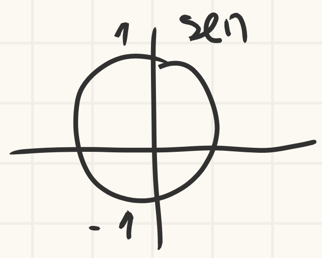
Ficha 1

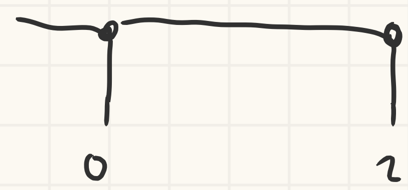



a)

b) $f(x) = \frac{\pi}{2} - \frac{2\arcsen(1-x)}{3}$

$D_f = \{ x \in \mathbb{R} : -1 \leq 1-x \leq 1 \}$ CA $1-x \leq 1 \Leftrightarrow -x \leq 1-1 \Leftrightarrow x \geq 0$

$1-x \geq -1 \Leftrightarrow -x \geq -2 \Leftrightarrow x \leq 2$




$D_f = ]0, 2] = CD_{f^{-1}}$

$CD_f$) $-\frac{\pi}{2} \leq \arcsen(x) \leq \frac{\pi}{2} \Leftrightarrow -\frac{\pi}{2} \leq \arcsen(1-x) \leq \frac{\pi}{2} \Leftrightarrow$

$\Leftrightarrow -\frac{\pi}{3} \leq 2\arcsen(1-x) \leq \frac{\pi}{3} \Leftrightarrow \frac{\pi}{2} + \frac{\pi}{3} \geq \frac{\pi}{2} - 2\arcsen \geq \frac{\pi}{2} - \frac{\pi}{3}$

$\Leftrightarrow \frac{3\pi}{6} + \frac{2\pi}{6} \geq \frac{\pi}{2} - 2\arcsen \geq \frac{3\pi}{6} - \frac{2\pi}{6} \Leftrightarrow \frac{5\pi}{6} \geq \frac{\pi}{2} - 2\arcsen \geq \frac{\pi}{6}$

$CD_f = \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] = D_{f^{-1}}$

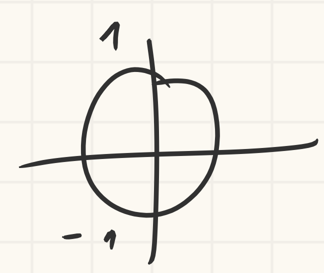
$f^{-1}$) $\frac{\pi}{2} - \frac{2\arcsen(1-x)}{3} = y \Leftrightarrow -\frac{2\arcsen(1-x)}{3} = y - \frac{\pi}{2} \Leftrightarrow$

$\Leftrightarrow \arcsen(1-x) = \left(y - \frac{\pi}{2}\right) \cdot \frac{3}{-2} \Leftrightarrow 1-x = \sen\left(\frac{3y - \frac{3\pi}{2}}{-2}\right)$

$\Leftrightarrow x = -\sen\left(-\frac{3y}{2} + \frac{3\pi}{4}\right) + 1$

$f^{-1} = 1 - \sen\left(\frac{3\pi}{4} - \frac{3y}{2}\right)$

d) $f(x) = e^{\arcsen x}$




$D_f = \{ x \in \mathbb{R} : -1 \leq x \leq 1 \}$ $D_f = [-1, 1] = CD_{f^{-1}}$

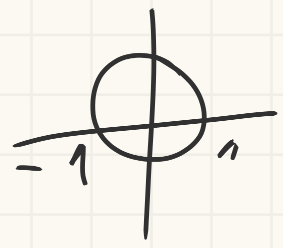
$CD_f$) $-\frac{\pi}{2} \leq \arcsen x \leq \frac{\pi}{2} \Leftrightarrow e^{-\frac{\pi}{2}} \leq e^{\arcsen x} \leq e^{\frac{\pi}{2}}$

$CD_f = \left[e^{-\frac{\pi}{2}}, e^{\frac{\pi}{2}}\right]$

$$f^{-1})\quad e^{\operatorname{arcsen} x} = y \iff \operatorname{arcsen} x = \ln y \iff$$

$$\iff x = \operatorname{sen}(\ln y)$$

$$f^{-1} = \operatorname{sen}(\ln(x))$$



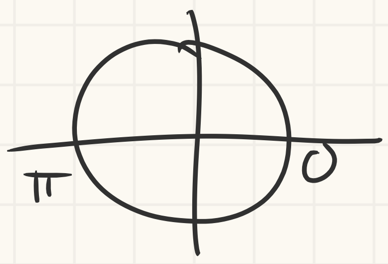

$$f)\ f(x) = 3\arccos(\sqrt{x+4}) - \frac{\pi}{2}$$

$$D_f = \{ x \in \mathbb{R} : 0 \leq x+4 \ \wedge\ -1 \leq \sqrt{x+4} \leq 1 \}$$

CA

- $x+4 \geq 0 \iff x \geq -4$
- $\sqrt{x+4} \geq -1$ cod. universal
- $\sqrt{x+4} \leq 1 \iff x+4 \leq 1 \iff x \leq -3$

$$D_f = [-4, -3] = CD_{f^{-1}}$$




$CD_f)$

$$0 \leq \arccos(x) \leq \pi \iff 0 \leq \arccos(\sqrt{x+4}) \leq \frac{\pi}{2} \iff$$

$$\iff 0 \leq 3\arccos(\sqrt{x+4}) \leq \frac{3\pi}{2} \iff -\frac{\pi}{2} \leq 3\arccos(\sqrt{x+4}) - \frac{\pi}{2} \leq \pi$$

$$CD_f = \left[-\frac{\pi}{2}, \pi\right] = D_{f^{-1}}$$

$$f^{-1})\quad 3\arccos(\sqrt{x+4}) - \frac{\pi}{2} = y \iff 3\arccos(\sqrt{x+4}) = y + \frac{\pi}{2}$$

$$\iff \arccos(\sqrt{x+4}) = \frac{y}{3} + \frac{\pi}{6} \iff \sqrt{x+4} = \cos\left(\frac{y}{3} + \frac{\pi}{6}\right)$$

$$\iff x+4 = \cos^2\left(\frac{y}{3} + \frac{\pi}{6}\right) \iff x = \cos^2\left(\frac{y}{3} + \frac{\pi}{6}\right) - 4$$

$$g)\ f(x) = \frac{1}{\pi + \arccos(x-2)}$$

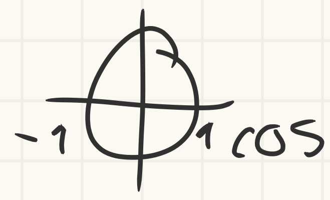
$$D_f = \{ x \in \mathbb{R} : -1 \leq x-2 \leq 1 \ \wedge\ x + \arccos(x-2) \neq 0 \}$$

C.A

- $x-2 \geq -1 \Leftrightarrow x \geq 1$
- $x-2 \leq 1 \Leftrightarrow x \leq 3$
- $\pi + \arccos(x-2) \neq 0 \Leftrightarrow \arccos(x-2) \neq -\pi \Leftrightarrow x-2 \neq \cos(-\pi)$

$\Leftrightarrow x \neq \cos(-\pi) - 2 \Leftrightarrow x \neq -3$

$$D_f = [1, 3] = CD_{f^{-1}}$$




$CD_f)$ $0 \leq \arccos(x-2) \leq \pi \Leftrightarrow \pi \leq \pi + \arccos(x-2) \leq 2\pi$

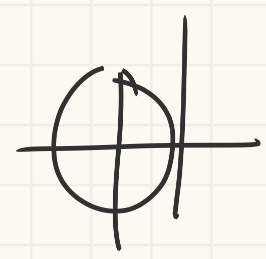
$\Leftrightarrow \frac{1}{\pi} \geq \frac{1}{\pi + \arccos(x-2)} \geq \frac{1}{2\pi}$

$$CD_f = \left[\frac{1}{2\pi}, \frac{1}{\pi}\right]$$

$f^{-1})$ $y = \frac{1}{\pi + \arccos(x-2)} \Leftrightarrow \frac{1}{y} = \pi + \arccos(x-2) \Leftrightarrow$

$\Leftrightarrow \frac{1}{y} - \pi = \arccos(x-2) \Leftrightarrow \cos\left(\frac{1}{y} - \pi\right) = x-2$

$\Leftrightarrow \cos\left(\frac{1}{y} - \pi\right) + 2 = x$ $\qquad f^{-1} = \cos\left(\frac{1}{x} - \pi\right) + 2$



h) $f(x) = \pi - 3\operatorname{arctg}\left(\frac{x-1}{2}\right)$

$$D_f = \{x \in \mathbb{R}\} = CD_{f^{-1}}$$

$CD_f)$ $-\frac{\pi}{2} \leq \operatorname{arctg}\left(\frac{x-1}{2}\right) \leq \frac{\pi}{2} \Leftrightarrow -\frac{3\pi}{2} \leq 3\operatorname{arctg}\left(\frac{x-1}{2}\right) \leq \frac{3\pi}{2}$

$\Leftrightarrow \pi + \frac{3\pi}{2} \geq \pi - 3\operatorname{arctg}\left(\frac{x-1}{2}\right) \geq \pi - \frac{3\pi}{2} \Leftrightarrow$

$\Leftrightarrow \frac{5\pi}{2} \geq \pi - 3\operatorname{arctg}\left(\frac{x-1}{2}\right) \geq -\frac{\pi}{2}$

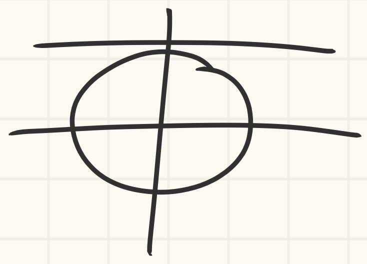
$$CD_f = \left[-\frac{\pi}{2}, \frac{5\pi}{2}\right] = D_{f^{-1}}$$

$F^{-1})$ $\pi - 3\,\text{arctg}\left(\frac{x-1}{2}\right) = y \Leftrightarrow -3\,\text{arctg}\left(\frac{x-1}{2}\right) = y - \pi$

$\Leftrightarrow \text{arctg}\left(\frac{x-1}{2}\right) = \frac{y-\pi}{-3} \Leftrightarrow \frac{x-1}{2} = \text{tg}\left(\frac{y-\pi}{-3}\right)$

$\Leftrightarrow x - 1 = 2\,\text{tg}\left(\frac{-y+\pi}{3}\right) \Leftrightarrow x = 2\,\text{tg}\left(\frac{-y+\pi}{3}\right) + 1$

$$F^{-1} = 2\,\text{tg}\left(\frac{-x+\pi}{3}\right) + 1$$



i) $f(x) = \text{arccotg}(\ln(x+1))$

$D_f = \{ x \in \mathbb{R} : x + 1 > 0 \}$ $\quad D_f = ]-1, +\infty[ = CD_{f^{-1}}$

$CD_f)$

$0 < \text{arccotg}(x) < \pi \Leftrightarrow 0 < \text{arccotg}(\ln(x)) < \pi$

$CD_f = ]0, \pi[ = D_{f^{-1}}$

$f^{-1})$ $\text{arccotg}((\ln(x+1)) = y \Leftrightarrow \ln(x+1) = \text{cotg}(y) \Leftrightarrow x+1 = e^{\text{cotg}(y)}$

$\Leftrightarrow x = e^{\text{cotg}(y)} - 1$ $\quad f^{-1}(x) = e^{\text{cotg}(x)} - 1$

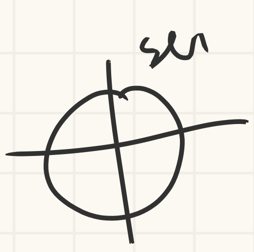
7) $f(x) = \text{arcsen}(x^2 - 1)$

$D_f = \{ x \in \mathbb{R} : -1 \leq x^2 - 1 \leq 1 \}$

C.A

- $x^2 - 1 \geq -1 \Leftrightarrow x^2 \geq 0$ cond univ.
- $x^2 - 1 \leq 1 \Leftrightarrow x^2 \leq 2 \Leftrightarrow x \leq \pm\sqrt{2}$

$\Leftrightarrow x \geq -\sqrt{2} \wedge x \leq \sqrt{2}$

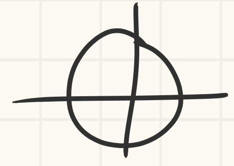



$D_f = [-\sqrt{2}, \sqrt{2}]$

$CD_f)$ $-\frac{\pi}{2} \leq \text{arcsen}(x) \leq \frac{\pi}{2} \Leftrightarrow 0 \leq \text{arcsen}(x^2) \leq \frac{\pi}{2}$

$\Leftrightarrow -\frac{\pi}{2} \leq \text{arcsen}(x^2 - 1) \leq \frac{\pi}{2}$ $\quad CD_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Interseção de f com os eixos?



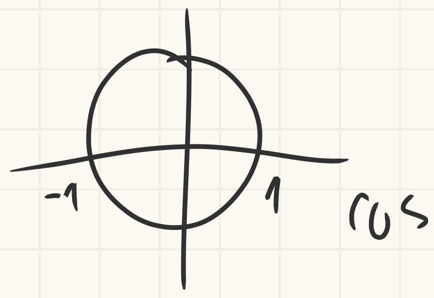
$f(0) = \arcsin(0^2 - 1) = \arcsin(-1) = -\frac{\pi}{2}$ interseta em $y = -\frac{\pi}{2}$

$f(x) = 0 \Leftrightarrow \arcsin(x^2 - 1) = 0 \Leftrightarrow x^2 - 1 = \text{sen}(0) \Leftrightarrow x^2 = 0 + 1 \Leftrightarrow x^2 = 1$

interseta em $f(1)$ e $f(-1)$

em $(-1, 0)$ , $(1, 0)$ e $(0, -\frac{\pi}{2})$

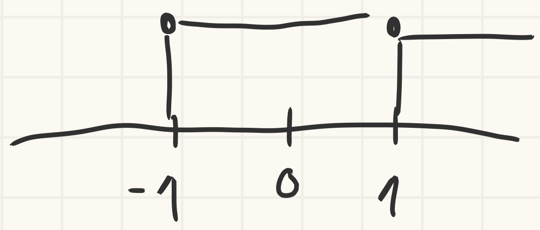
8) $g(x) = \arccos\left(\frac{1}{x}\right)$ , $D_g$, $CD_g$ e zeros?




$D_g = \{x \in \mathbb{R} : -1 \leq \frac{1}{x} \leq 1 \wedge x \neq 0\}$

C.A

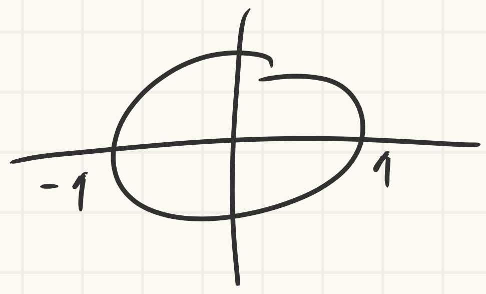
- $\frac{1}{x} \geq -1 \Leftrightarrow x \leq -1 \wedge x > 0$ $\quad ]-\infty, -1] \cup [1, +\infty[$
- $\frac{1}{x} \leq 1 \Leftrightarrow x \geq 1 \wedge x < 0$




como $D_f = ]-\infty, -1] \cup [1, +\infty[$ nunca vai ser 0

$CD_g$) $0 \leq \arccos \leq \pi \Leftrightarrow \arccos\left(\frac{1}{x}\right)$

$CD_g = [0, \pi] / \{\frac{\pi}{2}\}$




zeros) $g(x) = 0 \Leftrightarrow \arccos\left(\frac{1}{x}\right) = 0 \Leftrightarrow \frac{1}{x} = \cos(0) \Leftrightarrow \frac{1}{x} = 1 \Leftrightarrow x = 1$

10) $g(x) = \begin{cases} \sqrt{-x} \cdot \text{sen}\left(\frac{1}{x^3}\right) & , x < 0 \\ \text{arctg}(x^2) & , x \geq 0 \end{cases}$

$\lim_{x \to 0} g(x) = ?$

$\lim_{x \to 0^+} \operatorname{arctg}(0^?) = 0$

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} \sqrt{-x} \cdot \operatorname{sen}\left(\frac{1}{x^3}\right) = 0 \cdot \lim_{x \to 0^-} \operatorname{sen}\left(\frac{1}{x^3}\right) = 0$ ✓

13- $x^5 - 3x = 1$ admite pelo menos uma solução em $]1,2[$ ?

$f(x)\ x^5 - 3x - 1 = 0$

1) $1 - 3 - 1 = -3$

2) $32 - 6 - 1 = 32 - 7$

$f(1) \cdot f(2) < 0$ então admite um zero em $]1,2[$

17) $f(x) = \begin{cases} 1 - x & \text{se } x < 0 \\ x^2 + 3 & \text{se } x \geq 0 \end{cases}$ tem mínimo global em $[-1, 1]$?

$f(-1) = 1 - (-1) = 2$

$\lim_{x \to -1} f(x) = 2$

$f(1) = 1 + 3 = 4$

$\lim f(1) = 4$

$f(0) = 3$

$\lim_{x \to 0^+} = 3$

$\lim_{x \to 0^-} = 1$

22) $f(x) = (x - 1)(x^2 + 3x)$ $f'(x) = ?$

$f'(x) = (x-1)' \cdot (x^2 + 3x) + (x^2 + 3x)'(x - 1) =$

$= 1 \cdot (x^2 + 3x) + (2x + 3)(x - 1) = x^2 + 3x + 2x^2 - 2x + 3x - 3$

$= 2x^2 + x^2 + 3x - 2x + 3x - 3 = 3x^2 + 4x - 3$

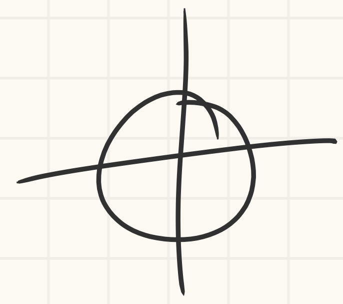
c) $f(x) = \dfrac{\cos x}{1 - \operatorname{sen} x}$ $f'(x) = ?$

$f'(x) = \dfrac{\cos' x\,(1 - \operatorname{sen} x) - \cos x\,(1 - \operatorname{sen} x)'}{(1 - \operatorname{sen} x)^2} =$

$= \dfrac{-\operatorname{sen} x\,(1 - \operatorname{sen} x) - \cos x\,(-\cos x)}{(1 - \operatorname{sen} x)^2} =$

$$= \frac{-\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{1 - 2\operatorname{sen} x + \operatorname{sen}^2 x} = \frac{-\operatorname{sen} x + 1}{(1 - \operatorname{sen} x)^2} =$$

$$= \frac{1}{1 - \operatorname{sen} x}$$



33) $f(x) = \operatorname{arcsen}(1-x) + \sqrt{2x - x^2}$

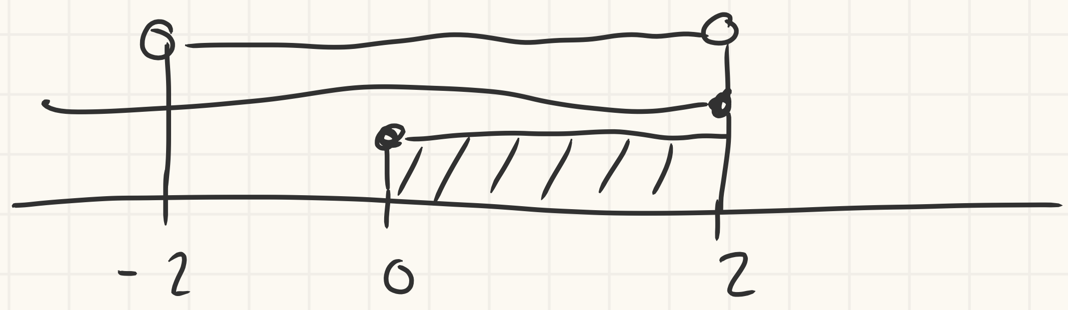
a) $D_f = ?$

$D_f = \{ x \in \mathbb{R} : 2x - x^2 > 0 \ \land \ -1 \leq 1 - x \leq 1$

CA

- $2x - x^2 > 0 \Leftrightarrow 2x > x^2 \Leftrightarrow x \in ]-2, 2[$
- $1 - x \geq -1 \Leftrightarrow -x \geq -2 \Leftrightarrow x \leq 2$
- $1 - x \leq 1 \Leftrightarrow -x \leq 0 \Leftrightarrow x \geq 0$

$D_f = [0, 2]$



b) $f'(x) = -\frac{x}{\sqrt{2x - x^2}}$ ?

$f'(x) = \operatorname{arcsen}'(1-x) + \left(\sqrt{2x - x^2}\right)' =$

$= \frac{-1}{\sqrt{1-(1-x)^2}} + \frac{1}{2}(2x - x^2)^{-\frac{1}{2}} \cdot (2 - 2x)$

$= -\frac{1}{\sqrt{2x - x^2}} + \frac{1}{2\sqrt{2x - x^2}}(2 - 2x) = \frac{-2}{2\sqrt{2x - x^2}} + \frac{2 - 2x}{2\sqrt{2x - x^2}}$

$= \frac{-2 + 2 - 2x}{2\sqrt{2x - x^2}} = \frac{-2x}{2\sqrt{2x - x^2}} =$

$= -\frac{x}{\sqrt{2x - x^2}}$ ✓

CA

$1 - (1-x)^2 = 1 - (1^2 - 2x + x^2)$

$= + 2x - x^2$

c) Max e min globais ?

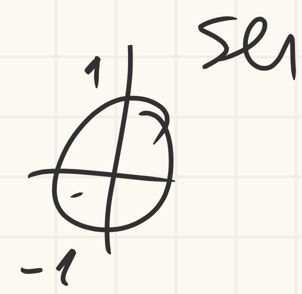
Como $f$ é contínua em $[0, 2]$, ela atinge max e mínimo globais nesse intervalo (T. Weierstrass)

Como $f'(x) < 0$ em $]0,2[ \rightarrow f$ é estrita/ decrescente

então maximizante $= f(0)$ e minimizante $= f(2)$

máximo) $f(0) = \arcsin(1-0) + \sqrt{2.0 - 0^2} = \arcsin(1) + 0$

$f(0) = \frac{\pi}{2}$




mínimo) $f(2) = \arcsin(1-2) + \sqrt{2.2 - 2^2} = \arcsin(-1) + \sqrt{0}$

$= -\frac{\pi}{2} + 0$

- mínimo global $-\frac{\pi}{2}$ em $f(2)$
- máximo global $\frac{\pi}{2}$ em $f(0)$

d) $CD_f$? $CD_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

34) $h(x) = \operatorname{arctg}(x^2 - 4x)$ Extremos e monotonia?

$D_h = \{x \in \mathbb{R}\}$

$h'(x) = \frac{2x-4}{1+(x^2-4x)^2}$

$h'(x) = 0 \Leftrightarrow \frac{2x-4}{1+(x^2-4x)^2} = 0 \Leftrightarrow 2x - 4 = 0 \wedge 1 + (x^2-4x)^2 \neq 0$

cond. universal

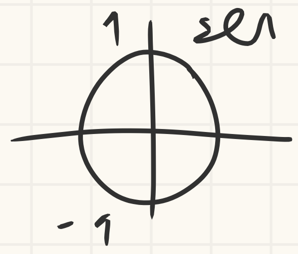
$2x - 4 = 0 \Leftrightarrow 2x = 4 \Leftrightarrow x = 2$

| | $-\infty$ | 2 | $+\infty$ |
|---|---|---|---|
| $h'(x)$ | $-$ | 0 | $+$ |
| $h(x)$ | $\searrow$ | min | $\nearrow$ |

h é estritamente decrescente em $]-\infty, 2[$ e est. cresc. em $]2, +\infty[$

2 é minimizante global de h

$h(2) = \operatorname{arctg}(2^2 - 4 . 2) = \operatorname{arctg}(-4)$ é o mínimo global de $h$




35) $g(x) = \operatorname{arcsen}((x-1)^2)$

a) $Dg = ?$ $\quad Dg = \{x \in \mathbb{R} : -1 \leq (x-1)^2 \leq 1\}$

C.A

- $(x-1)^2 \geq -1$ cond. univ $\qquad Dg = [0, 2]$

- $(x-1)^2 \leq 1 \Leftrightarrow x - 1 \leq 1^2 \rightarrow x \leq +1+1 \Leftrightarrow x \leq 2$

  $x \leq -1+1 \Leftrightarrow x \geq 0$

c) Extremos e monotonia?

$$g'(x) = \frac{(2x-2) \cdot 1}{\sqrt{1-(x-1)^4}} = \frac{2x-2}{\sqrt{1-(x-1)^4}}$$

$$g'(x) = 0 \Leftrightarrow \frac{2x-2}{\sqrt{1-(x-1)^4}} = 0 \Leftrightarrow 2x - 2 = 0 \wedge 1-(x-1)^4 > 0$$

C.A

- $2x - 2 = 0 \Leftrightarrow 2x = 2 \Leftrightarrow x = 1$

- $1-(x-1)^4 > 0 \Leftrightarrow 1 > (x-1)^4$

  $\Leftrightarrow |x-1| < 1 \Leftrightarrow x - 1 < \pm 1$

  $\Leftrightarrow x - 1 < 1 \wedge x - 1 > -1$

  $\Leftrightarrow x < 2 \wedge x > 0 \rightarrow x \in ]0, 2[$

| | 0 | | 1 | | 2 |
|---|---|---|---|---|---|
| $g'$ | | $-$ | 0 | $+$ | |
| $g$ | $\frac{\pi}{2}$ | ↘ | 0 | ↗ | $\frac{\pi}{2}$ |


$g(0) = \operatorname{arcsen}((0-1)^2) = \operatorname{arcsen}(1) = \frac{\pi}{2}$

$g(1) = \operatorname{arcsen}(0) = 0$

$g(2) = \operatorname{arcsen}((2-1)^2) = \operatorname{arcsen}(1) = \frac{\pi}{2}$

$g$ é: estritamente decrescente em $[0, 1]$

" crescente em $[1, 2]$

máximo global $= \frac{\pi}{2}$, maximizantes $= 0$ e $2$

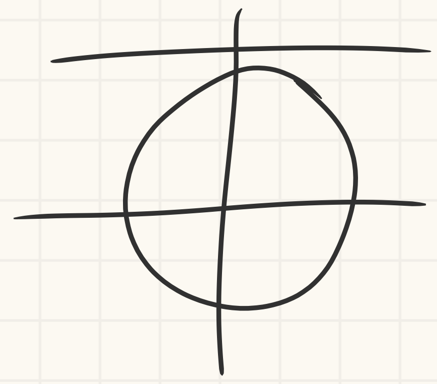
mínimo global $= 0$, minimizante $= 1$

d) Invertível?

Como $g(0) = g(2) \Rightarrow g$ n é injetiva, logo tb n é invertível

34) $h(x) = \operatorname{arctg}(x^2 - 4x)$ Extremos? Monotonia?

$D_h = \mathbb{R}$



$$h'(x) = \operatorname{arctg}(x^2-4x)' = \frac{2x-4}{1+(x^2-4x)^2}$$

$$h'(x) = 0 \Leftrightarrow \frac{2x-4}{1+(x^2-4x)^2} = 0 \Leftrightarrow 2x-4 = 0 \wedge \underbrace{1+(x^2-4x)^2 \neq 0}_{\text{cond. univ.}}$$

$$\Leftrightarrow x = 2$$

| | $-\infty$ | $2$ | $+\infty$ |
|---|---|---|---|
| $h'$ | $-$ | $0$ | $+$ |
| $h$ | $\searrow$ | | $\nearrow$ |

$$h(2) = \operatorname{arctg}(2^2 - 4.2) = \operatorname{arctg}(4-8)$$
$$= \operatorname{arctg}(-4)$$

35) $g(x) = \operatorname{arcsen}((x-1)^2)$

a) $D_g$?

$$D_g = \{x \in \mathbb{R} : -1 \leq (x-1)^2 \leq 1\}$$

C.A

- $(x-1)^2 \geq -1$ cond. universal
- $(x-1)^2 \leq 1 \Leftrightarrow |x-1| \leq 1 \Leftrightarrow \begin{cases} x-1 \leq 1 \Leftrightarrow x \leq 2 \\ x-1 \geq -1 \Leftrightarrow x \geq 0 \end{cases}$

$$D_g = [0, 2]$$

b) $g(x) = \frac{\pi}{6}$ tem pelo menos uma solução em $[1, 2]$?

$$\arcsin((x-1)^2) = \frac{\pi}{6} \Leftrightarrow \arcsin((x-1)^2) - \frac{\pi}{6} = 0$$

em 1)
$$\arcsin((1-1)^2) - \frac{\pi}{6} \Leftrightarrow \arcsin(0) - \frac{\pi}{6} = -\frac{\pi}{6}$$

em 2)
$$\arcsin((2-1)^2) - \frac{\pi}{6} \Leftrightarrow \arcsin(1) - \frac{\pi}{6} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi}{6} - \frac{\pi}{6}$$
$$= \frac{2\pi}{6} = \frac{\pi}{3}$$

Pelo T. Bolzano como $g(1) - \frac{\pi}{6} \cdot g(2) - \frac{\pi}{6} < 0$
então $g(x) = \frac{\pi}{6}$ tem pelo menos uma solução em $[1,2]$

c) Extremos e monotonia?

$$g'(x) = \arcsin((x-1)^2)' = \frac{((x-1)^2)'}{\sqrt{1-(x-1)^4}} = \frac{2(x-1)\cdot(x-1)'}{\sqrt{1-(x-1)^4}}$$

$$= \frac{2(x-1)}{\sqrt{1-(x-1)^4}}$$

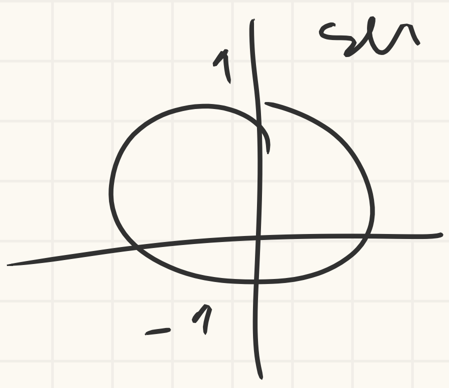
$$g'(x) = 0 \Leftrightarrow \frac{2(x-1)}{\sqrt{1-(x-1)^4}} = 0 \Leftrightarrow x-1=0 \land 1-(x-1)^4 > 0$$

$$\Leftrightarrow x = 1 \land -(x-1)^4 > -1 \Leftrightarrow x = 1 \land (x-1)^4 < 1$$

$$|x-1| < 1$$

$$x - 1 < 1 \Leftrightarrow x < 2$$
$$x - 1 > -1 \Leftrightarrow x > 0$$
$$x \in ]0,2[$$

| | 0 | | 1 | | 2 |
|---|---|---|---|---|---|
| $g'$ | | $-$ | 0 | $+$ | |
| $g$ | $\frac{\pi}{2}$ | $\searrow$ | 0 | $\nearrow$ | $\frac{\pi}{2}$ |




$$g(0) = \arcsin((0-1)^2) = \frac{\pi}{2}$$

$$g(1) = \arcsin(0) = 0$$

$$g(2) = \arcsin(1) = \frac{\pi}{2}$$

d) Inverntível? Não pq $g(0) = g(2) \to g$ n é injetiva $\to g$ n é inverntível

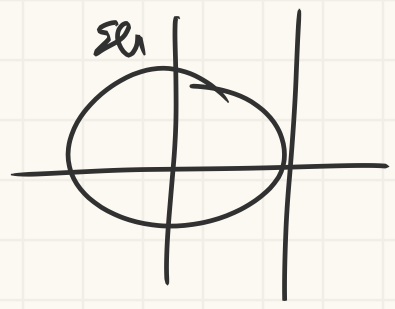
36

a) $$\lim_{x \to +\infty} \left(\ln(3x^2 + 2) - \ln(x^2)\right) \overset{\infty - \infty}{=} \lim \left(\ln\left(\frac{3x^2+2}{x^2}\right)\right) = \lim\left(\ln\left(\frac{3x^2}{x^2} + \frac{2}{x^2}\right)\right)$$

$$= \lim\left(\ln\left(3 + \frac{2}{x^2}\right)\right) = \ln(3 + 0) = \ln(3)$$

38) $$g(x) = \begin{cases} \operatorname{arctg}(x^2) & , \ x \leq 0 \\ x^2 \operatorname{sen}\left(\frac{1}{x}\right) & , \ x > 0 \end{cases}$$

a) diferenciavel em $x = 0$? $g'(0) = ?$




$$g(0) = \operatorname{arctg}(0) = 0$$

$$\lim_{x \to 0^-} g(x) = \operatorname{arctg}(0) = 0$$

$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} x^2 \operatorname{sen}\left(\frac{1}{x}\right) \overset{0 \cdot 0}{=} 0$$

42)

a) $$T_0^3 (x^3 + 2x + 1) = ? \quad = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!}(x - 0)^k$$

$f(x) = x^3 + 2x + 1 \qquad f(0) = 1$

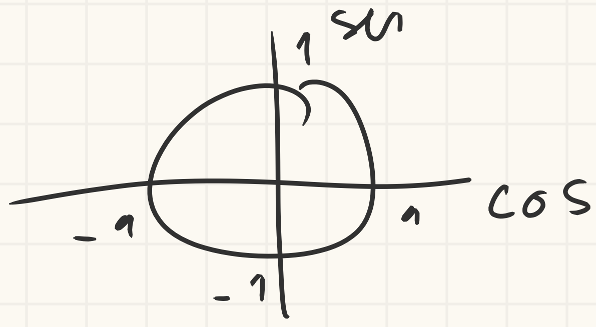
$f'(x) = 3x^2 + 2 \qquad f'(0) = 2$

$f''(x) = 6x \qquad f''(0) = 0$

$f'''(x) = 6 \qquad f'''(0) = 6$

$$= 1 + 2(x - 0) + 0 + \frac{6}{3!}(x - 0)^3 = 1 + 2x + x^3$$

b) $T_{\pi}^{3}(\cos x) = ?$




$f(x) = \cos x \qquad f(\pi) = \cos(\pi) = -1$

$f'(x) = -\text{sen } x \qquad f'(\pi) = -\text{sen}(\pi) = 0$

$f''(x) = -\cos x \qquad f''(\pi) = -\cos(\pi) = 1$

$f'''(x) = \text{sen } x \qquad f'''(\pi) = \text{sen}(\pi) = 0$

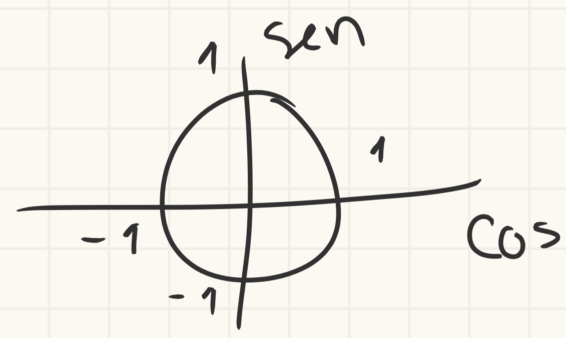
$$T_{\pi}^{3} = -1 + \frac{0}{1}(x-\pi)^1 + \frac{1}{2!}(x-\pi)^2 + \frac{0}{3!}(x-\pi)^3$$

$$= -1 + 0 + \frac{1}{2}(x-\pi)^2 + 0 = \frac{(x-\pi)^2}{2} - 1$$

Ex miniteste tipo:

5. Usando o polinómio de MacLaurin de ordem 2 de $f(x) = \cos x$, $T_0^2(f(x))$, podemos concluir que um valor aproximado de $\cos(\frac{1}{2})$ é igual a:

$T_0^2(\cos x) = ?$




$f(x) = \cos x \qquad f(0) = \cos 0 = 1$

$f'(x) = -\text{sen } x \qquad f'(0) = -\text{sen } 0 = 0$

$f''(x) = -\cos x \qquad f''(0) = -\cos 0 = -1$

$$T_0^2(\cos x) = 1 + 0 - \frac{1}{2!}(x-0)^2 = 1 - \frac{x^2}{2}$$

$$x = 1/2 \to 1 - \frac{(1/2)^2}{2} = 1 - \frac{1/4}{2} = 1 - \frac{1}{4} \cdot \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$$

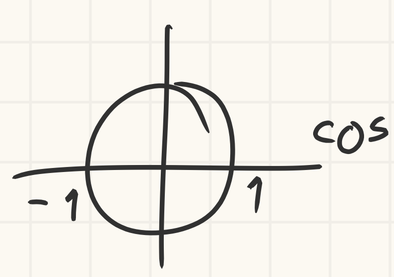
47) zeros de $g(x) = \begin{cases} \arccos(x^2) & , \quad -1 \leq x < 0 \\ e^{-x+1} & , \quad x \geq 0 \end{cases}$

$D_g = \{x \in \mathbb{R} : x^2 \leq 1\} \to [-1, 1]$

C.A

$x^2 \leq 1 \Leftrightarrow x \leq 1 \wedge x \geq -1$

$\arccos(x^2) = 0 \Leftrightarrow x^2 = \cos 0 \Leftrightarrow x^2 = 1 \Leftrightarrow \cancel{x = 1} \wedge x = -1$




$e^{-x+1} = 0 \quad \Leftrightarrow$
imp.

44) $f(x) = e^x$

a) $T_0^n = ?$ $\quad T_0^n = 1 + 1.(x-0)^1 + \frac{1}{2!}(x-0)^2 + \dots + \frac{x^n}{n!} + \frac{e^\theta}{(n+1)!} x^{n+1}, \theta \in [0, x]$

$f(x) = e^x \quad f(0) = 1$

$f'(x) = e^x \quad f'(0) =$

48) $h(x) = \arccos(e^{x-1}) + \pi$, invertível? $h^{-1} = ?$ $Dh^{-1} = ?$ $CDh^{-1} = ?$

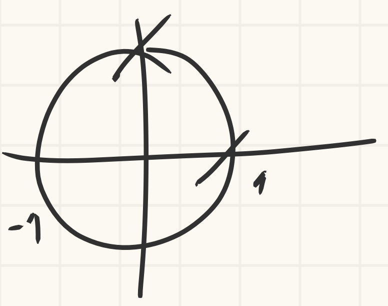
$Dh = \{x \in \mathbb{R} : -1 \leq e^{x-1} \leq 1\}$ $\qquad Dh = ]-\infty, 1] = CDh^{-1}$

CA

- $e^{x-1} \geq -1$ cond. univ
- $e^{x-1} \leq 1 \Leftrightarrow x-1 \leq \ln(1) \Leftrightarrow x-1 \leq 0 \Leftrightarrow x \leq 1$

$e^{-\infty}$ a $e^0 = 0$ a $1$

cDh) $\quad 0 \leq \arccos(x) \leq \pi \Leftrightarrow 0 \leq \arccos(e^{x-1}) \leq \frac{\pi}{2} \Leftrightarrow$



$\Leftrightarrow \pi \leq \arccos(e^{x-1}) + \pi \leq \frac{3\pi}{2}$

$CDh = [\pi, \frac{3\pi}{2}] = Dh^{-1}$

$h^{-1}$) $\arccos(e^{x-1}) + \pi = y \Leftrightarrow e^{x-1} = \cos(y-\pi) \Leftrightarrow x-1 = \ln(\cos(y-\pi))$

$\Leftrightarrow x = \ln(\cos(y-\pi)) + 1$

$h^{-1}(x) = \ln(\cos(x-\pi)) + 1$

50) $4x^3 - 6x^2 + 1 = 0$ tem 3 raizes em que intervalos?

$(4x^3 - 6x^2 + 1)' = 12x^2 - 12x$

$12x^2 - 12x = 0 \Leftrightarrow 12x^2 = 12x \Leftrightarrow x^2 = x \Leftrightarrow \overbrace{x = 0 \wedge x = 1}^{\text{entre } ]0,1[ \text{ tem no max um zero}}$

em $x = 0$) $\quad 4.0 - 6.0 + 1 = 1$

em $x = 1$) $\quad 4 - 6 + 1 = -1$

em $x = -1$) $\quad 4.(-1) - 6 + 1 = -4 - 6 + 1 = 9$

| | $-\infty$ | $-1$ | | $0$ | | $1$ | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $g'$ | $-$ | $0$ | $-$ | $0$ | $-$ | $0$ | $+$ |
| $g$ | ↘ | $9$ | ↘ | $1$ | ↘ | $-1$ | ↗ |

$12x^2 = 12x \iff 12x = 12 \iff$