1. $f'(x;u) = \lim_{h \to 0} \frac{f(x+hu)-f(x)}{h} < 0$

$f(x+hu) = f(x) + h \cdot f'(x;u) + o(h)$

$f(x+hu) - f(x) = h \cdot [f'(x;u) - \alpha(h)]$, $\lim_{h \to 0} \alpha(h) = 0$.

Because $f'(x;u) < 0$, $\exists h > 0$ ($\alpha(h) > f'(x;u)$)

then $h \cdot [f'(x;u) - \alpha(h)] < 0$. Therefore, $f(x+hu) < f(x)$.

2. According to Cauchy-Schwarz inequality:

$\|f'(x;u)\| = \|\nabla f(x) \cdot u\| \le \|\nabla f(x)\|_2 \|u\|_2 = \|\nabla f(x)\|_2$

The equation holds when $u = \frac{\nabla f(x)}{\|\nabla f(x)\|_2}$ or $u = -\frac{\nabla f(x)}{\|\nabla f(x)\|_2}$

When $u = \frac{\nabla f(x)}{\|\nabla f(x)\|_2}$, $f'(x;u) = \|\nabla f(x)\|_2$. When $u = -\frac{\nabla f(x)}{\|\nabla f(x)\|_2}$, $f'(x;u) = -\|\nabla f(x)\|_2$.

So the oppisite direction of $\nabla f(x)$ is the steepest descent direction

and the gradient direction is the steepest ascent direction.