Линдеман Никита, 874

$$\int_0^{\pi/2} \left(\dot{x}^2(t) - 20x(t)\dot{x}(t) - 74x^2(t) \right) dt + x^2(0) + x\left(\frac{\pi}{2}\right) \longrightarrow \min$$

1) $L = \dot{x}^2 - 20x\dot{x} - 74x^2$

$l = x^2(0) + x\left(\frac{\pi}{2}\right)$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0$$

$$-20\dot{x} - 148x - 2\ddot{x} + 20\dot{x} = 0$$

$$\ddot{x} + 74x = 0 \quad \Rightarrow \quad x(t) = C_1 \cos\left(\sqrt{74}\, x\right) + C_2 \sin\left(\sqrt{74}\, x\right)$$

2) Условие трансверсальности:

$$\begin{cases} \left.\frac{\partial L}{\partial \dot{x}}\right|_{t=0} = \frac{\partial l}{\partial x(0)} \\ \left.\frac{\partial L}{\partial \dot{x}}\right|_{t=\frac{\pi}{2}} = -\frac{\partial}{\partial x\left(\frac{\pi}{2}\right)} \end{cases} \Rightarrow$$

$$\begin{cases} 2\dot{x}(0) - 20x(0) = 2x(0) \\ 2\dot{x}\left(\frac{\pi}{2}\right) - 20x\left(\frac{\pi}{2}\right) = -1 \end{cases} \Rightarrow \begin{cases} \dot{x}(0) = 11\, x(0) \\ 2\dot{x}\left(\frac{\pi}{2}\right) - 20x\left(\frac{\pi}{2}\right) + 1 = 0 \end{cases}$$

$$-2C_1\sqrt{74}\sin\left(\sqrt{74}\,\frac{\pi}{2}\right) + 2C_2\sqrt{74}\cos\left(\sqrt{74}\,\frac{\pi}{2}\right)$$

3)

$$\begin{cases} \sqrt{74}\, C_2 = 11 C_1 \\ -2\cdot\sqrt{74}\, C_1 \sin\left(\sqrt{74}\,\frac{\pi}{2}\right) + 2\sqrt{74}\, C_2 \cos\left(\sqrt{74}\,\frac{\pi}{2}\right) - 20 C_1 \cos\left(\sqrt{74}\,\frac{\pi}{2}\right) - 20 C_2 \sin\left(\sqrt{74}\,\frac{\pi}{2}\right) = -1 \end{cases}$$

$\Rightarrow C_1$ и C_2. Далее подставляем их в

$x(t) = C_1 \cos\left(\sqrt{74}\, x\right) + C_2 \sin\left(\sqrt{74}\, x\right)$ и это будет ответ.