ДЗ 2.

N1. $E_z^2 - \frac{2E_2}{E_1}\cos\alpha E_z E_y + \frac{E_2^2}{E_1^2}E_y^2 = E_2^2\sin^2\alpha$

$E_y = E_1\cos(\omega t - kx)$ $E_z = E_2\cos(\omega t - kx + \alpha)$

$E_y = E_y'\cos\theta + E_z'\sin\theta$ $E_z = -E_y'\sin\theta + E_z'\cos\theta$

$$\frac{E_z^2}{E_2^2\sin\alpha} - \frac{2E_2\cos\alpha E_z E_y}{E_2^2\sin^2\alpha} + \frac{E_2^2 E_y^2}{E_1^2 E_2^2\sin^2\alpha} = 1.$$

$$\frac{E_z'^2\cos^2\theta - E_y' E_z\sin 2\theta + E_y'^2\sin^2\theta}{E_2^2\sin^2\alpha} +$$

$$+ \frac{E_y'^2\sin 2\theta + 2E_y' E_z'\sin^2\theta - 2E_y' E_z\cos^2\theta}{\frac{E_1 E_2\sin^2\alpha}{\cos\alpha}} -$$

$$-2\frac{E_z'^2\cdot\frac{1}{2}\sin 2\theta}{\frac{E_1 E_2\sin^2\alpha}{\cos\alpha}} + \frac{E_y'^2\cos^2\theta + E_y' E_z'\sin 2\theta + E_z'^2\sin^2\theta}{E_1^2\sin^2\alpha} = 1.$$

$$\Rightarrow \frac{E_y'^2}{\sin^2\alpha}\left(\frac{\sin^2\theta}{E_2^2} + \frac{2\cos\theta\sin\theta}{E_1 E_2}\cos\alpha + \frac{\cos^2\theta}{E_1^2}\right) +$$

$$+ \frac{2E_y' E_z'}{\sin^2\alpha}\left(-\frac{\sin 2\theta}{2E_2^2} - \frac{1}{E_1 E_2}\cos\alpha\cos 2\theta + \frac{\sin 2\theta}{2E_1^2}\right) +$$

$$+ \frac{E_z'^2}{\sin^2\alpha}\left(\frac{\cos^2\theta}{E_2^2} - 2\frac{\sin\theta\cos\theta}{E_1 E_2}\cos\alpha + \frac{\sin^2\theta}{E_1^2}\right) = 1.$$

1. $$a = \frac{\sin\alpha}{\sqrt{\frac{\sin^2\theta}{E_1^2} - \frac{\sin 2\theta}{E_1E_2}\cos\alpha + \frac{\cos^2\theta}{E_2^2}}}$$

$$b = \frac{\sin\alpha}{\sqrt{\frac{\cos^2\theta}{E_1^2} + \frac{\sin 2\theta}{E_1E_2}\cos\alpha + \frac{\sin^2\theta}{E_2^2}}}$$

2. $$\frac{1}{2}\sin 2\theta\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right) - \frac{1}{E_1E_2}\cos\alpha\cos 2\theta = 0$$

$$\frac{1}{2}\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right)\cdot tg 2\theta = \frac{1}{E_1E_2}\cos\alpha$$

$$tg 2\theta = \frac{2E_1E_2}{E_2^2 - E_1^2}\cos\alpha$$

3. Круговая поляризация:

$E_2 = E_1$ и $\cos\alpha = 0$ $\quad \alpha = \pi/2 + 2\pi n \quad n \in \mathbb{Z}$

<u>№2.</u>

$t \in [-\pi; \pi]$

$f(t) = t^2$

$$f(t) = \sum_{-\infty}^{+\infty} \hat{f}_k e^{ikt}$$

$$\hat{f}_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 e^{-ikt}\,dt = \left| \begin{array}{ll} u = t^2 & dv = e^{-ikt}dt \\ du = 2t\,dt & V = -\frac{e^{-ikt}}{ik} \end{array} \right| =$$

$$= \frac{1}{2\pi}\left(-\frac{t^2 e^{-ikt}}{ik}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \frac{2t}{ik} e^{-ikt}\,dt \right) =$$

$$= \left| \begin{matrix} u = t & dv = e^{-ikt}dt \\ du = dt & v = -\frac{1}{ik} e^{-ikt} \end{matrix} \right| = \frac{1}{2\pi}\left(-2\frac{t e^{-ikt}}{i^2k^2}\Big|_{-\pi}^{+\pi} \right.$$

$$\left. + \underbrace{\int_{-\pi}^{\pi} \frac{2 e^{-ikt}}{i^2k^2}\,dt}_{0} \right) = \frac{2\pi\left(e^{-i\pi k} + e^{+i\pi k}\right)}{2\pi k^2} = \underline{\frac{2}{k^2}(-1)^k}.$$

$$\Rightarrow f(t) = \sum_{-\infty}^{+\infty} \frac{2}{k^2}(-1)^k e^{ikt}$$

$$\hat{f}_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2\,dt = \pi^3/3$$

$$\Rightarrow f(t) = \frac{\pi^2}{3} + \sum_{-\infty}^{-1} \frac{2}{k^2}(-1)^k e^{ikt} + \sum_{+1}^{+\infty} \frac{2}{k^2}(-1)^k e^{ikt} =$$

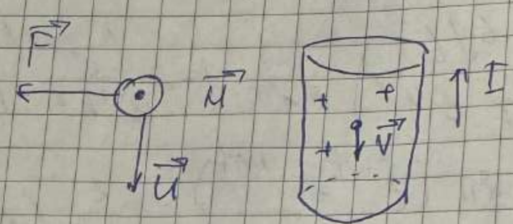
$$= \frac{\pi^2}{3} + \sum_{+1}^{+\infty} \frac{4}{k^2}(-1)^k \cos kt$$

2. $f(\pi) = \pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{+\infty} 4/n^2 \, (-1)^n \cos \pi k = \pi^2/3 +$

$$+ \sum_{n=1}^{\infty} 4/n^2 \, (-1)^n \cdot (-1)^n = \pi^2/3 + \sum_{k=1}^{+\infty} 4/k^2 \Rightarrow \sum_{k=1}^{\infty} 4/k^2 = 2\pi^2/3$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \pi^2/6.$$

№3. Неподвижн. СО λ – лин. плотность



$|\lambda_+| = |\lambda_-| = \lambda$ (т.к. стержень нейтрален. оп.)

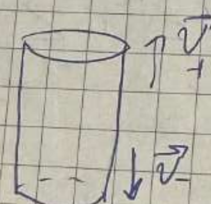
$H = \frac{2I}{cR}$ $I = \frac{dq}{dt} = \lambda v_-$

$H = \frac{2\lambda v_-}{cR}$ $F = \frac{q}{c} u \cdot H = 2q \frac{uv}{c^2} \cdot \frac{\lambda}{R}$

Подвижная СО

$\vec{F} = q\vec{E}$

q



$|v_+'| = u$

скорость положит. зарядов скорость СО

$v_-' = \frac{u - v_-}{1 - \frac{uv_-}{c^2}}$ $v_- = \frac{v_-' - u}{1 + \frac{uv_-'}{c^2}}$

$Q = const \Rightarrow \lambda l = const \Rightarrow \lambda \gamma^{-1} = const$.

$\lambda_+' = \lambda_+ \frac{1}{\sqrt{1 - (\frac{u}{c})^2}}$ $\lambda_-' = \lambda_- \frac{\sqrt{1 - (\frac{v_-}{c})^2}}{\sqrt{1 - (\frac{v_-'}{c})^2}}$

$$= \frac{\lambda}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \left(1 - 1 + \frac{uv}{c^2}\right) = \lambda \frac{uv}{c^2}\gamma \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

$$\Rightarrow E \cdot 2\pi R l = 4\pi \lambda' l \Rightarrow E = \frac{2\lambda'}{R} \qquad \lambda' = \lambda \frac{uv}{c^2}\gamma$$

$$E = 2\frac{uv}{c^2}\frac{\lambda}{R}\gamma \qquad F = 2q\frac{uv}{c^2}\frac{\lambda}{R}\gamma$$

№5. 1. $\nabla \times f(\vec{r})\cdot\vec{r} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial_x & \partial_y & \partial_z \\ f(r)x & f(r)y & f(r)z \end{vmatrix} =$

$$= \bar{i}\left(z\frac{\partial f}{\partial y} - y\frac{\partial f}{\partial z}\right) +$$

$$+ \vec{j}\left(x\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial x}\right) + \bar{k}\left(y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y}\right) =$$

$$= \bar{i}\left(z\frac{\partial f}{\partial r}\frac{\partial r}{\partial y} - y\frac{\partial f}{\partial r}\frac{\partial r}{\partial z}\right) + \ldots = \bar{i}\,0 + \bar{j}\,0 + \bar{k}\,0 = 0.$$

2. $\nabla \times (\vec{a} \times \vec{r}) = -\nabla \times (\vec{r} \times \vec{a}) = -(\vec{a} - 3\vec{a}) = 2\vec{a}$

№7. $\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

$$(\nabla \cdot [\nabla \times \vec{H}]) = \frac{4\pi}{c}(\nabla \cdot \vec{j}) + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \cdot \vec{E})$$

$\operatorname{div}\operatorname{rot} = 0 \qquad \nabla \cdot \vec{E} = 4\pi\rho$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0.$$

№8. $\vec{A}' = \vec{A} + \text{grad}\, f \quad \varphi' = \varphi - \frac{1}{c}\frac{\partial f}{\partial t}$

f - произв ф-ла

$\nabla\cdot \vec{A}' + \frac{1}{c}\frac{\partial \varphi'}{\partial t} = 0$

$\nabla \vec{A} + \nabla\, \text{grad}\, f + \frac{1}{c}\frac{\partial \varphi}{\partial t} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} =$

$= \nabla \vec{A} + \Delta f + \frac{1}{c}\frac{\partial \varphi}{\partial t} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} =$

$= \nabla \vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} + \Box f = \nabla \vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = -\Box f = 0$

$\Rightarrow \nabla \vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$ чтд.

№9.

1. $B_z = B_0 - \alpha z$

$\text{div}\, \vec{B} = \partial_x B_x + \partial_z B_z = \partial_x B_x - \alpha = 0$

$B_x = \alpha x + C$

$B_z(B_x) - 0 = \text{tg}\frac{\pi}{4}(B_x - 0)$ $\quad \frac{B_z}{B_x} = 1$ в начале коорд.

$\frac{B_z}{B_x} = \frac{B_0}{C} = 1 \Rightarrow C = B_0 \Rightarrow \begin{cases} B_z = B_0 - \alpha z \\ B_x = B_0 + \alpha x \end{cases}$