Esercizio 1.

$\lambda(\theta) = \lambda_0 \cos(\theta)$ $\quad Q = 10\, nC.$

a) $Q = \int \lambda_0(\theta)\, d\ell = \int_0^{\pi} \lambda_0 \cos\theta\, R\, d\theta \quad \boxed{d\ell = R\, d\theta}$

$$= \lambda_0 R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta$$

$$= \lambda_0 R \left[\operatorname{sen}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\lambda_0 R.$$

$Q = 2\lambda_0 R$

$$\lambda_0 = \frac{Q}{2R} = \frac{10 \cdot 10^{-9}\, C}{2 \cdot 0{,}1\, m} = 5 \cdot 10^{-8}\, \frac{C}{m}$$

b) Il filo è simmetrico rispetto all'asse y, ~~le forze~~ la forza risultante sarà ~~solo~~ lungo l'asse ~~y~~ y (~~lung~~ lungo x le forze sono bilanciate a vicenda). Il filo provoca una forza attrattiva alla carica $-Q$, ~~quindi il vettore~~ essendo loro di carica opposta, quindi il verso della F_e è $+\hat{y}$.

$$E_y = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda \cdot y}{R^3}\, dl \qquad \text{con } y = R \cdot \cos\theta, \quad dl = R\, d\theta, \quad \lambda = \lambda_0 \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda_0 \cos\theta \cdot \cancel{R} \cos\theta}{R^{\cancel{3}}}\, \cancel{R}\, d\theta$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0 R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos 2\theta + 1}{2}\, d\theta$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0 R} \left[\frac{\operatorname{sen} 2\theta}{4} + \frac{1}{2}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\lambda_0}{4\cancel{\pi}\varepsilon_0 R} \cdot \frac{\cancel{\pi}}{2} = \frac{\lambda_0}{8\varepsilon_0 R}\, (-\hat{y})$$

$$F_y = -Q \cdot E_y$$

$$= -Q \cdot \frac{\lambda_0}{8\varepsilon_0 R}(-\hat{y}) = \frac{Q\lambda_0}{8\varepsilon_0 R}\hat{y} = \frac{1 \cdot \cancel{10^{-9}} \, C \cdot 5 \cdot 10^{\cancel{-8}\,-5} \frac{C}{m}}{8 \cdot 8{,}85 \cdot 10^{\cancel{-12}\,-3} \frac{C^2}{N \cdot m^2} \cdot 0{,}1\,m}$$

$$= 7{,}06 \, N \, (\hat{y})$$

c) $\vec{p} = q \cdot \vec{d}$

$d\vec{p} = dq \cdot \vec{d}$ $\qquad d \equiv R.$

$$p = \int \lambda \cdot R \, dl = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \lambda_0 \cos\theta \cdot R^2 \, d\theta$$

$$= \lambda_0 R^2 \cdot \left[\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\lambda_0 R^2$$

$$= 1 \cdot 10^{-9} \, C \cdot m$$

d) $A_1 = (0, D)$

$B_1 = \left(0, \frac{D}{2}\right)$

~~$V_{A1} =$~~

~~V_B~~

la distanza ~~da~~ A è molto maggiore rispetto allo braccio del dipolo. Il potere risentito ~~solo attrazione~~ solo la forza ~~rispetto~~ dovuta dal dipolo essendo la carica complessiva del sistema nulla.

~~$V_{A1} - V_{B1} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3} -$~~

$$V = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3}$$

$$\Delta V = V_{A1} - V_{B1} = \frac{1}{4\pi\varepsilon_0} \left(\frac{p \cdot D}{D^3} - \frac{p \cdot \frac{D}{2}}{\frac{D^3}{8}} \right)$$

$$= -\frac{3p}{4\pi\varepsilon_0 D^2}$$

$$-\Delta U_1 = \Delta K_1 \implies -q_p \Delta V = \frac{m V_f^2}{2} - \frac{m V_i^2}{2}$$

d)

$$v_f^2 = \left(\frac{m v_i^2}{2} - q\,\Delta V\right)\cdot\frac{2}{m}$$

$\cancel{\Delta V < 0}$

$$v_{f1} = \sqrt{v_i^2 - 2\frac{q}{m}\Delta V} = \sqrt{v_i^2 + 2\frac{q}{m}|\Delta V|}$$

$$= \sqrt{v_i^2 - \cancel{2}\frac{q}{m}\frac{3}{\cancel{4}_{2}}\frac{P}{\pi\varepsilon_0 D^2}} = 1{,}40\cdot 10^4 \text{ m/s}$$

$$\cancel{\sqrt{v_i^2 - 2\frac{q}{m}\frac{3}{4}\frac{P}{\pi\varepsilon_0 D^2}} = 4{,}9\cdot 10^4} \text{ m/s}$$

Per il secondo protone

$A_2 = (0, -D)$

$B = (0, -\frac{D}{2})$

$\cancel{V_{A2} - V_{B2}} =$

$$-\Delta V_2 = V_{A2} - V_{B2} = +\Delta V_1 = \frac{3P}{4\pi\varepsilon_0 D^2}$$

$$v_{f2} = \sqrt{v_i^2 + \frac{3}{2}\frac{q}{m}\frac{P}{\pi\varepsilon_0 D^2}} = \cancel{4{,}9\cdot 10^4 \text{ m/s}}$$

$2{,}46\cdot 10^4$ m/s

Esercizio 2

a)

$$E = \begin{cases} = 0 & r < R_1 \\ \dfrac{Q_1}{4\pi r^2 \varepsilon_0} & R_1 < r < R_2 \\ \dfrac{Q_1}{4\pi r^2 \varepsilon_0 \varepsilon_r} & R_2 < r < R_3 \\ \cancel{Q_1 + Q} & \\ \dfrac{Q_1}{4\pi r^2 \varepsilon_0} & R_3 < r < R_4 \\ \dfrac{Q_1 + Q_4}{4\pi r^2 \varepsilon_0} & R_4 < r \end{cases}$$

Con il teorema di Gauss

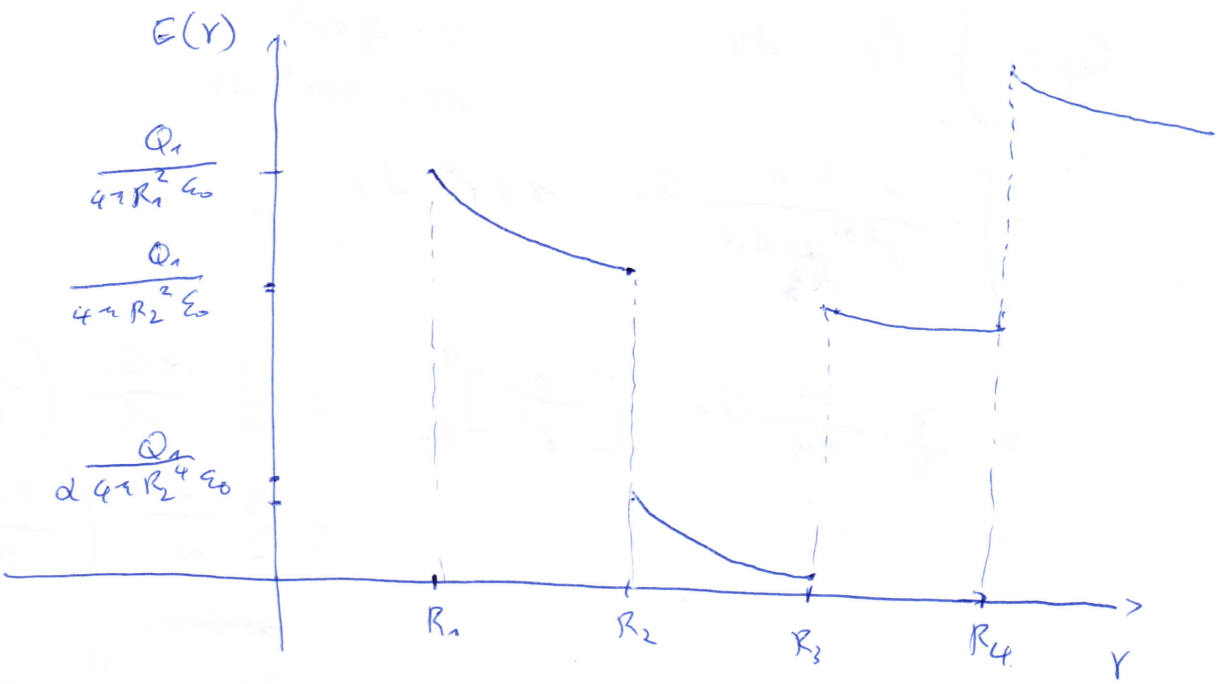
$$4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

~~In caso che si sia~~ In presenza di un materiale dielettrico

$$E_r = \frac{E_0}{\varepsilon_r} \quad \text{con } \varepsilon_r = \chi + 1$$

$$\varepsilon_r = \chi + 1 = \alpha r^2$$

$$\frac{Q_1}{4\pi r^2 \varepsilon_0 \varepsilon_r} = \frac{Q_1}{\alpha 4\pi r^4 \varepsilon_0}$$



b)

$~~\vec{\nabla} \cdot \vec{E} = \frac{\rho + \rho_p}{\varepsilon_0}~~$

$R_2 < r < R_3$

il dielettrico è presente per $~~R_2 < r~~ < R_3$

$$\vec{P} = \chi \varepsilon_0 \vec{E} = (\alpha r^2 - 1)\, \varepsilon_0 \cdot \frac{Q_1}{4\pi r^2 \varepsilon_0 \varepsilon_r}$$

$$= \frac{\alpha r^2 - 1}{\alpha r^2} \cdot \frac{1}{4\pi r^2} Q_1$$

$~~\rho_p~~$ $\rho_p = -\vec{\nabla} \cdot \vec{P}$

$$= ~~\vec{\nabla}~~ ($$

$$-\frac{1}{r^2} \frac{\partial (r^2 P)}{\partial r} = -\frac{1}{r^2} \cdot \frac{\partial \left(\alpha \frac{1}{4\pi} Q_1 - \frac{1}{4\pi \alpha r^2} Q_1 \right)}{\partial r}$$

$$= -\frac{1}{r^2} \left(+ \frac{3}{4} \frac{1}{\pi \alpha r^3} Q_1 \right) = -\frac{1}{r^5} \frac{3}{4} \frac{1}{\pi \alpha} Q_1$$

$$Q_p = \int \rho_p \, d\tau \qquad \tau = \frac{4}{3}\pi r^3 \qquad d\tau = 4\pi r^2 \, dr$$

$$= \int -\frac{1}{r^{3}} \frac{3}{4} \frac{1}{\pi \alpha} Q_1 \cdot \cancel{4\pi r^2}\, dr$$

$$= \frac{3}{2} \cdot \frac{1}{\pi \alpha} Q_1 \cdot \left[\frac{1}{r^2} \right]_{R_2}^{R_3} = \frac{3}{2} \frac{Q_1}{\alpha} \left(\frac{1}{R_3} - \frac{1}{R_2} \right)$$

$$= \frac{3}{2} \frac{Q_1}{\alpha} \left(\frac{R_2 - R_3}{R_3 R_2} \right)$$

$~~75 \cdot 10~~$

$75 \cdot 10^{-13}\, C = 7{,}5 \cdot 10^{-12}\, C$

c) $V_\infty = 0$

Qui considero $V_\infty = 0$

$$\cancel{V_{R_4} = -\int_\infty^{R_4} E_4 \, dr = \int_{R_4}^{\infty}}$$

$V_{R_1} = V(r)$ per $r < R_1$ essendo il campo interno alla prima sfera nullo.

$$V_{R_1} = -\left\{\int_\infty^{R_4} E(r)\,dr + \int_{R_4}^{R_3} E(r)\,dr + \int_{R_3}^{R_2} E(r)\,dr + \int_{R_2}^{R_1} E(r)\,dr\right\}$$

$$= \int_{R_4}$$

$$-\int_\infty^{R_4} E(r)\,dr = \int_{R_4}^{\infty} \frac{Q_1+Q_4}{4\pi r^2 \varepsilon_0}\,dr = \frac{Q_1+Q_4}{4\pi\varepsilon_0}\frac{1}{R_4}$$

$$-\int_{R_4}^{R_3} = \int_{R_3}^{R_4} \frac{Q_1}{4\pi r^2 \varepsilon_0}\,dr = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

$$-\int_{R_3}^{R_2} = \int_{R_2}^{R_3} \frac{Q_1}{4\pi r^2 \varepsilon_0 \varepsilon_r}\,dr = \cancel{\frac{Q_1}{4\pi\varepsilon_0\varepsilon_r}} \frac{Q_1}{4\pi\varepsilon_0\alpha}\int_{R_2}^{R_3} \frac{1}{r^4}\,dr$$

$$= \frac{Q_1}{4\pi\varepsilon_0\alpha}\left(\frac{1}{3R_2^3} - \frac{1}{3R_3^3}\right)$$

$$-\int_{R_2}^{R_1} = \int_{R_1}^{R_2} \frac{Q_1}{4\pi r^2 \varepsilon_0}\,dr = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$V_{R_1} = \frac{Q_4}{4\pi\varepsilon_0}\frac{1}{R_4} + \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_3} - \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{3\alpha R_2^3} - \frac{1}{3\alpha R_3^3}\right)$$

d) ~~La forza~~

La pressione P. è uguale in modulo alla densità di ~~carica~~ energia elettrostatica

~~P =~~

$$p = u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\cancel{\varepsilon_0}\cdot\frac{Q_1+Q_4}{4\pi R_4^2 \cancel{\varepsilon_0}} = \frac{Q_1+Q_4}{8\pi R_4^2}$$

$$= 5{,}47\cdot 10^{-9}\ \frac{N}{m^2}$$