

# Teste 1 - 2024

①

a)  $]a, c[$ ,  $f(a) < 0$ ,  $f(c) > 0$   
 $\exists x_0 \in ]a, c[$  tal que  $f(x_0) = 0$

$]c, b[$ ,  $f(c) > 0$ ,  $f(b) < 0$   
 $\exists y_0 \in ]c, b[ : f(y_0) = 0$

②

b)  $g : [0, +\infty[ \rightarrow \mathbb{R}$  e  $\varphi : D_\varphi \rightarrow \mathbb{R}$   $\varphi(x) = g(1 - x^2)$   
 $f(x) = 1 - x^2$   $\varphi(x) = (g \circ f)(x)$

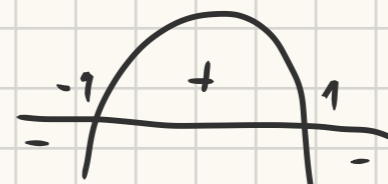
$$D_\varphi = \{ x \in \mathbb{R} : 1 - x^2 \in [0, +\infty[ \}$$

$$= \{ x \in \mathbb{R} : 1 - x^2 \geq 0 \} = [-1, 1] \text{ Pelo T. Weierstrass } \varphi \text{ tem m\u00e1x e min globais no Dom\u00ednio}$$

C.A

$$1 - x^2 \geq 0 \Leftrightarrow x^2 \leq 1$$

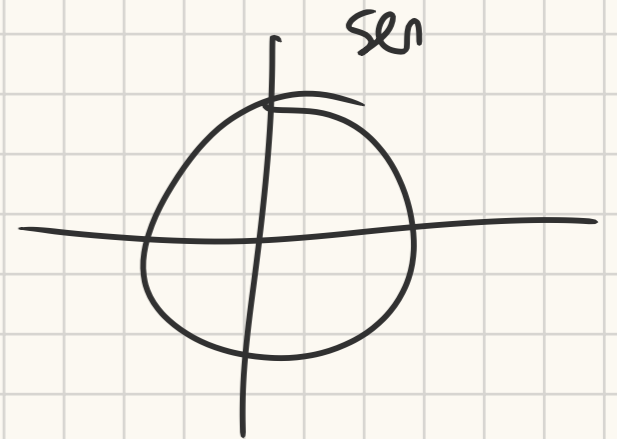
$$x = \pm 1$$



(por ser fechado e limitado)

c)  $h(x) = (x+1) \arcsen(\sqrt{x} - 1)$ ,  $x \in [0, 4]$   $\uparrow$  Lagrange existe um  $c$   
 $\in ]0, 4[$  tal que:

$$h'(x) = \frac{h(4) - h(0)}{4 - 0} = \frac{5 \arcsen(1) - \arcsen(-1)}{4} =$$
$$= \frac{5 \frac{\pi}{2} + \frac{\pi}{2}}{4} = \frac{6 \frac{\pi}{2}}{4} = \frac{3\pi}{4}$$



d)  $g(x) = x^{21} - x + a$ ,  $a \in \mathbb{R}$ . Numero máx de zeros?

$$g'(x) = 21x^{20} - 1$$

$21x^{20} - 1 = 0 \Leftrightarrow x^{20} = \frac{1}{21}$  tem 2 soluções  $\rightarrow$  Pelo T. Rolle entre  
dois zeros de  $g'$  há no max um zero de  $g$

No máx  $g$  tem 3 zeros

e)  $\lim_{n \rightarrow 0} (n^4 + 1)^{\frac{1}{n^2}} = ?$

$$\lim_{n \rightarrow 0} (n^4 + 1)^{\frac{1}{n^2}} = \lim_{n \rightarrow 0} e^{\ln((n^4 + 1)^{\frac{1}{n^2}})} = \lim_{n \rightarrow 0} e^{\frac{1}{n^2} \ln(n^4 + 1)} = e^{\lim_{n \rightarrow 0} \frac{\ln(n^4 + 1)}{n^2}}$$

R. Cauchy

$$= e^{\lim_{n \rightarrow 0} \left( \frac{\frac{4n^3}{n^4 + 1}}{2n} \right)} = e^{\lim_{n \rightarrow 0} \left( \frac{2n^2}{n^4 + 1} \right)} = e^0 = 1$$

$$\lim_{n \rightarrow 0} (\cos(2n))^{\frac{1}{n}} = \lim_{n \rightarrow 0} e^{\ln(\cos(2n))^{\frac{1}{n}}} = \lim_{n \rightarrow 0} e^{\frac{\ln(\cos(2n))}{n}}$$

$$= e^{\lim_{n \rightarrow 0} \frac{\ln(\cos(2n))}{n}} \stackrel{\text{R. Cauchy}}{=} e^{\lim_{n \rightarrow 0} \left( \frac{\frac{-2\sin(2n)}{\cos(2n)}}{1} \right)} = e^{\lim_{n \rightarrow 0} (-2\operatorname{tg}(2n))} = e^0 = 1$$

Deve aparecer em escolha múltipla

f)  $x \in [2, 4]$ ,  $g(x) = \int_{2x+1}^2 \ln t \, dt$ ,  $g'(x) = ?$

f. Fundamental do Cálculo.

$$g'(x) = g(2) \cdot (2)' - g(2x+1) \cdot (2x+1)' =$$

$$= \ln(2) \cdot 0 - \ln(2n+1) \cdot 2 = \left( -2\ln(2n+1) = g'(n) \right)$$

T. Fermat

$f$  diferenciável

- se  $c$  é extremo (maximizador ou minimizador) então  $f'(c) = 0$
- Mas  $f'(c) = 0 \nRightarrow c$  é extremo

Ex:  $f(x) = x^3$

$$f'(x) = 0 \Leftrightarrow 3x^2 = 0 \Leftrightarrow x = 0 \quad \text{Mas } 0 \text{ não é extremo de } f(x)$$

$$(\sin^2(x))' = 2\sin(x) \cdot \cos(x) = \sin(2x)$$

Exs de revisão

①  $f(x) = \arccos(1 - e^x) + \pi$

a)  $D_f = ?$   $D_f = \{x \in \mathbb{R} : -1 \leq 1 - e^x \leq 1\}$   $\Delta_f = ]-\infty, \ln(2)]$

CA  $\cdot 1 - e^x \geq -1 \Leftrightarrow -e^x \geq -2 \Leftrightarrow e^x \leq 2 \Leftrightarrow x \leq \ln(2)$

$\cdot 1 - e^x \leq 1 \Leftrightarrow -e^x \leq 0 \Leftrightarrow e^x \geq 0$  cond. universal

b) Monotonia e extremos?

$$f'(x) = -\frac{(1 - e^x)'}{\sqrt{1 - (1 - e^x)^2}} = \frac{e^x}{\sqrt{1 - (1 - e^x)^2}} > 0 \quad \forall x \in D_f \rightarrow \text{f estrita/ crescente}$$

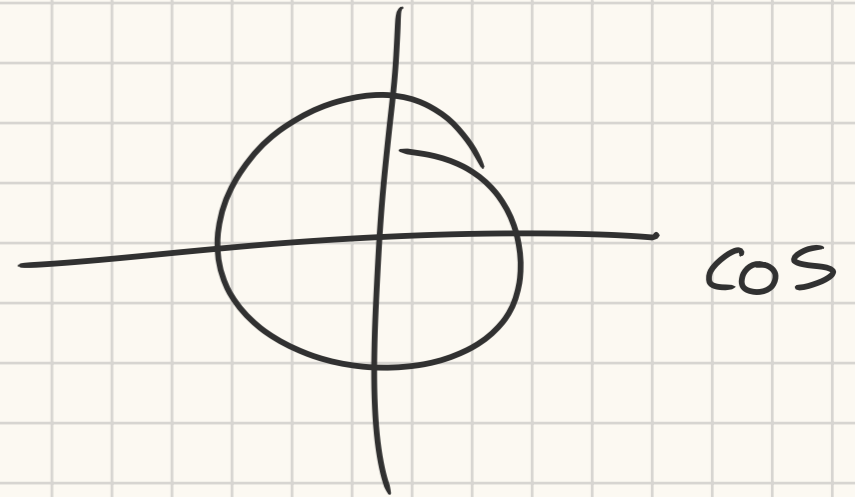
$$f(\ln(2)) = \arccos(1 - e^{\ln(2)}) + \pi = \arccos(1 - 2) + \pi = \arccos(-1) + \pi \\ = \pi + \pi = 2\pi$$

( $\ln(2)$  é maximizante,  $2\pi$  é o máximo absoluto)

c) Caracterizar  $f^{-1}(u)$

$$D_f = CD_f^{-1} = ]-\infty, \ln(2)]$$

$$CD_f = D_f^{-1}$$



$CD_f$ )

$$0 \leq \arccos(u) \leq \pi \quad \text{sempre } \neq 1$$

$$0 < \arccos(1 - e^u) \leq \pi$$

$$\pi < \arccos(1 - e^u) + \pi \leq 2\pi$$

$$CD_f = ]\pi, 2\pi] = D_f^{-1}$$

$$y = \arccos(1 - e^u) + \pi \Leftrightarrow y - \pi = \arccos(1 - e^u) \Leftrightarrow$$

$$\Leftrightarrow \cos(y - \pi) = 1 - e^u \Leftrightarrow \cos(y - \pi) + 1 = e^u \Leftrightarrow \ln(\cos(y - \pi) + 1) = u$$

$$f^{-1}(u) = \ln(\cos(u - \pi) + 1)$$

② Determinar  $f$  tal que  $f'(x) = \frac{3\cos(\ln x)}{x}$  e  $f(1) = 2$

$$\int \frac{3\cos(\ln(x))}{x} dx = 3 \int \frac{1}{x} \cos(\ln(x)) dx = 3\sin(\ln(x)) + c$$

$$f(1) = 2 \Rightarrow 3\sin(\ln(1)) + c = 2 \Rightarrow 3\sin(0) + c = 2$$

$$\Rightarrow c = 2$$

$$f(x) = 3\sin(\ln(x)) + 2$$

③

a)  $\int \arcsin(2x) dx = \arcsin(2x) \cdot x - \int x \cdot \frac{2}{\sqrt{1-(2x)^2}} dx$

$$u = \arcsin(2x) \quad du = \frac{2}{\sqrt{1-(2x)^2}} dx$$
$$v = x$$
$$= x \arcsin(2x) - 2 \int \frac{x}{\sqrt{1-4x^2}} dx$$

$$\begin{aligned}
 &= x \arcsin(2x) - 2 \int x (1-4x^2)^{-1/2} = x \arcsin(2x) + \frac{2}{8} \int -8x (1-4x^2)^{-1/2} \\
 &= x \arcsin(2x) + \frac{1}{4} \left( \frac{(1-4x^2)^{1/2}}{1/2} \right) = x \arcsin(2x) + \frac{\sqrt{1-4x^2}}{2} + C, C \in \mathbb{R}
 \end{aligned}$$

$$\text{b) } \int \frac{1}{x^2 \sqrt{x^2+4}} dx, \quad \text{use } x = 2 \operatorname{tg}(t)$$

$$\int \frac{1}{x^2 \sqrt{x^2+4}} dx \quad x = 2 \operatorname{tg}(t) \Rightarrow \operatorname{arctg}\left(\frac{x}{2}\right) = t, \quad t \in ]0, \frac{\pi}{2}[$$

$$dx = 2 \sec^2(t) dt$$

$$= \int \frac{2 \sec^2(t)}{4 \operatorname{tg}^2 t \sqrt{4 \operatorname{tg}^2 t + 4}} dt = \int \frac{2 \sec^2(t)}{4 \operatorname{tg}^2 t \sqrt{4 \sec^2 t}} dt = \int \frac{2 \sec^2 t}{4 \operatorname{tg}^2 t \cdot 2 \sec t}$$

$$= \int \frac{2 \sec^2 t}{8 \operatorname{tg}^2 t \cdot \sec t} = \frac{1}{4} \int \frac{\sec t}{\operatorname{tg}^2 t} dt =$$

$$= \frac{1}{4} \int \frac{1}{\cos t} \cdot \frac{\cos^2 t}{\sin^2 t} dt = \frac{1}{4} \int \frac{\cos t}{\sin^2 t} dt =$$

$$= \frac{1}{4} \int \cotg t \cdot \operatorname{cosec} t dt = -\frac{1}{4} \operatorname{cosec} t + C$$

$$= -\frac{1}{4} \operatorname{cosec} \left( \arctg \left( \frac{x}{2} \right) \right) + C, \quad C \in \mathbb{R}$$

CA OU \*

$$\bullet \operatorname{tg}^2 t + 1 = \sec^2 t$$

$$\bullet \cotg^2 t + 1 = \operatorname{cosec}^2 t$$

$$\frac{x}{2} = \operatorname{tg} t \Leftrightarrow \cotg t = \frac{2}{x}$$

$$\cotg^2 t + 1 = \operatorname{cosec}^2 t$$

$$\left( \frac{2}{x} \right)^2 + 1 = \operatorname{cosec}^2 t \Leftrightarrow \frac{4}{x^2} + 1 = \operatorname{cosec}^2 t \Leftrightarrow \operatorname{cosec}^2 t = \frac{4 + x^2}{x^2}$$

$$\Rightarrow \operatorname{cosec} t = \sqrt{\frac{4 + u^2}{u^2}}$$

$$\ast -\frac{1}{4} \cdot \sqrt{\frac{4 + u^2}{u^2}} + C, C \in \mathbb{R}$$

④  $f(u) = e^{2u}$ . Polinômio de MacLaurin de ordem 2 para determinar uma aproximação de  $\sqrt{e}$

$$f(0) = 1$$

$$f'(u) = 2e^{2u}$$

$$f''(u) = 4e^{2u}$$

$$f'(0) = 2$$

$$f''(0) = 4$$

$$\begin{aligned} T_0^2 f(u) &= \frac{1}{0!} (u-0)^0 + \frac{2}{1!} (u-0)^1 + \frac{4}{2!} (u-0)^2 \\ &= 1 + 2u + 2u^2 \end{aligned}$$

$$e^{2n} = \sqrt{e} \quad (\Leftrightarrow) \quad 2n = \frac{1}{2} \quad (\Leftrightarrow) \quad n = \frac{1}{4}$$

$$\text{Se } f(n) = e^{2n} \quad \rightarrow \quad \sqrt{e} = f\left(\frac{1}{4}\right)$$

$$\begin{aligned} T_0^2 f\left(\frac{1}{4}\right) &= 1 + 2 \cdot \frac{1}{4} + 2 \cdot \left(\frac{1}{4}\right)^2 = \frac{3}{2} + \frac{2}{16} = \frac{3}{2} + \frac{1}{8} = \frac{12}{8} + \frac{1}{8} \\ &= \frac{13}{8} \end{aligned}$$