \longrightarrow \min$$

$$\int_0^{\pi} x(t)\sin(t)\,dt = 1, \quad x(0) = 0 \quad и \quad x(\pi) = 0.$$

$$L = \lambda_0 \dot{x}^2 + \lambda_1 x \sin t$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0$$

$$\lambda_1 \sin t - \frac{d}{dt} 2\lambda_0 \dot{x} = 0$$

$$\lambda_1 \sin t - 2\lambda_0 \ddot{x} = 0$$

$$\lambda_0 = 0 \Rightarrow \lambda_1 = 0$$

$$\lambda_0 = \frac{1}{2} \Rightarrow \ddot{x} - \lambda_1 \sin t = 0$$

$$x(t) = -\lambda_1 \sin t + C_1 t + C_2$$

$$\begin{cases} x(t) = -\lambda_1 \sin t + C_1 t + C_2 \\ x(0) = 0, \quad x(\pi) = 0 \\ \int_0^{\pi} x(t)\sin(t)\,dt = 1 \end{cases} \Rightarrow$$

$$0 = -\lambda_1 \sin 0 + C_1 \cdot 0 + C_2 \Rightarrow C_2 = 0$$

$$0 = -\lambda_1 \cdot \sin(\pi) + C_1 \cdot \pi \Rightarrow C_1 = 0$$

$$-\lambda_1 \int_0^\pi \sin^2 t \, dt = -\frac{\lambda_1}{2}\pi = 1 \Rightarrow \lambda_1 = -\frac{2}{\pi}$$

$$\hat{x} = \frac{2}{\pi} \sin t$$