Homework 1

September 22, 2016

(i) Give at least 2 examples of machine learning applications in your life. For each example, please describe how you think this real-world application can be reduced to a machine learning problem.

(ii) Prove that the following inequality

$$\|x\|_\infty \leq \|x\|_2 \leq \|x\|_1$$

holds true for any $x \in \mathbb{R}^n$,

(iii) Show function $q(x) = \|Ax - b\|_2^2$ is convex, where $A \in \mathbb{R}^{m \times n}$, $x, b \in \mathbb{R}^n$. Assume $rank(A) = n$, find the minimizer of $q(x)$. Is it a global minimizer? Why?

(iv) f is twice continuously differentiable. At a point $x \in \mathbb{R}^n$, direction d is a descent direction, i.e., $\nabla f(x)^T d < 0$. Show that we can decrease f by moving (a sufficiently small distance) along such a direction.

(v) Consider the function $f(x_1, x_2) = (x_1 + x_2^2)^2$. At the point $x^T = (1, 0)$ we consider the search direction $p^T = (-1, 1)$. Show that p is a descent direction and find all minimizers of the problem $\min_{\alpha > 0} f(x + \alpha p)$.