Ficha 2

9

a) $\int (x+1) \operatorname{sen} x \, dx = \int v \cdot u' = u.v - \int u \cdot v' = -\cos x \cdot (x+1) + \int \cos x$

$u' = \operatorname{sen} x \qquad v = x+1 \qquad = -\cos x (x+1) + \operatorname{sen} x + C, \; C \in \mathbb{R}$

$u = -\cos x \qquad v' = 1$

b) $\int x \cos(x) \, dx = \int v \cdot u' = u.v - \int v' . u = \operatorname{sen}(x) \cdot x - \int 1 . \operatorname{sen} x \, dx$

$u' = \cos x \qquad v = x \qquad = x \operatorname{sen} x + \cos x$

$u = \operatorname{sen} x \qquad v' = 1$

c) $x^2 \cos(x) dx = \int v \cdot u' = u.v - \int v' . u = x^2 \operatorname{sen} x - \int 2x \operatorname{sen} x$

$u' = \cos x \qquad v = x^2 \qquad = x^2 \operatorname{sen} x - 2 \int x \operatorname{sen} x = \ldots$

$u = \operatorname{sen} x \qquad v' = 2x$

10) a) $\int x\sqrt{x+1}$

$t = \sqrt{x+1} \Leftrightarrow t^2 = x+1 \Leftrightarrow x = t^2 - 1$

$dx = 2t\,dt$

$\int (t^2-1).t\,.\,2t\,dt = \int t^3 - t\,.\,2t\,dt = \int 2t^4 - 2t^2\,dt$

$= 2\left(\frac{t^5}{5} - \frac{t^3}{3}\right)$

$= 2\left(\frac{\sqrt{x+1}^5}{5} - \frac{\sqrt{x+1}^3}{3}\right) + c\,,\ c \in \mathbb{R}$

b) $\int \frac{x}{1+\sqrt{x}}\,dx \longrightarrow \int \frac{(t-1)^2}{t}\,.\,2(t-1)\,dt = 2\int \frac{(t-1)^3}{t}\,dt$

$t = 1+\sqrt{x} \Leftrightarrow (t-1)^2 = x$

$dx = 2(t-1)\,dt$

$= 2\int \frac{(t-1)(t^2-2t-1)}{t}\,dt$

⑥ $\int \frac{1}{x^2} + 1$ que se anula em $x=2$?

$\int \frac{1}{x^2} + 1\,dx = \int x^{-2} + 1\,dx = -\frac{1}{2}\int -2x^{-2}\,dx + \int 1\,dx = -\frac{1}{2}\left(\frac{x^{-1}}{-1}\right) + x + c$

$$= -\frac{1}{2}\left(-\frac{1}{x}\right) + x + C = \frac{1}{2x} + x + C, \; C \in \mathbb{R}$$

$$\frac{1}{2.2} + 2 + C = 0 \Leftrightarrow \frac{1}{4} + 2 = -C \Leftrightarrow \frac{1}{4} + \frac{8}{4} = -C \Leftrightarrow \frac{9}{4} = -C \rightarrow \frac{1}{2x} + x - \frac{9}{4}$$

7) $\int \frac{3\cos(\ln x)}{x} = 2$ em $x = 1$

$$\int \frac{3\cos(\ln x)}{x}\,dx = 3\int \cos(\ln x)\cdot\frac{1}{x}\,dx = 3\,\text{sen}(\ln x) + C$$

$$3\,\text{sen}(\ln(1)) + C = 2 \Leftrightarrow 3\,\text{sen}(0) - 2 = -C \Leftrightarrow C = 2 \rightarrow 3\,\text{sen}(\ln x) + 2$$

1) a) $\int 3x^2 + 5x + 7\,dx = \int 3x^2\,dx + \int 5x\,dx + \int 7\,dx =$

$$= x^3 + \frac{5x^2}{2} + 7x + C, \; C \in \mathbb{R}$$

b) $\int \sqrt[3]{x}\, dx = \int x^{1/3}\, dx = \dfrac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + C = -\dfrac{3}{2} \cdot \dfrac{1}{\sqrt[3]{x^2}} + C$

c) $\int (x^3+1)^2\, dx = \int x^6 + 2x^3 + 1\, dx = \dfrac{x^7}{7} + \dfrac{2x^4}{4} + x + C,\ C \in \mathbb{R}$

d) $\int \dfrac{\operatorname{arctg} x}{1+x^2}\, dx = \int \underbrace{\dfrac{1}{1+x^2}}_{u'} \underbrace{\operatorname{arctg} x}_{u}\, dx = \dfrac{(\operatorname{arctg} x)^2}{2} + C,\ C \in \mathbb{R}$

e) $\int \dfrac{3x^2}{1+x^3}\, dx \overset{\frac{u'}{u}}{=} \ln|1+x^3| + C,\ C \in \mathbb{R}$

f) $\int \dfrac{1}{x^7}\, dx = \int x^{-7}\, dx = \dfrac{x^{-6}}{-6} + C = \dfrac{1}{-6x^6} + C,\ C \in \mathbb{R}$

g) $\int \dfrac{x+1}{2+4x^2} = \dfrac{1}{2} \int \dfrac{x+1}{1+2x^2}\, dx = \dfrac{1}{2} \int \dfrac{x}{1+2x^2}\, dx + \dfrac{1}{2} \int \dfrac{1}{1+2x^2}\, dx =$

$= \dfrac{1}{2} \cdot \dfrac{1}{4} \int \dfrac{4x}{1+2x^2}\, dx + \dfrac{1}{2} \cdot \dfrac{1}{2} \int \dfrac{2}{1+2x^2}\, dx = \dfrac{1}{8} \ln(1+2x^2) + \dfrac{1}{4} \operatorname{arctg}(2x)$

h) $\int 4x^3 \cos x^4\, dx = \operatorname{sen}(x^4) + C,\ C \in \mathbb{R}$

i) $\int \frac{x}{\sqrt{1-x^2}}\,dx = \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx = \int x \cdot (1-x^2)^{-1/2} = -\frac{1}{2}\int -2x \cdot (1-x^2)^{-1/2}\,dx$

$= -\frac{1}{2}\frac{(1-x^2)^{1/2}}{\frac{1}{2}} + c = -(1-x^2)^{1/2} = -\sqrt{1-x^2}$

j) $\int \underbrace{\operatorname{sen} x}_{-u'} \underbrace{\cos^5 x}_{u^5}\,dx = -\frac{\cos^6 x}{6}$

k) $\int \operatorname{tg} x\,dx = \int \frac{\operatorname{sen} x}{\cos x}\,dx = -\ln|\cos x| + c,\ c \in \mathbb{R}$

l) $\int \frac{\ln x}{x}\,dx = \int \frac{1}{x}\ln x\,dx = \frac{\ln^2 x}{2} + c,\ c \in \mathbb{R}$

m) $\int e^{\overbrace{\operatorname{tg} x}^{u}} \underbrace{\sec^2 x}_{u'}\,dx = e^{\operatorname{tg} x}$

n) $\int x \cdot 7^{x^2}\,dx = \frac{1}{2}\int 2x \cdot 7^{x^2} = \frac{7^{x^2}}{2\ln(7)}$

o) $\int \operatorname{sen}(\sqrt{2}\,x)\,dx = \frac{1}{\sqrt{2}}\int \sqrt{2}\operatorname{sen}(\sqrt{2}\,x)\,dx = \frac{-\cos(\sqrt{2}\,x)}{\sqrt{2}} + c,\ c \in \mathbb{R}$

p) $\int \frac{x^2+1}{x}\,dx = \int \frac{x^2}{x} + \frac{1}{x}\,dx = \int x + \frac{1}{x}\,dx = \frac{x^2}{2} + \ln|x| + c,\ c \in \mathbb{R}$

q) $\int \frac{x}{(7+5x^2)^{3/2}}\,dx = \int x\,.\,(7+5x^2)^{-3/2}\,dx = \frac{1}{10}\int 10x\,.\,(7+5x^2)^{-3/2}\,dx$

$= \frac{(7+5x^2)^{-1/2}}{-1/2}\cdot\frac{1}{10} + c = -\frac{2}{10\sqrt{7+5x^2}} + c,\ c \in \mathbb{R}$

r) $\int \frac{x^3}{1+x^8}\,dx = \frac{1}{4}\int \frac{4x^3}{1+{x^4}^2}\,dx = \frac{1}{4}\operatorname{arctg}(x^4) + c,\ c \in \mathbb{R}$

2

a) $\int \frac{e^{\operatorname{arcsen} x}}{\sqrt{1-x^2}}\,dx = e^{\operatorname{arcsen} x} + c,\ c \in \mathbb{R}$

b) $\int \operatorname{tg}^2 x\,dx = \int \sec^2 x - 1\,dx = \int \sec^2 x\,dx - \int 1\,dx$

$= \operatorname{tg} x - x + c, \quad c \in \mathbb{R}$

c) $\int \frac{1}{x} \cos(\ln x)\, dx = \operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$

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a) $\int x\sqrt{x+1}\, dx \longrightarrow \int t^2 - 1\,(t)\; 2t\; dt = \int (t^2-1) \cdot 2t^2\, dt = \int 2t$

$t = \sqrt{x+1} \Leftrightarrow t^2 - 1 = x$

$dx = 2t\, dt$

b) $\int \frac{x}{1+\sqrt{x}}\, dx = \int \frac{t^2}{1+t}\, 2t\; dt = 2\int \frac{t^3}{1+t}\, dt = 2\int \frac{t^2}{\frac{1}{t}+1}\, dt$

$t = \sqrt{x} \Leftrightarrow t^2 = x$

$2t\, dt = dx$

9) a) $\int (x+1) \text{sen } x = (x+1)(-\cos x) - \int -\cos x \, dx$

$u = x+1 \qquad dv = \text{sen } x \qquad = -\cos(x)(x+1) - \text{sen } x + C, \; C \in \mathbb{R}$

$du = 1 \qquad v = -\cos x$

b) $\int x \cos(x) \, dx = x \,\text{sen } x - \int \text{sen}(x) \, dx = x \,\text{sen } x + \cos x + C$

$u = x \qquad dv = \cos x$

$du = 1 \qquad v = \text{sen } x$

c) $\int x^2 \cos x \, dx = x^2 \text{sen } x - \int 2x \,\text{sen } x$

$u = x^2 \qquad dv = \cos x$

$du = 2x \qquad v = \text{sen } x$

$u = 2x \qquad dv = \text{sen } x$

$du = 2 \qquad v = -\cos x$

$= x^2 \text{sen } x - \left(-2x \cos(x) - \int 2(-\cos x) \, dx \right) =$

$= x^2 \operatorname{sen} x + 2x \cos x + 2 \operatorname{sen} x + C =$

d) $\int e^{-3x}(2x+3)\,dx = (2x+3)\left(\frac{e^{-3x}}{3}\right) - \int \frac{e^{-3x}}{3} \cdot 2 \,dx$

$dv = e^{-3x}$ $\quad u = 2x+3$

$v = \frac{e^{-3x}}{3}$ $\quad du = 2$

$= (2x+3)\frac{e^{-3x}}{3} - \frac{2}{3}\int e^{-3x}\,dx = (2x+3)\left(\frac{e^{-3x}}{3}\right) + \frac{2}{9}\int -3e^{-3x}\,dx$

$= (2x+3)\left(\frac{e^{-3x}}{3}\right) + \frac{2}{9}e^{-3x} + C$

e) $\int \ln^2 x \,dx = \int 1 \cdot \ln^2 x \,dx = x\ln^2 x - \int x \cdot 2\ln x \cdot \frac{1}{x}\,dx$

$dv = 1$ $\quad u = \ln^2 x$

$v = x$ $\quad du = 2\ln x \cdot \frac{1}{x}$

$$= x\ln^2(x) - 2\int \ln x \, dx = x\ln^2(x) - 2\left(x\ln x - \int \frac{1}{x} \cdot x \, dx\right)$$

$u = \ln x \qquad dv = 1$

$du = \frac{1}{x} \qquad v = x$

$$= x\ln^2 x - 2x\ln x + 2x + C$$

g) $\int \ln(x^2+1)\,dx = x \cdot \ln(x^2+1) - \int x \cdot \frac{2x}{x^2+1}\,dx =$

$u = \ln(x^2+1) \qquad dv = 1$

$du = \frac{2x}{x^2+1} \qquad v = x$

$$= x\ln(x^2+1) - \int \frac{2x}{x^2+1} \cdot x\,dx = x\ln(x^2+1) - 2\int \frac{x^2+1-1}{x^2+1}\,dx =$$

$$= x\ln(x^2+1) - 2\int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx = x\ln(x^2+1) - 2x + 2\arctg(x) + C$$

h) $\int x \operatorname{arctg} x = \frac{x^2}{2} \operatorname{arctg} x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} \, dx =$

$u = \operatorname{arctg}$ $\quad dv = x$

$du = \frac{1}{1+x^2}$ $\quad v = \frac{x^2}{2}$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} \, dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int 1 - \frac{1}{x^2+1} \, dx$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{\operatorname{arctg} x}{2} + C$

(10) a) $\int x \sqrt{x+1} \, dx = \int (t^2-1) \cdot t \cdot 2t \, dt = \int (t^2-1) \cdot 2t^2 \, dt$

$t = \sqrt{x+1} \Leftrightarrow t^2 - 1 = x$ $\quad = \int 2t^4 - 2t^2 \, dt = \frac{2t^5}{5} - \frac{2t^3}{3} + C$

$2t \, dt = dx$

$$= \frac{2\sqrt{x+1}^5}{5} - 2\frac{\sqrt{x+1}^3}{3} + C, \quad C \in \mathbb{R}$$

b) $\int \frac{x}{1+\sqrt{x}}\,dx = \int \frac{(t-1)^2}{t} \cdot 2t^2 - 2t \;dt = 2\int \frac{(t-1)^2}{t} \cdot t^2 - t \;dt$

$t = \sqrt{x} + 1$

$x = (t-1)^2$

$dx = 2(t-1).t =$

$= (2t-2).t$

$= 2t^2 - 2t \; dt$

$$= 2\int (t-1)^2 . t \;dt - 2\int t \;dt = 2\int (t-1)^2 . t \;dt - t^2$$

$$= 2\int (t^2 - 2t - 1) . t \;dt - t^2 =$$

$$= 2\int t^3 - 2t^2 - t \;dt - t^2 = 2\frac{t^4}{4} - 4\frac{t^3}{3} - 2\frac{t^2}{2} - t^2$$

$$= \frac{t^4}{2} - \frac{4t^3}{3} - 2t^2 + C$$

$$= \frac{(\sqrt{x}+1)^4}{2} - \frac{4(\sqrt{x}+1)^3}{3} - 2(\sqrt{x}+1)^2 + C, \quad C \in \mathbb{R}$$

c) $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = \int \frac{\cos(u)}{\sin^2(u)\cdot\sqrt{1-\operatorname{sen}^2(u)}}\,du = \int \frac{\cos(u)}{\sin^2(u)\cdot\sqrt{\cos^2(u)}}\,dx$

$x = \sin(u) \Leftrightarrow \arcsin(x) = u$

$dx = \cos(u)\,du$

$= \int \frac{1}{\sin^2(u)}\,du =$??

d) $\frac{1}{x^2\sqrt{x^2+4}}\,dx = \int \frac{1}{x^2\sqrt{x^2+2^2}}\,dx =$

$x = 2\operatorname{tg}(t) \Leftrightarrow \operatorname{arctg}\left(\frac{x}{2}\right) = t$

$dx = 2\sec^2(t)\,dt$

$$= \int \frac{2\sec^2(t)}{2tg^2(t) \cdot \sqrt{4tg^2(t)+4}}\, dt = \int \frac{\sec^2 t}{tg^2(t) \cdot \sqrt{4\sec^2(t)}}\, dt$$

$$= \int \frac{\sec^2 t}{tg^2(t) \cdot 2 \cdot \sqrt{\sec^2 t}}\, dt = \int \frac{\sec t}{tg^2(t)}\, dt = \frac{1}{2} \int \frac{2\sec(t)}{tg^2(t)}\, dt$$

??\}\}

Copiando da aula)

j) $\frac{1}{x^2\sqrt{9-x^2}}\, dx = \int \frac{1}{9sen^2 t \sqrt{9-9sen^2 t}} \cdot 3\cos t$

$x = 3\,sen(t) \Leftrightarrow arcsen\left(\frac{x}{3}\right) = t$

$dx = 3\cos(t)$

$$= \int \frac{3\cos t}{9\operatorname{sen}^2 t \sqrt{9(1-\operatorname{sen}^2 t)}}\, dt = \int \frac{3\cos t}{9\operatorname{sen}^2 t \cdot 3\sqrt{\cos^2 t}}\, dt =$$

$$= \frac{3\cos t}{9\operatorname{sen}^2 t \cdot 3\cos t}\, dt = \int \frac{1}{9}\operatorname{cosec}^2 t \; dt = -\frac{1}{9}\operatorname{cotg}(t) + C$$

$$= -\frac{1}{9}\sqrt{\operatorname{cosec}^2 t - 1} + C =$$

Ficha 2 - Parte 2

2- a) $F(x) = \int_0^{x^2} e^{t^2}\, dt$ $\qquad F'(x) = ?$

T. fundamental do calculo

$$F'(x) = e^{(x^2)^2} \cdot (x^2)' - e^{0^2} \cdot (0)' = e^{x^4} \cdot 2x$$

b) $F(x) = \int_0^x \frac{t^2}{t^2+1}\, dt$ $\qquad F'(x) = \frac{x^2}{x^2+1} \cdot 1 - 0 \cdot 0 = \frac{x^2}{x^2+1}$

c) $F(x) = \int_x^0 e^{-s^2}\, ds$ $\qquad F'(x) = e^{-0^2} \cdot 0 - e^{-x^2} \cdot 1 = -e^{-x^2}$

d) $F(x) = \int_1^x (\text{sen } t^2 + e^{-t^2})\, dt$

$$F'(x) = (\text{sen } x^2 + e^{-x^2}) \cdot 1 - \text{sen } 1 + e^{-1^2} \cdot 0$$

$$= \text{sen } x^2 + e^{-x^2}$$

4) $F(x) = \int_0^{x^2} \text{sen } t^2\, dt$ $\qquad F'\left(\sqrt[4]{\frac{\pi}{4}}\right) = ?$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sen	1	$\frac{\sqrt{2}}{2}$	3
cos	3	2	1

tan

$$F'(x) = \operatorname{sen}(x^4).(x^2)' = 2x.\operatorname{sen}(x^4$$

$$F'\left(\sqrt[4]{\frac{\pi}{4}}\right) = 2.\sqrt[4]{\frac{\pi}{4}}.\operatorname{sen}\left(\frac{\pi}{4}\right) = 2\sqrt[4]{\frac{\pi}{4}}.\frac{\sqrt{2}}{2} = \sqrt[4]{\frac{\pi}{4}}.\sqrt{2}$$

$$= \sqrt[4]{\frac{\pi}{4}}.\sqrt[4]{2^2} = \sqrt[4]{\frac{4\pi}{4}} = \sqrt[4]{\pi}$$

6) $G = \int_0^x e^{3t^4+4t^3}\,dt$

a) monotonia?

$$G'(x) = e^{3x^4+4x^3}.1 = e^{3x^4+4x^3}$$

??

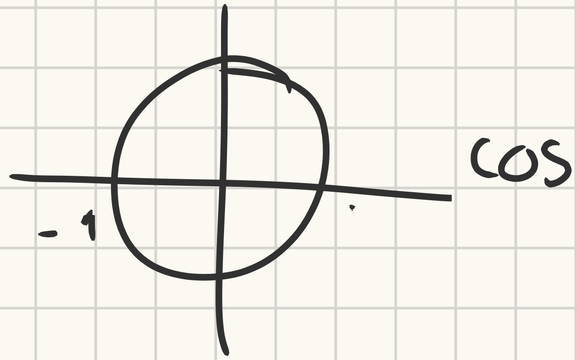
$G'(x) = 0 \iff$ imp.

		0	
$G'(x)$			
$G(x)$			

8) $$\lim_{x\to 1} \frac{\int_1^{x^2} t\cos(1-e^{1-t})\,dt}{x^2-1} = \frac{2x\cdot x^2\cos(1-e^{1-x^2}) - \cos(1-e^0)\cdot 0}{2x}$$

$$= \lim_{x\to 1}\frac{x^2\cdot 2x\cdot\cos(1-e^{1-x^2})}{2x} = \lim_{x\to 1} x^2\cos(1-e^{1-x^2})$$

$$= 1\cdot\cos(1-e^0) = 1\cdot\cos(0) = 1$$



1) São integraveis?

a) $f:[0,4]\to\mathbb{R}$ $\quad f(x)=\cos(\underbrace{x^2-2x}_{0 \text{ a } \pi})$

f é continua em $[0,4]$ logo é integravel em $[0,4]$

b) $f\ [-2,1] \to \mathbb{R}$ $\quad f(x) \begin{cases} x+1 & , \ x \in [-2, 0[\\ 2 & , \ x = 0 \\ x & , \ x \in]0, 1] \end{cases}$

limitada em $[-2, 1]$ ✓

$\lim\limits_{x \to 0^-} x+1 = 0$ $\qquad f(0) = 2$

$\lim\limits_{x \to 0} 2 = 2$

$\lim\limits_{x \to 0^+} x = 0$

Só n é contínua em $f(0)$, logo é integrável em $[-2, 1]$

12)

a) $\displaystyle\int_0^2 6x^4 \, dx = \left[6 \frac{x^5}{5} \right]_0^2 = 6 \cdot \frac{2^5}{5} - 6 \cdot 0 = 6 \cdot \frac{32}{5} = \frac{192}{5}$

b) $\int_3^2 \frac{t^2}{3} - \sqrt{t} \quad dt = \int_3^2 \frac{t^2}{3} \, dt - \int_3^2 t^{1/2} \, dt = \left[\frac{1}{3} \frac{t^3}{3}\right]_3^2 - \left[\frac{t^{3/2}}{\frac{3}{2}}\right]_3^2$

$= \left(\frac{2^3}{9} - \frac{3^3}{9}\right) - \left(\frac{2\sqrt{2^3}}{3} - \frac{2\sqrt{3^3}}{3}\right) = \frac{8}{9} - \frac{9}{9} - \left(\frac{2}{3}\left(\sqrt{2^3} - \sqrt{3^3}\right)\right)$

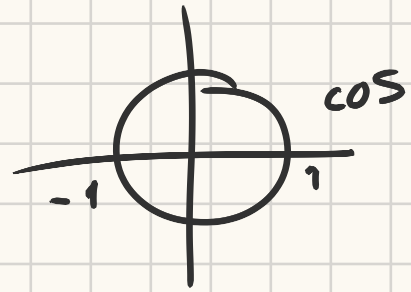
$= \cdot \frac{1}{9} - \frac{2}{3}\left(2\sqrt{2} - 3\sqrt{3}\right) = -\frac{1}{9} - \frac{4\sqrt{2}}{3} + 2\sqrt{3}$

c) $\int_{-4}^{-3} \frac{e^x}{3} \, dx = \left[\frac{1}{3} e^x\right]_{-4}^{-3} = \frac{1}{3}\left(e^{-3} - e^{-4}\right) = \frac{1}{3} e^3\left(1 - e^{-1}\right) = \frac{e^3}{3}\left(1 - \frac{1}{e}\right)$

d) $\int_1^3 \frac{x^3}{\sqrt{x}} \, dx = \int_1^3 x^3 \cdot x^{-1/2} \, dx = \int_1^3 x^{\frac{6}{2} - 1/2} = \int_1^3 x^{5/2} = \left[\frac{x^{7/2}}{7/2}\right]_1^3$

$= \frac{3^{7/2}}{7/2} - \frac{1^{7/2}}{7/2} = \frac{2\sqrt{3^7}}{7} - \frac{2}{7}$

e) $$\int_0^1 \frac{1}{1+t^2}\,\partial t = \operatorname{arctg}(1) - \operatorname{arctg}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$



i) $$\int_{-\pi}^{0} \operatorname{sen}(3x)\,dx = \left[-\frac{1}{3}\cos(3x)\right]_{\pi}^{0} = -\frac{1}{3}\Big(\cos(0) - \cos(\pi)\Big)$$

$$= -\frac{1}{3}(1+1) = -\frac{2}{3}$$

n) $$\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx = \int_e^{e^2} \frac{1}{x}\cdot(\ln x)^{-2}\,dx = \left[\frac{(\ln x)^{-1}}{-1}\right]_e^{e^2} = -\frac{1}{\ln e^2} + \frac{1}{\ln e}$$

$$= -\frac{1}{2} + \frac{1}{1} = \frac{1}{2}$$

p) $$\int_1^2 \frac{1}{x^2+2x+5}\,dx = *$$

C A/

$$x^2+2x+5 = x^2+2x+1+4 = (x+1)^2+4 = 4\left(\frac{(x+1)^2}{4}+1\right) =$$

$$= 4\left(\left(\frac{x+1}{2}\right)^2+1\right)$$

$$= * \int_1^2 \frac{1}{4\left(\left(\frac{x+1}{2}\right)^2+1\right)} dx = \frac{1}{4}\int_1^2 \frac{1}{\left(\frac{x+1}{2}\right)^2+1} = \frac{1}{2}\int_1^2 \frac{1/2}{1+\left(\frac{x+1}{2}\right)^2} dx = \frac{1}{2}\left[\operatorname{arctg}\left(\frac{x+1}{2}\right)\right]_1^2$$

$$= \frac{1}{2}\left(\operatorname{arctg}\left(\frac{3}{2}\right) - \operatorname{arctg}\left(\frac{2}{2}\right)\right) = \frac{1}{2}\operatorname{arctg}\left(\frac{3}{2}\right) - \frac{\pi}{4}$$

13

a) $\displaystyle\int_{-\ln 2}^{\ln 2} \frac{1}{e^x+4}\,dx$

$t = e^x+4 \Leftrightarrow e^x = t-4 \Leftrightarrow x = \ln(t-4)$

$\Leftrightarrow dx = \frac{1}{t-4}\,dt$

x	t
$-\ln 2$	$e^{-\ln 2}+4 = e^{\ln 2^{-1}}+4 = \frac{1}{2}+4 = \frac{1}{2}+\frac{8}{2} = \frac{9}{2}$
$\ln 2$	$e^{\ln 2}+4 = 6$

$$= \int_{\frac{9}{2}}^{6} \frac{1}{t} \cdot \frac{1}{t-4}\, dt = \int_{\frac{9}{2}}^{6} \frac{1}{(t-4)\,t}\, dt = *$$

CA

$$\frac{1}{(t-4)t} = \frac{A}{t-4} + \frac{B}{t} = \frac{A(t) + B(t-4)}{(t-4)\,t}$$

$$\begin{cases} A + B = 0 \\ -4B = 1 \end{cases} \qquad \begin{cases} A = 1/4 \\ B = -1/4 \end{cases}$$

$$=* \int_{9/2}^{6} \frac{1/4}{t-4} + \frac{-1/4}{t}\, dt = \frac{1}{4} \int \frac{1}{t-4} - \frac{1}{t}\, dt = \frac{1}{4}\Big[\ln|t-4| - \ln|t|\Big]_{9/2}^{6}$$

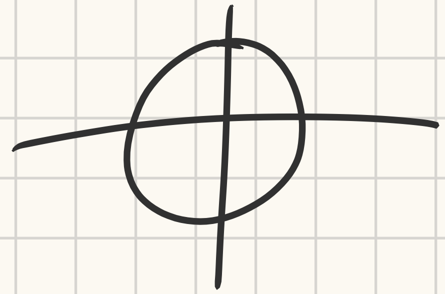
$$= \frac{1}{4}\Big(\big(\ln(2) - \ln(6)\big) - \big(\ln(1/2) - \ln(9/2)\big)\Big) = \frac{1}{4}\left(\ln\left(\frac{2}{6}\right) - \ln\left(\frac{1/2}{9/2}\right)\right)$$

$$= \frac{1}{4}\ln\left(\frac{1/3}{9/4}\right) = \frac{1}{4}\ln\left(\frac{9}{12}\right) = \frac{1}{4}\ln\left(\frac{3}{4}\right)$$

b) $$\int_0^1 \frac{x}{1+x^4}\,dx = \frac{1}{2}\int_0^1 \frac{2x}{1+(x^2)^2} = \frac{1}{2}\Big[\operatorname{arctg}(x^2)\Big]_0^1 =$$

sen $\frac{1}{2}$

cos



$$= \frac{1}{2}\Big(\operatorname{arctg}(1) - \operatorname{arctg}(0)\Big) = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$$

c) $$\int_0^1 \sqrt{4-x^2}\,dx \xrightarrow[dx = 2\cos(t)\,dt]{x = 2\sin(t) \Leftrightarrow t = \arcsin\left(\frac{x}{2}\right)} \int_0^{\frac{\pi}{6}} \sqrt{4-4\sin^2(t)}\cdot 2\cos(t)\,dt$$

x	t
1	$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
0	$\arcsin(0) = 0$

$$4 - 4\sin^2 = 4\cos^2$$

$$= \int_0^{\pi/6} 2\cos t \cdot \sqrt{4\cos^2(t)}\,dt = \int_0^{\pi/6} 2\cos t \cdot 2\cos t\,dt = 4\int_0^{\pi/6} \cos^2 t\,dt$$

$$= 4\int_0^{\pi/6} \frac{1+\cos(2t)}{2}\, dt = 2\int_0^{\pi/6} 1+\cos(2t)\, dt$$

$$= 2\int_0^{\pi/6} 1\, dt + \int_0^{\pi/6} 2\cos(2t)\, dt = \left[2t\right]_0^{\pi/6} + \left[\operatorname{sen}(2t)\right]_0^{\pi/6} =$$

$$= 2\frac{\pi}{6} - 0 + \operatorname{sen}\left(\frac{2\pi}{6}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2} + C, \quad C \in \mathbb{R}$$

d) $\displaystyle\int_1^e x\ln x\, dx = \ln x \cdot \frac{x^2}{2}\Big|_1^e - \int_1^e \frac{x^2}{2}\cdot\frac{1}{x}\, dx =$

$u = \ln x \qquad dv = x$

$du = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$$= \frac{x^2}{2}\ln x\Big|_1^e - \int_1^e \frac{x^2}{2x}\, dx = \frac{e^2}{2}\cdot 1 - \frac{1}{2}\cdot 0 - \frac{1}{2}\int_1^e x\, dx$$

$$= \frac{e^2}{2} - \frac{1}{2} - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e = \frac{e^2}{2} - \frac{1}{2} - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1}{2}\right) = \frac{e^2}{2} - \frac{1}{2} - \frac{e^2}{4} + \frac{1}{4}$$

$$= \frac{2e^2 - e^2}{4} - \frac{2}{4} + \frac{1}{4} = \frac{e^2}{4} - \frac{1}{4} = \frac{e^2-1}{4}$$

e) $$\int_1^e \ln^2 x \, dx = \int_1^e 1 \cdot \ln^2 x \, dx = \ln^2(x) \cdot x \Big|_1^e - \int_1^e 2 \cdot \ln x \cdot \frac{1}{x}$$

$$u = \ln^2 x \qquad dv = 1$$

$$du = 2\ln x \cdot \frac{1}{x} \qquad v = x$$

$$= \ln^2(e) \cdot e - \ln^2(1) - 1 - 2\int_1^e \frac{1}{x} \cdot \ln(x)\, dx = e - 0 - 1 - 2\left[\frac{\ln^2(x)}{2}\right]_1^e$$

$$= e - 1 - 2\left(\frac{\ln^2(e)}{2} - \frac{\ln^2(1)}{2}\right) = e - 1 - 2\left(\frac{1}{2} - \frac{0}{2}\right) = e - 1 - \frac{2}{2} = e - 2 \quad + C$$

14

a) $\int_0^2 f(x)\, dx$, $f(x) = \begin{cases} 2 & \text{se } 0 \leq x < 1 \\ \frac{1}{x} & \text{se } 1 \leq x \leq 2 \end{cases}$

$$\int_0^2 f(x) = \int_0^1 f(x) + \int_1^2 f(x) =$$

$$= \int_0^1 2 \, dx + \int_1^2 \frac{1}{x}\, dx = \left[2x\right]_0^1 + \left[\ln x\right]_1^2$$

$$= 2 \cdot 1 + \ln(2) - \ln(1) = 2 + \ln(2) + C, \quad C \in \mathbb{R}$$

b) $\int_{-1}^1 f(x)$, $f(x) \begin{cases} \frac{2}{1+x^2} & , \quad x \in [-1, 0[\\ 7 & , \quad x = 0 \\ \frac{1}{1+x} & , \quad x \in]0, 1] \end{cases}$

$$\int_{-1}^{1} f(x) = \int_{-1}^{0} \frac{2}{1+x^2} + \int_0^1 \frac{1}{1+x}$$

$$= 2\Big[\arctan(x)\Big]_{-1}^{0} + \Big[\ln(1+x)\Big]_0^1$$

$$= 2(\arctan(0) - \arctan(-1) + \ln(2) - \ln(1)$$

$$= 2\left(0 - \frac{\pi}{4}\right) + \ln(2) - 0 = \frac{\pi}{2} + \ln(2)$$

c) $$\int_{-1}^{3} f(x) \quad , \quad f(x) = \begin{cases} \dfrac{x}{1+x^2} & , \; x \neq 1 \\ 5 & , \; x = 1 \end{cases}$$

$$\int_{-1}^{3} f(x) = \int_{-1}^{1} \frac{x}{1+x^2} + \int_1^3 \frac{x}{1+x^2} = \cancel{\ }$$

C.A

$$\int \frac{x}{1+x^2} = \int \frac{\sqrt{u-1}}{u} \cdot \frac{1}{2\sqrt{u-1}}\, du = \frac{1}{2}\int \frac{1}{u}\, du = \frac{1}{2}\ln(u)$$

$$u = x^2+1 \Leftrightarrow \sqrt{u-1} = x \qquad = \frac{1}{2}\ln(x^2+1) + C$$

$$\frac{1}{2}(u-1)^{-1/2} = dx$$

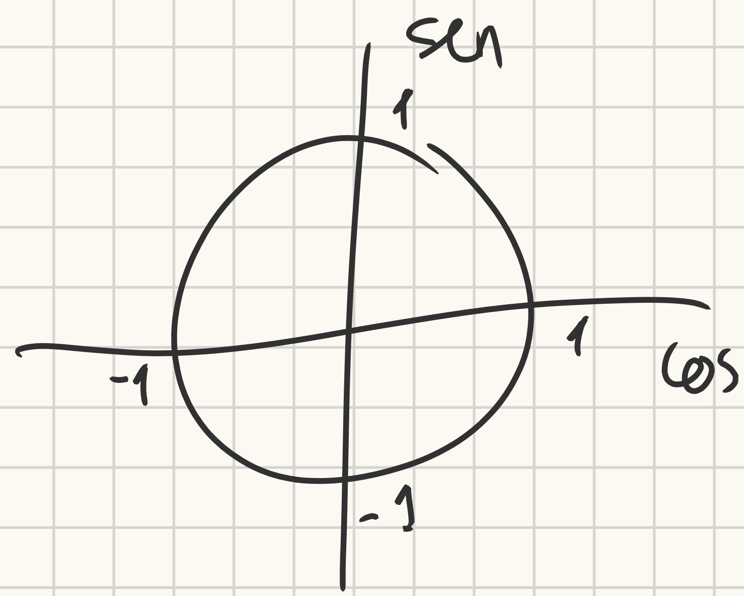
$$\Leftrightarrow \frac{1}{2\sqrt{u-1}}\, du = dx$$

$$* \left[\frac{\ln(x^2+1)}{2}\right]_{-1}^{1} + \left[\frac{\ln(x^2+1)}{2}\right]_{1}^{3} =$$

$$= \frac{1}{2}\Big(\ln(2) - \ln(2) + \ln(10) - \ln(2)\Big) = \frac{1}{2}\ln\left(\frac{10}{2}\right) = \frac{1}{2}\ln(5)$$

d) $$\int_0^{2\pi} f(x)\, dx \quad , \quad f(x) \begin{cases} -2 \;, & x \in [0, \frac{\pi}{2}] \\ \cos x \;, & x \in [\frac{\pi}{2}, \frac{3\pi}{2}[\\ \operatorname{sen} x \;, & x \in]\frac{3\pi}{2}, 2\pi] \end{cases}$$

$$\int_0^{2\pi} f(x) = \int_0^{\pi/2} 2 + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x + \int_{\frac{3\pi}{2}}^{2\pi} \operatorname{sen} x$$



$$= \left[2x\right]_0^{\pi/2} + \left[\operatorname{sen} x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[-\cos\ \right]_{\frac{3\pi}{2}}^{2\pi}$$

$$= 2\frac{\pi}{2} + \operatorname{sen}\left(\frac{3\pi}{2}\right) - \operatorname{sen}\left(\frac{\pi}{2}\right) - \cos\left(2\pi\right) + \cos\left(\frac{3\pi}{2}\right)$$

$$= \pi - 1 - 1 - 1 + 0 = \pi - 3$$

14 (De novo)

a) $\int_0^2 f(x)$, $f(x) = \begin{cases} 2 & \text{se} \quad 0 \leq x < 1 \\ \frac{1}{x} & \text{se} \quad 1 \leq x \leq 2 \end{cases}$

$$\int_0^2 f(x) = \int_0^1 2\,dx + \int_1^2 \frac{1}{x}\,dx = \left[2x\right]_0^1 + \left[\ln(x)\right]_1^2 =$$

$$= 2.1 - 2.0 + \ln(2) - \ln(1) = 2 + \ln(2)$$

b) $\displaystyle\int_{-1}^{1} f(x)$, $f(x) = \begin{cases} \dfrac{2}{1+x^2} & , \quad x \in [-1, 0[\\ 7 & , \quad x = 0 \\ \dfrac{1}{1+x} & , \quad x \in]0, 1] \end{cases}$

$$\int_{-1}^{1} f(x) = \int_{-1}^{0} \frac{2}{1+x^2}\,dx + \int_0^1 \frac{1}{1+x}\,dx$$

sen

cos

tan $\quad \frac{\sqrt{3}}{3} \quad 1 \quad \sqrt{3}$

$$= 2\left[\operatorname{arctg}(x)\right]_{-1}^{0} + \left[\ln(1+x)\right]_0^1 = 2(\operatorname{arctg}(0) - \operatorname{arctg}(-1)) + \ln(2) - \ln(1)$$

$$= 2\left(0 + \frac{\pi}{4}\right) + \ln(2) - 0 = \frac{2\pi}{4} + \ln(2) = \frac{\pi}{2} + \ln(2)$$

c) $\int_{-1}^{3} f(x)$, $f(x) = \begin{cases} \frac{x}{1+x^2} & , \quad x \neq 1 \\ 5 & , \quad x = 1 \end{cases}$

$$\int_{-1}^{3} f(x) = \int_{-1}^{1} \frac{x}{1+x^2} + \int_{1}^{3} \frac{x}{1+x^2} = *$$

CA

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2x}{1+x^2} = \frac{1}{2}\ln(1+x^2)$$

$$* \frac{1}{2}\left(\left[\ln(1+x^2)\right]_{-1}^{1} + \left[\ln(1+x^2)\right]_{1}^{3}\right) =$$

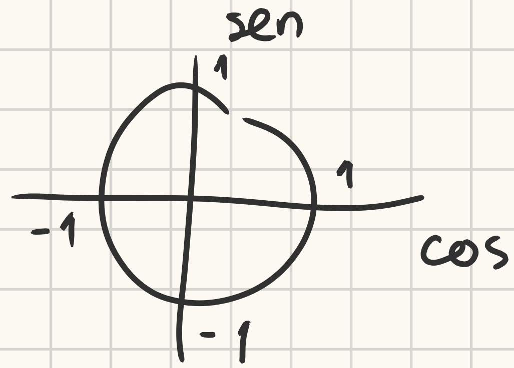
$$= \frac{1}{2}\left(\ln(2) - \ln(2) + \ln(10) - \ln(2)\right) = \frac{1}{2}\left(\ln(10) - \ln(2)\right) = \frac{1}{2}\left(\ln\left(\frac{10}{2}\right)\right)$$

$= \frac{1}{2} \ln(5) + C, \quad C \in \mathbb{R}$

d) $\displaystyle\int_0^{2\pi} f(x)$, $\quad f(x) = \begin{cases} -2 & , \; x \in \left[0, \frac{\pi}{2}\right] \\ \cos x & , \; x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \\ \operatorname{sen} x & , \; x \in \left]\frac{3\pi}{2}, 2\pi\right] \end{cases}$

$$\int_0^{2\pi} f(x)\,dx = \int_0^{\pi/2} -2 \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx + \int_{\frac{3\pi}{2}}^{2\pi} \operatorname{sen} x \, dx$$

$$= \left[-2x\right]_0^{\pi/2} + \left[\operatorname{sen} x\right]_{\pi/2}^{\frac{3\pi}{2}} + \left[-\cos x\right]_{\frac{3\pi}{2}}^{2\pi}$$



$$= -2\frac{\pi}{2} - 0 + \operatorname{sen}\frac{3\pi}{2} - \operatorname{sen}\frac{\pi}{2} + (-\cos 2\pi) + \cos\left(\frac{3\pi}{2}\right)$$

$$= -\pi - 1 - 1 - 1 + 0 = -\pi - 3$$

(15) $f(x) = \frac{1}{x\ln(x)}$

a) primitiva que se anula em $x = e^2$?

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{x} \cdot \frac{1}{\ln(x)}\,dx = \int \frac{\overbrace{\frac{1}{x}}^{u'}}{\underbrace{\ln(x)}_{u}}\,dx = \ln|\ln(x)| + C$$

$$\ln|\ln(e^2)| + C = 0 \Leftrightarrow \ln|2| + C = 0 \Leftrightarrow \ln(2) = -C \Leftrightarrow -\ln(2) = C$$

$$\therefore \ln|\ln|x|| - \ln(2)$$

b) Area entre $x = e$, $x = e^3$, eixo de abcissas e f ?

$$f(x) = \frac{1}{x\ln(x)} > 0$$

$$\int_{e}^{e^3} \frac{1}{x\ln(x)}$$, pela alínea anterior $\rightarrow \left[\ln|\ln|x||\right]_{e}^{e^3}$

$$\ln(\ln(e^3)) - \ln(\ln(e)) = \ln(3) - \ln(1) = \ln\left(\frac{3}{1}\right) = \ln(3)$$

(16) Area entre $x=0$, $x=2$, eixo das abcissas e por $g(x)$

$g(x) = x\ln(x+1) \geq 0$

$$\int_0^2 x\ln(x+1)\,dx = \ln(x+1)\cdot\frac{x^2}{2}\Big|_0^2 - \int_0^2 \frac{x^2}{2}\cdot\frac{1}{x+1}\,dx$$

$u = \ln(x+1) \qquad dv = x$

$du = \frac{1}{x+1} \qquad v = \frac{x^2}{2}$

$$= \ln(x+1)\cdot\frac{x^2}{2}\Big|_0^2 - \frac{1}{2}\int_0^2 \frac{x^2}{x+1}\,dx = \ln(x+1)\cdot\frac{x^2}{2}\Big|_0^2 - \frac{1}{2}\int_0^2 x-1+\frac{1}{x+1}$$

$$
\begin{array}{r|l}
x^2 & x+1 \\ \cline{2-2}
-x^2-x & x-1 \\ \cline{1-1}
-x & \\
+x+1 & \\ \cline{1-1}
+1 &
\end{array}
$$

$$= \ln(x+1)\,\frac{x^2}{2}\Big|_0^2 - \frac{1}{2}\left(\int_0^2 x-1 + \int_0^2 \frac{1}{x+1}\right)$$

$$= \ln(3)\,\frac{4}{2} - \ln(1)\cdot\frac{1}{2} - \frac{1}{2}\left(\left[\frac{x^2}{2}-x\right]_0^2 + \left[\ln(x+1)\right]_0^2\right) =$$

$$= 2\ln(3) - 0 - \frac{1}{2}\left(\frac{4}{2} - 2 - 0 + \ln(3) - \ln(1)\right) =$$

$$= 2\ln(3) - \frac{1}{2}\left(\ln(3)\right) = 2\ln(3) - \frac{\ln(3)}{2} = \frac{1}{2}\left(4\ln(3) - \ln(3)\right) = \frac{3}{2}\ln(3)$$

treino de divisão

$$
\begin{array}{r|l}
x^3 + 0x^2 + px + q & x^2+2x+5 \\ \cline{2-2}
-x^3 - 2x^2 - 5x & x-2 \\ \cline{1-1}
-2x^2 + (p-5)x + q &
\end{array}
$$

$$+2x^2 + 4x + 10$$
$$(p-1)x + q10$$

$$\frac{-2x^2}{x} = -2x$$

$$\frac{3x}{x} = 3$$

$$\begin{array}{l|l} x^3 - 3x^2 + 5x - 3 & x - 1 \\ \hline -x^3 + x^2 & x^2 - 2x + 3 \\ \hline \quad -2x^2 + 5x - 3 & \\ \quad +2x^2 - 2x & \\ \hline \qquad 3x - 3 & \\ \qquad -3x + 3 & \\ \hline \qquad\quad 0 & \end{array}$$

$$\frac{x^3 - 3x^2 + 5x - 3}{x - 1} = x^2 - 2x + 3$$

$$\begin{array}{l|l} 6x^4 - x^3 + 3x^2 - x + 1 & 2x^2 + x - 3 \\ \hline -6x^4 - 3x^3 + 9x^2 & 3x^2 - 2x + 7 \\ \hline \quad -4x^3 + 12x^2 - x + 1 & \\ \quad 4x^3 + 2x^2 - 6x & \\ \hline \qquad 14x^2 - 7x + 1 & \\ \qquad -14x^2 - 7x + 21 & \\ \hline \qquad\quad -14x + 22 & \end{array}$$

$$\frac{6x^4}{2x^2} = 3x^2$$

$$\frac{-4x^3}{2x^2} = -2x$$

$$\frac{14x^2}{2x^2} = 7$$

$$\frac{6x^4 - x^3 + 3x^2 - x + 1}{2x^2 + x - 3} = 3x^2 - 2x + 7 + \frac{-14x + 22}{2x^2 + x - 3}$$

$$\begin{array}{r|l} x^2 + x - 2 & x - 1 \\ \cline{2-2} -x^2 + x & x + 2 \\ \hline 2x - 2 & \\ -2x + 2 & \\ \hline 0 & \end{array}$$

$$\frac{x^2}{x} = x$$

$$\frac{2x}{x} = 2$$

$$\frac{x^2 + x - 2}{x - 1} = x + 2$$

(17) Area entre $x = 0$, $x = 2$, eixo das abcissas e $g(x) = \dfrac{e^{2x} + 1}{e^x + 1}$

$$\frac{e^{2x} + 1}{e^x + 1} > 0$$

x	t
0	$e^0 + 1 = 2$
2	$e^2 + 1$

$$\int_0^2 \frac{e^{2x} + 1}{e^x + 1}\, dx = \longrightarrow \int_2^{e^2+1} \frac{(t-1)^2 + 1}{t(t-1)} \cdot dt$$

$$t = e^x + 1 \Leftrightarrow \ln(t-1) = x \Leftrightarrow t - 1 = e^x$$

$$\frac{1}{t-1}\, dt = dx$$

$$= \int_2^{e^2+1} \frac{t^2-2t+1+1}{t^2-t}\,dt = \int_2^{e^2+1} \frac{t^2-2t+2}{t^2-t} = \int_2^{e^2+1} 1 - \frac{t-2}{t^2-t}$$

$$\begin{array}{l|l} t^2-2t+2 & t^2-t \\ \cline{2-2} -t^2+t & 1 \\ \hline -t+2 & \end{array}$$

$$= \int_2^{e^2+1} 1\,dt - \int_2^{e^2+1} \frac{t-2}{t(t-1)}$$

$$= [t]_2^{e^2+1} - \int_2^{e^2+1} \frac{t-2}{t(t-1)}\,dt \quad *$$

CA

$$\frac{t-2}{t(t-1)} = \frac{A}{t} + \frac{Bt+C}{t-1} = \frac{A(t-1)+B(t^2)+C(t)}{t(t-1)} =$$

$$= \frac{B(t^2)+A+C(t)+A(-1)}{t(t-1)}$$

$$\begin{cases} B=0 \\ A+C=1 \\ A=2 \end{cases} \quad \begin{cases} B=0 \\ C=1-2 \\ A=2 \end{cases} \quad \begin{cases} B=0 \\ C=-1 \\ A=2 \end{cases} \longrightarrow \frac{2}{t} + \frac{-1}{t-1}$$

$$* \quad [t]_2^{e^2+1} - \int_2^{e^2+1} \frac{2}{t}\, dt - \int_2^{e^2+1} \frac{1}{t-1} =$$

$$= [t]_2^{e^2+1} - \left[2\ln|t|\right]_2^{e^2+1} - \left[\ln|t-1|\right]_2^{e^2+1} =$$

$$= \left[e^x + 1\right]_0^2 - \left[2\ln(e^x+1)\right]_0^2 - \left[\ln|e^x)\right]_0^2 =$$

$$= e^2 + 1 - (e^0 + 1) - 2\left(\ln(e^2+1) - \ln(e^0+1)\right) - (2 - 0)$$

$$= e^2 + 1 - 1 - 1 - 2\ln\left(\frac{e^2+1}{2}\right) - 2 = e^2 - 1 - 2 - 2\ln\left(\frac{e^2+1}{2}\right)$$

$$= e^2 - 3 - 2\ln\left(\frac{e^2+1}{2}\right)$$

(18) $f(x) = x^3 - 3x^2 + 2x$. Area entre $f(x)$, $x=0$, $x=2$ e O_x?

$$x^3 - 3x^2 + 2x = 0 \Leftrightarrow (x-1)(x^2 - 2x) = 0 \Leftrightarrow$$

$$\Leftrightarrow (x-1) = 0 \vee (x^2 - 2x) = 0$$

$$\begin{array}{c|ccc} & 1 & -3 & 2 \\ 1 & & 1 & -2 \\ \hline & 1 & -2 & 0 \end{array}$$

$$\Leftrightarrow x = 1 \vee x^2 - 2x + 0 = 0$$

$$\Leftrightarrow \underline{x = 1} \vee \underline{x = 0} \vee \underline{x = 2}$$

CA $x^2 - 2x + 0 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1.0)}}{2} \Leftrightarrow x = \frac{2 \pm \sqrt{4}}{2} \Leftrightarrow x = \frac{2 \pm 2}{2}$$

$$\Leftrightarrow x = \frac{2-2}{2} \vee x = \frac{2+2}{2} \Leftrightarrow x = 0 \vee x = 2$$

em $x = -1$) $(-1)^3 - 3(-1)^2 + 2(-1) = -1 - 3 - 2 < 0$

em $x = \frac{1}{2}$) $\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) = \frac{1}{8} - \frac{3}{4} + \frac{2}{2} =$

$= \frac{1}{8} - \frac{6}{8} + \frac{8}{8} = \frac{3}{8} > 0$

em $x = \frac{3}{2}$) $\left(\frac{3}{2}\right)^3 - 3.\left(\frac{3}{2}\right)^2 + 2\left(\frac{3}{2}\right) = \frac{27}{8} - \frac{27}{4} + 3 =$

$= \frac{27}{8} - \frac{54}{8} + \frac{24}{8} = -\frac{3}{8} < 0$

$$\int_0^2 f(x) = \int_0^1 f(x) - \int_1^2 f(x) =$$

$$= \int_0^1 x^3 - 3x^2 + 2x - \int_1^2 x^3 - 3x^2 + 2x$$

$$= \left[\frac{x^4}{4} - \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} \right]_0^1 - \left[\frac{x^4}{4} - x^3 + x^2 \right]_1^2$$

$$= \frac{1^4}{4} - 1^3 + 1^2 - 0 - \left(\frac{2^4}{4} - 2^3 + 2^2 - \left(\frac{1}{4} - 1 + 1 \right) \right)$$

$$= \frac{1}{4} - \frac{16}{4} + 8 - 4 + \frac{1}{4} = -\frac{14}{4} + 4 = -\frac{14}{4} + \frac{16}{4} = \frac{2}{4} = \frac{1}{2}$$

(19) Area entre $f(x) = \frac{4 + \operatorname{sen}^2 x}{1+4x^2}$, $g(x) = \frac{\operatorname{sen}^2 x}{1+4x^2}$, $x = 0$ e 0 , $x = \frac{1}{2}$

$$\frac{4 + \operatorname{sen}^2 x}{1+4x^2} > 0 \quad , \quad \frac{\operatorname{sen}^2 x}{1+4x^2} \geq 0 \quad , \quad \frac{4 + \operatorname{sen}^2 x}{1+4x^2} > \frac{\operatorname{sen}^2 (x)}{1+4x^2}$$

$$\int_0^{1/2} \frac{4+\operatorname{sen}^2 x}{1+4x^2} - \frac{\operatorname{sen}^2 x}{1+4x^2} = \int_0^{1/2} \frac{4+\operatorname{sen}^2 x - \operatorname{sen}^2 x}{1+4x^2}\, dx$$

$$= \int_0^{1/2} \frac{4}{1+4x^2}\, dx = 2\int_0^{1/2} \frac{2}{1+(2x)^2}\, dx = 2\Big[\operatorname{arctg}(2x)\Big]_0^{1/2}$$

$$= 2\Big(\operatorname{arctg}(1) - \operatorname{arctg}(0)\Big) = 2\left(\frac{\pi}{4} - 0\right) = 2\,\frac{\pi}{4} = \frac{\pi}{2}$$

(20) Area entre $f(x) = x^2$ e $g(x) = x$?

$$x^2 = x \iff x^2 - x = 0 \iff x(x-1) = 0 \iff x = 0 \lor x = 1$$

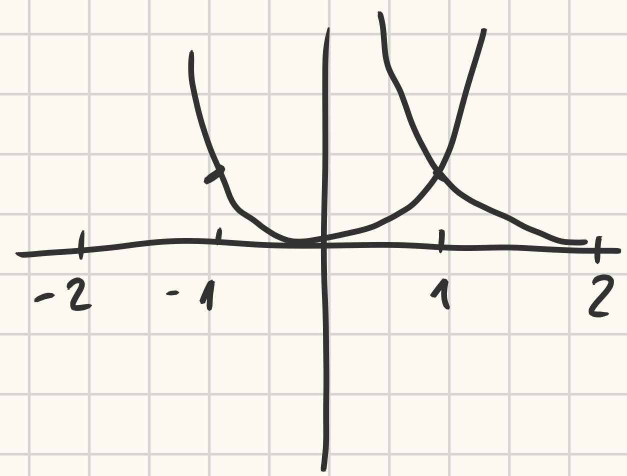
$$x^2 > x$$

em $x = \frac{1}{2}$) $\left(\frac{1}{2}\right)^2 = \frac{1}{4} < \frac{1}{2}$

$$\int_0^1 x - x^2 \, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

22 Area entre $f(x) = \frac{1}{x}$, $g(x) = x^2$, $x = 2$ e $y = 0$?

O_x



$$\frac{1}{x} = x^2 \Leftrightarrow 1 = x^3 \Leftrightarrow x = 1$$

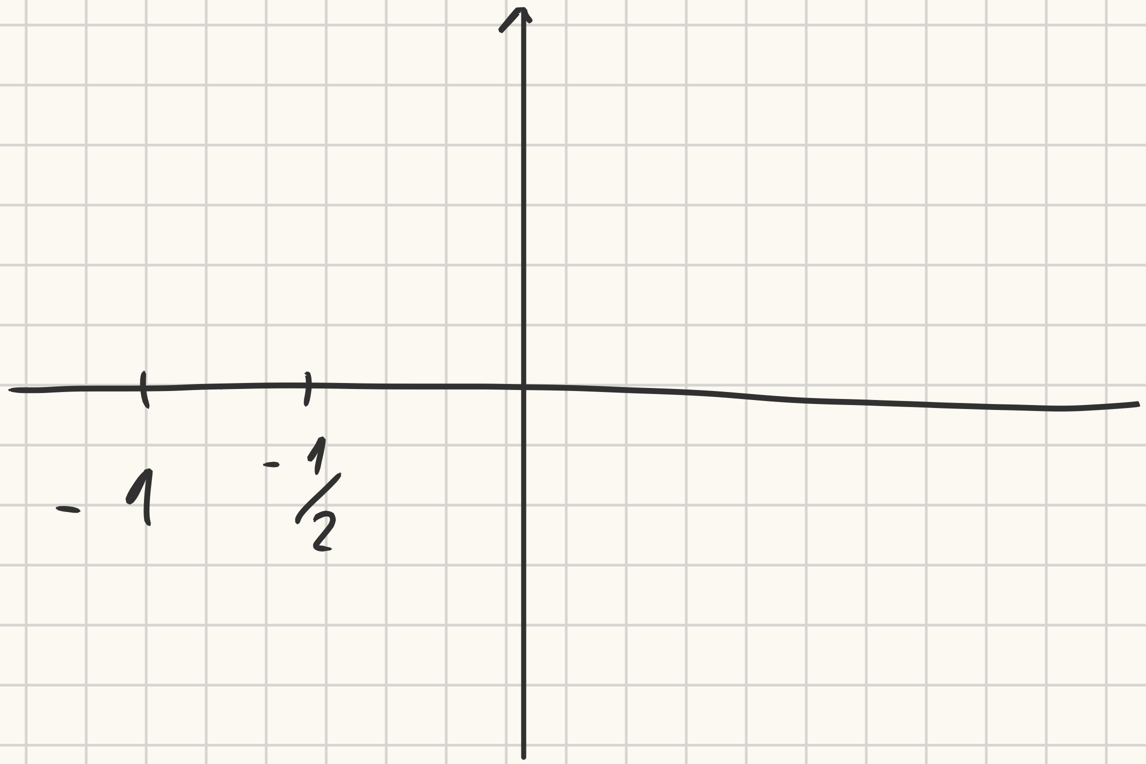
$1 \to 2$

$$\int_0^1 x^2 + \int_1^2 \frac{1}{x} = \left[\frac{x^3}{3}\right]_0^1 + \Big[\ln|x|\Big]_1^2 = \frac{1}{3} + \ln(2) - \ln(1)$$

$$= \frac{1}{3} + \ln(2)$$

(21) Area entre $f(x) = e^{2x+1}$, $g(x) = xe^{2x+1}$, $x = -1$ e $x = -\frac{1}{2}$

$e^{2x+1} = xe^{2x+1} \Leftrightarrow x = 1$



$e^{-2+1} = e^{-1} = \frac{1}{e}$

$-1e^{-2+1} = -1e^{-1} = -\frac{1}{e} \rightarrow$

em $x < 0$) $f(x) > g(x)$

em $x = -\frac{2}{3}$)

$f(-\frac{2}{3}) = e^{-\frac{4}{3}+\frac{3}{3}} = e^{-1/3} = \frac{1}{\sqrt{e^3}}$ $\quad f(x) > g(x)$

$g(-\frac{2}{3}) = -\frac{2}{3} \cdot \frac{1}{\sqrt{e^3}} = -\frac{2}{3\sqrt{e^3}}$ $\quad g(x) < 0 < f(x)$

$\int_{-1}^{-1/2} f(x) + g(x)$ $\quad \int f(x) - g(x)$

$$\int_{-1}^{-1/2} e^{2x+1}\,dx + \int_{-1}^{-1/2} x\, e^{2x+1}\,dx = e\left(\int_{-1}^{-1/2} e^{2x}\,dx + \int_{-1}^{-1/2} x e^{2x}\,dx\right)$$

$$= e\left(\left[\frac{1}{2}\, e^{2x}\right]_{-1}^{-1/2} + x\cdot\frac{e^{2x}}{2}\Big|_{-1}^{-1/2} - \int_{-1}^{-1/2}\frac{1}{2}e^{2x}\,dx\right) =$$

$$u = x \qquad dv = e^{2x}$$

$$du = 1 \qquad v = \frac{1}{2}e^{2x}$$

$$= \left[e\left(\frac{1}{2}\left(e^{-1} - e^{-2}\right)\right] + \left[\frac{1}{2}\left(-\frac{1}{2}\cdot e^{-1} - \left(-1\cdot e^{-2}\right)\right) - \frac{1}{4}\left(e^{-1} - e^{-2}\right)\right)\right]$$

$$= e\left(\frac{1}{2}\left(\frac{1}{e} - \frac{1}{e^2}\right) + \frac{1}{2}\left(-\frac{1}{2e} + \frac{1}{e^2}\right) - \frac{1}{4}\left(\frac{1}{e} - \frac{1}{e^2}\right)\right)$$

$$= e\left(\frac{1}{2e} - \frac{1}{2e^2} - \frac{1}{4e} + \frac{1}{2e^2} - \frac{1}{4e} + \frac{1}{4e^2}\right) =$$

$$= \cancel{\frac{1}{2}} - \cancel{\frac{1}{2e}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{2e}} - \cancel{\frac{1}{4}} + \frac{1}{4e} = \frac{1}{4e}$$

$$\int_{-1}^{-1/2} xe^{2x+1} = \frac{1}{2}\int_{-1}^{-1/2} 2e^{2x+1} = \left[\frac{1}{2}e^{2x+1}\right]_{-1}^{-1/2}$$

$$\frac{1}{2}\left(e^{0} - e^{-1}\right) = \frac{1}{2}\left(1 - e^{-1}\right) = \frac{1}{2}\cdot 1 - \frac{1}{2}e^{-1} = \frac{1}{2} - \frac{e^{-1}}{2}$$

$$\int_{-1}^{-1/2} xe^{2x+1} = \frac{e^{2x+1}}{2}\cdot x \Big|_{-1}^{-1/2} - \int_{-1}^{-1/2} \frac{e^{2x+1}}{2}\cdot dx =$$

$u = x \qquad dv = e^{2x+1}$

$du = 1 \qquad v = \frac{e^{2x+1}}{2}$

$$= \frac{e^{2x+1}}{2} \cdot x \Big|_{-1}^{-1/2} - \frac{1}{2}\int_{-1}^{-1/2} e^{2x+1} = \frac{1}{2}\left(e^0 \cdot -\frac{1}{2} - \left(e^{-1} \cdot -1\right)\right) - \frac{1}{2}\left[\frac{e^{2x+1}}{2}\right]_{-1}^{-1/2}$$

$$= \frac{1}{2}\left(-\frac{1}{2} + e^{-1}\right) - \frac{1}{4}\left(e^0 - e^{-1}\right) = \frac{1}{2}\left(-\frac{1}{2} + e^{-1}\right) - \frac{1}{4}\left(1 - e^{-1}\right)$$

$$= -\frac{1}{4} + \frac{e^{-1}}{2} - \frac{1}{4} + \frac{e^{-1}}{4} = -\frac{1}{2} + \frac{2e^{-1}}{4} + \frac{e^{-1}}{4} = -\frac{1}{2} + \frac{3e^{-1}}{4}$$

$$\frac{1}{2} - \frac{e^{-1}}{2} - \left(-\frac{1}{2} + \frac{3e^{-1}}{4}\right)$$

$$= \frac{1}{2} - \frac{e^{-1}}{2} + \frac{1}{2} - \frac{3e^{-1}}{4} = 1 - \frac{2e^{-1}}{4} - \frac{3e^{-1}}{4} = 1 - \frac{5e^{-1}}{5}$$

$$\int_a^b f(x) = -\int_b^a f(x)$$

Slide 25

(1) Se f é par $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$?

$$\int_{-a}^{a} f(x) = \int_{-a}^{0} f(x)\,dx \int_{0}^{a} f(x)\,dx = -\int_{0}^{-a} f(x) + \int_{0}^{a} f(x)\,dx \quad *$$

CA
$$\int_{0}^{-a} f(x)\,dx = \int_{0}^{a} f(-x).(-dt)$$

$x = -t$

$dx = -dt$

x	t
$-a$	a
0	0

Como f é par

$f(x) = f(-x)$

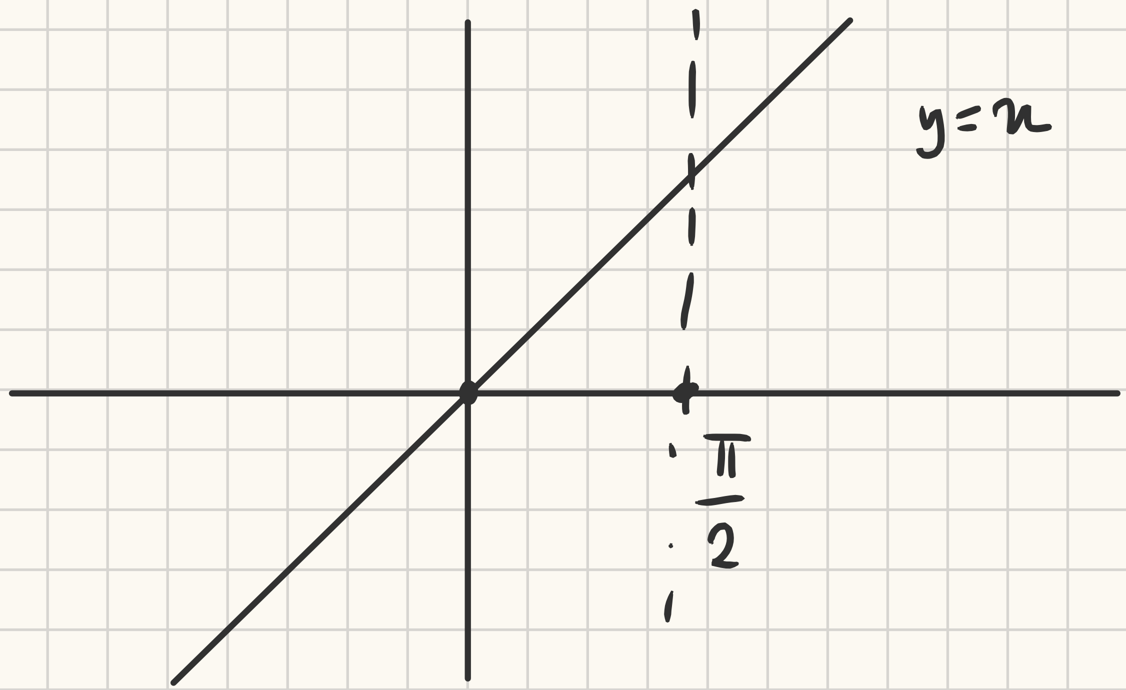
então:

$$\int_{0}^{a} f(-t).(-dt) = \int_{0}^{a} f(t)\,dt$$

$* \quad \int_0^a f(x)\,dx + \int_0^a f(x)\,dt = 2\int_0^a f(x)\,dx$

Ficha 2.2

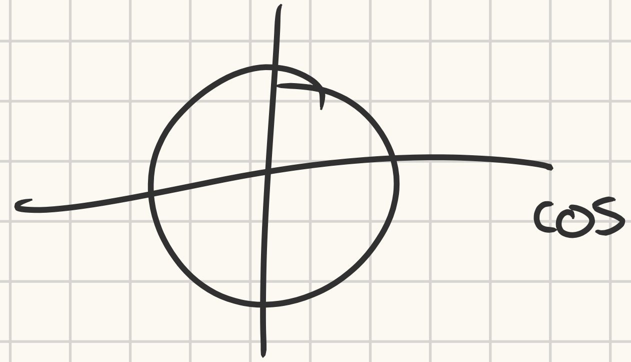
23 Area entre $f(x) = x\cos(x)$, $y = x$, $x = 0$, $x = \frac{\pi}{2}$



$$x = x\cos x \Leftrightarrow \cos x = 1 \Leftrightarrow x = 0$$

em $x = 0$)

$f(0) = 0$ $\qquad$ $y = 0$



em $x = \frac{\pi}{2}$

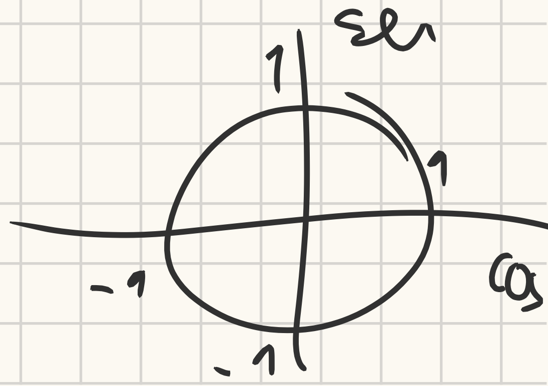
$F\left(\frac{\pi}{2}\right) = 0$ $\qquad$ $y = \frac{\pi}{2}$ $\rightarrow$ $y > f(x)$

$$\int_0^{\pi/2} x - \int_0^{\pi/2} x\cos x = \left[\frac{x^2}{2}\right]_0^{\pi/2} - x\,\text{sen}\,x\Big|_0^{\pi/2} - \int_0^{\pi/2} \text{sen}\,x\,dx$$

$u = x$ $\qquad$ $dv = \cos x$

$du = 1$ $\qquad$ $v = \text{sen}\,x$

$$= \left[\frac{x^2}{2}\right]_0^{\pi/2} - \left[x\,\text{sen}\,x\right]_0^{\pi/2} - \left[-\cos x\right]_0^{\pi/2} =$$



$$= \frac{\frac{\pi^2}{4}}{2} - 0 - \left(\frac{\pi}{2} - 0\right) - \left(0 - 1\right) =$$

$$= \frac{\pi^2}{8} - \frac{\pi}{2} + 1 = \frac{\pi^2}{8} - \frac{4\pi}{8} + \frac{8}{8} = \frac{\pi^2 - 4\pi + 8}{8}$$

25 !!

$$A = \left\{ (x, y) \in \mathbb{R}^2 : y \geq (x-3)^2, y \geq x - 1, y \leq 4 \right\}$$

a) Representar a região

$(x-3)^2 = x - 1 \Leftrightarrow x^2 - 6x + 9 = x - 1 \Leftrightarrow x^2 - 7x + 10 = 0 \Leftrightarrow x = 2 \vee x = 5$

$x - 1 = 4 \Leftrightarrow x = 5$

$(x-3)^2 = 4 \Leftrightarrow x^2 - 6x + 9 = 4 \Leftrightarrow x^2 - 6x + 5 = 0 \Leftrightarrow x = 1 \vee x = 5$

C.A

$$x^2 - 7x + 10 = 0 \rightarrow x = \frac{7 \pm \sqrt{7^2 - 4.10}}{2} \Leftrightarrow x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$\Leftrightarrow x = \frac{7 - 3}{2} \vee x = \frac{7+3}{2} \Leftrightarrow x = \frac{4}{2} \vee x = \frac{10}{2} \Leftrightarrow x = 2 \vee x = 5$$

$$x^2 - 6x + 5 = 0 \rightarrow x = \frac{6 \pm \sqrt{36 - 20}}{2} \Leftrightarrow x = \frac{6-4}{2} \vee x = \frac{6+4}{2}$$

$$\Leftrightarrow x = \frac{2}{2} \vee x = \frac{10}{2} \Leftrightarrow x = 1 \vee x = 5$$

$(x-3)^2$ em $x = 0 \rightarrow (-3)^2 = 9$

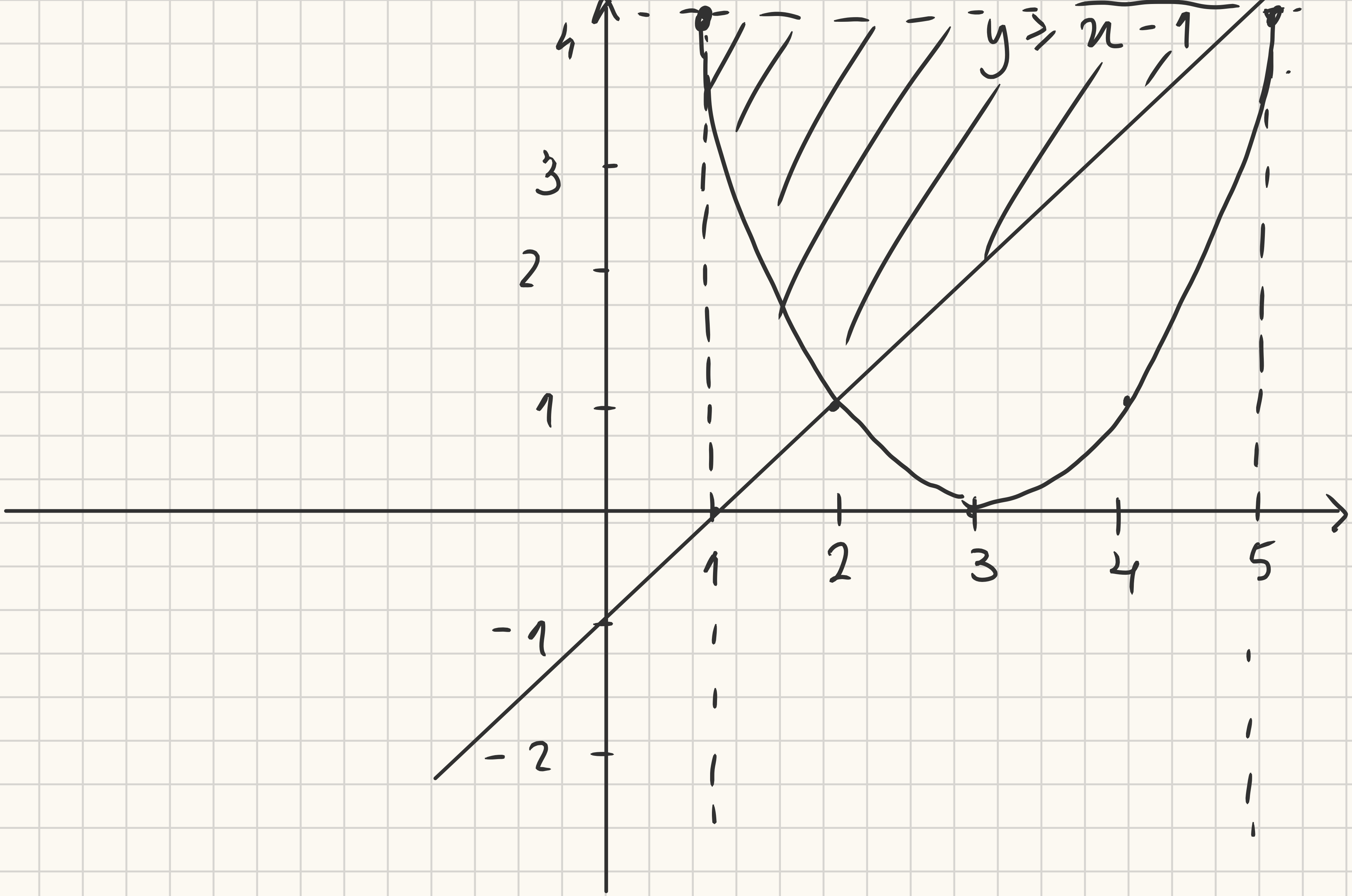
em $x = 1 \rightarrow (-2)^2 = 4$

em $x = 2 \rightarrow (-1)^2 = 1$

em $x = 3 \rightarrow (0)^2 = 0$

em $x = 4 \rightarrow (1)^2 = 1$

em $x = 5 \rightarrow (2)^2 = 4$



$$\int_1^2 4-(x-3)^2\,dx + \int_2^5 4-x+1 \quad dx =$$

$$= \left[4x\right]_1^2 - \left[\frac{(x-3)^3}{3}\right]_1^2 + \left[4x\right]_2^5 - \left[\frac{x^2}{2}\right]_2^5 + \left[x\right]_2^5$$

$$= 8-4 - \left(\frac{(-1)^3}{3} - \frac{(-2)^3}{3}\right) + 20 - 8 - \left(\frac{25}{2} - \frac{4}{2}\right) + 5 - 2$$

$$= 4 + \frac{1}{3} - \frac{8}{3} + 12 - \frac{21}{2} + 3 = 19 - \frac{7}{3} - \frac{21}{2} =$$

$$= \frac{114}{6} - \frac{14}{6} - \frac{63}{6} = \frac{37}{6}$$

29 $\quad f(x) = \left\{ \frac{x}{(x^2+1)^{3/2}} \right.$

a) $\int f(x)\, dx$?

$$\int \frac{x}{(x^2+1)^{3/2}}\,dx = \int \frac{\sqrt{t-1}}{t^{3/2}} \cdot \frac{1}{2\sqrt{t-1}}\,dt = \int \frac{1}{2t^{3/2}}\,dt$$

$$t = x^2+1 \Leftrightarrow \sqrt{t-1} = x$$

$$\frac{1}{2}(t-1)^{-1/2} = dx$$

$$\frac{1}{2\sqrt{t-1}} = dx$$

$$= \frac{1}{2}\int t^{-3/2}\,dt = \frac{1}{2}\,\frac{t^{-1/2}}{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{t}} : 2 = -\frac{1}{\sqrt{t}} + c = -\frac{1}{\sqrt{x^2+1}} + c,\ c \in \mathbb{R}$$

b) Area de f, abcissas, $x=-1$ e $x=\sqrt{3}$?

$$f(x) = \frac{x}{(x^2+1)^{3/2}}$$

$$f(-1) = \frac{-1}{(1+1)^{3/2}} = \frac{-1}{2^{3/2}} = \frac{-1}{\sqrt{2^3}} = \frac{-1}{2\sqrt{2}}$$

$$f(x) = 0 \Leftrightarrow \frac{x}{(x^2+1)^{3/2}} = 0 \Leftrightarrow x = 0 \wedge (x^2+1)^{3/2} \neq 0$$

cond. univ.

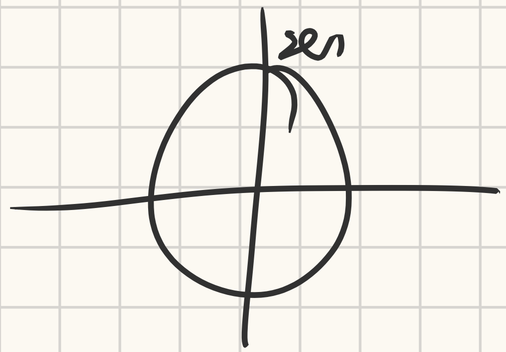
$$-\int_{-1}^{0} f(x) + \int_{0}^{\sqrt{3}} f(x) = -\int_{-1}^{0} \frac{x}{(x^2+1)^{3/2}} \, dx + \int_{0}^{\sqrt{3}} \frac{x}{(x^2+1)^{3/2}} \, dx$$

C.A $\displaystyle\int \frac{x}{(x^2+1)^{3/2}} \, dx = \int \frac{x}{\sqrt{(x^2+1)^3}} \, dx = \int$!!

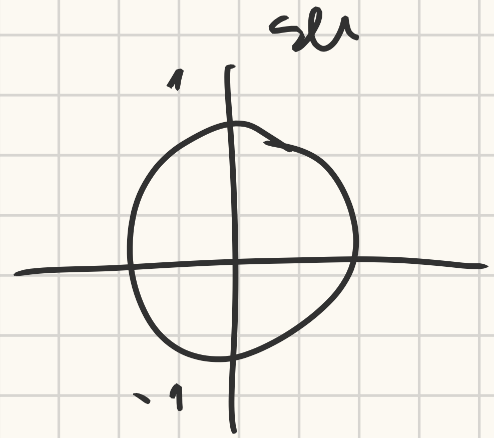
Revisões Cap 1

Ficha 1

(35) $g(x) = \text{arcsen}\left((x-1)^2\right)$



a) $D_g = ?$ $\qquad D_g = \{ x \in \mathbb{R} : -1 \leq (x-1)^2 \leq 1 \}$



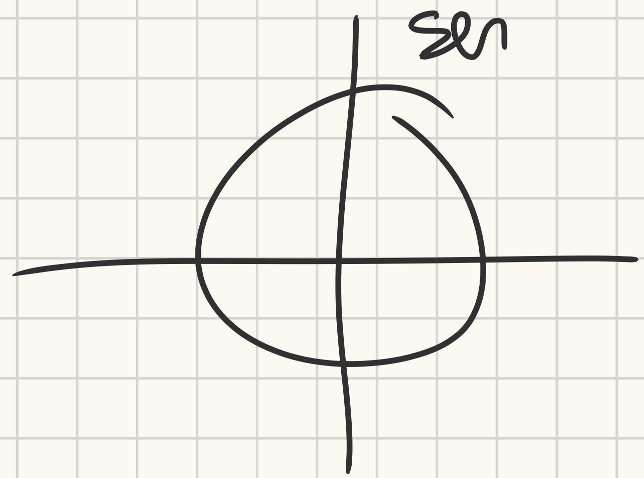
$-1 \leq (x-1)^2$ cond univ $\quad \wedge \quad (x-1)^2 \geq 1 \Leftrightarrow x-1 \geq \pm 1$

$x - 1 \leq 1 \wedge x - 1 \geq -1 \Leftrightarrow x \leq 2 \wedge x \geq 0$

$D_g = [0, 2]$

b) $g(x) = \frac{\pi}{6}$ tem pelo menos 1 solução em $[1, 2]$

$\text{arcsen}\left((x-1)^2\right) = \frac{\pi}{6} \Leftrightarrow \text{arcsen}\left((x-1)^2\right) - \frac{\pi}{6} = 0$



em $x = 1$)

$\text{arcsen}\left((0)^2\right) - \frac{\pi}{6} = -\frac{\pi}{6}$

em $x = 2$)

$\text{arcsen}\left((1)^2\right) - \frac{\pi}{6} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{6}$

como $\left(g(1) - \frac{\pi}{6}\right) . \left(g(2) - \frac{\pi}{6}\right) < 0$ então pelo T. Cauchy existe pelo menos uma solução em $[1,2]$

c) Extremos locais e monotonia?

em $D_g = [0,2]$

$$g'(x) = \arcsen\left((x-1)^2\right)' = \frac{\left((x-1)^2\right)'}{\sqrt{1-(x-1)^2}} = \frac{2x-2}{\sqrt{1-(x-1)^2}}$$

$$\frac{2x-2}{\sqrt{1-(x-1)^2}} = 0 \Leftrightarrow 2x-2=0 \wedge 1-(x-1)^2 > 0 \Leftrightarrow$$

$$\Leftrightarrow x = 1 \wedge (x-1)^2 < 1 \Leftrightarrow$$

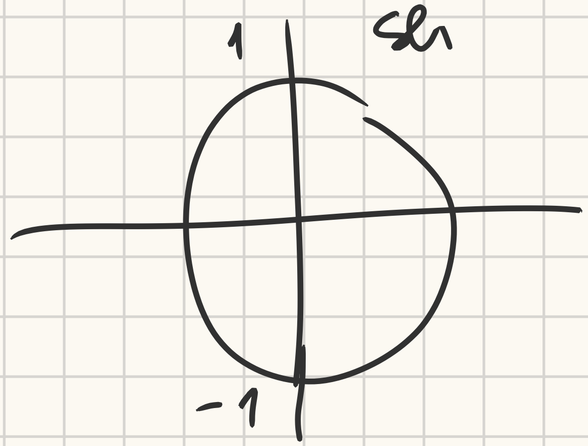
$$\Leftrightarrow x = 1 \wedge x-1 < 1 \wedge x-1 > -1$$

$$\Leftrightarrow x = 1 \wedge x < 2 \wedge x > 0$$

		0		1		2	
$g'(x)$			$-$	0	$+$		
$g(x)$		$\frac{\pi}{2}$	$\searrow$	0	$\nearrow$	$\frac{\pi}{2}$	

$$g'\left(\tfrac{1}{2}\right) = \frac{1-2}{\sqrt{1-\left(-\frac{1}{2}\right)^2}} = \frac{-1}{\sqrt{1-\frac{1}{4}}}$$

$$g'\left(\tfrac{3}{2}\right) = \frac{2\cdot\frac{3}{2}-2}{\sqrt{1-\left(\frac{3}{2}-1\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$$



$$g(0) = \text{arcsen}((0-1)^2) = \text{arcsen}(1) = \frac{\pi}{2}$$

$$g(1) = \text{arcsen}((1-1)^2) = \text{arcsen}(0) = 0$$

$$g(2) = \text{arcsen}((2-1)^2) = \text{arcsen}(1) = \frac{\pi}{2}$$

g é estritamente decrescente em $]0,1[$ e estritamente crescente em $]1,2[$
g tem um mínimo em $=1$, de valor 0

d) g é invertível?

$g(0) = g(2)$ logo g não é injetiva e por isso também não é invertível

Miniteste 1 2024/2025

1. $f(x) = \text{arccotg}(\ln(3x-1))$, D_f e CD_f = ?

$D_f = \{ x \in \mathbb{R} : 3x - 1 > 0 \}$

C.A

$3x - 1 > 0 \Leftrightarrow 3x > 1 \Leftrightarrow x > \frac{1}{3}$ $\qquad D_f =]\frac{1}{3}, +\infty[$

$0 < \text{arccotg}(x) < \pi$ $\qquad CD_f =]0, \pi[$

$0 < \text{arccotg}(3x-1) < \pi$

$0 < \text{arccotg}(\ln(3x-1) < \pi$

② $g(x) = \arccos(x-2) + \text{sen}(3-x)$, $x \in [2, 3]$. Lagrange permite concluir que existe um $c \in]2, 3[$ tal que $g'(c) = ?$

$$g'(c) = \frac{g(3) - g(2)}{3 - 2} = \frac{0 - \frac{\pi}{2} + \text{sen}(1)}{1} = -\frac{\pi + 2\,\text{sen}(1)}{2}$$

$g(2) = \arccos(2-2) + \text{sen}(3-2) = \arccos(0) + \text{sen}(1) = \frac{\pi}{2} + \text{sen}(1)$

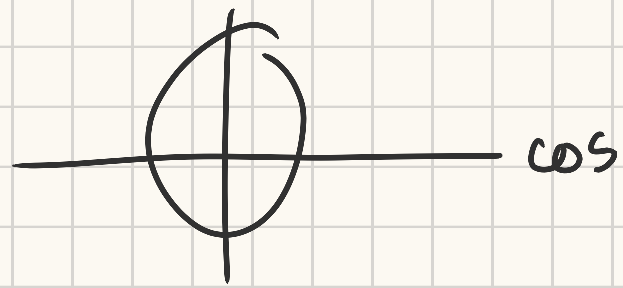
$g(3) = \arccos(3-2) + \text{sen}(3-3) = \arccos(1) + \text{sen}(0) = 0$

(3) Seja h diferenciavel tal que $h'(x) = 1 - e^{x^2 - 4}$. Numero máximo de zeros de h = ?

$$h'(x) = 0 \Leftrightarrow 1 - e^{x^2-4} = 0 \Leftrightarrow e^{x^2-4} = 1 \Leftrightarrow x^2 - 4 = \ln(1) \Leftrightarrow x^2 - 4 = 0$$

$$\Leftrightarrow x^2 = 4 \Leftrightarrow x = 2 \lor x = -2$$

3 zeros no máx



(4) $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x - \operatorname{sen} x}{x^2} = ?$

$$\lim_{x \to 0} \frac{\operatorname{tg}(x) - \operatorname{sen}(x)}{x^2} \overset{\text{Cauchy}}{=} \lim_{x \to 0} \frac{\sec^2 x - \cos x}{2x} \overset{\text{Cauchy}}{=} \lim_{x \to 0} \frac{\left(\frac{1}{\cos^2 x}\right)' + \operatorname{sen} x}{2}$$

$$= \lim_{x \to 0} \frac{-2\cos(x)(-\operatorname{sen} x) + \operatorname{sen} x}{2} = \frac{0}{2} = 0$$

(5) $T_0^2 f(x) = ?$, $f(x) = \cos x$, $\cos(1/2) = ?$

$f(x) = \cos x$ $\quad f(0) = 1$

$f'(x) = -\operatorname{sen} x$ $\quad f'(0) = 0$

$f''(x) = -\cos x$ $\quad f''(0) = -1$

$$T_0^2 f(x) = \frac{1}{0!}(x-0)^0 + \frac{0}{1!}(x-0)^1 + \frac{-1}{2!}(x-0)^2$$

$$= 1 + 0 - \frac{1}{2}x^2 = 1 - \frac{x^2}{2}$$

$\cos x = \cos(1/2) \Leftrightarrow x = 1/2 \rightarrow 1 - \frac{(1/2)^2}{2} = 1 - \frac{(1/4)}{2} = 1 - \frac{1}{8} = \frac{7}{8}$

Teste 1 - 202

(1) a)

$f(a) < 0$ e $f(b) < 0$, $a < b$

se $f(c) > 0$ então existem pelo menos dois zeros em $]a, b$

b) $\varphi(x) = g(1-x^2)$, $g : [0, +\infty[\to \mathbb{R}$ contínua

$D\varphi = \{ x \in \mathbb{R} : 1-x^2 \geq 0 \}$ $D\varphi = [-1, 1]$

CA.

$1-x^2 \geq 0 \Leftrightarrow x^2 \leq 1 \Leftrightarrow x \geq -1 \wedge x \leq 1$

Como $D\varphi$ é fechado e limitado, pelo T. Weierstrass φ tem maximos e mínimos globais nesse domínio

c) $h(x) = (x+1)\operatorname{arcsen}(\sqrt{x} - 1)$, $x \in [0,4]$. $c \in]0,4[$. $h'(c) = ?$

$$h'(c) = \frac{h(4) - h(0)}{4 - 0} = \frac{\frac{5\pi}{2} + \frac{\pi}{2}}{4} = \frac{6\pi}{2} \cdot \frac{1}{4} = \frac{3\pi}{4}$$

$h(4) = 5\operatorname{arcsen}(2-1) = 5\operatorname{arcsen}(1) = \frac{5\pi}{2}$

$h(0) = \operatorname{arcsen}(-1) = -\frac{\pi}{2}$

d) $g(x) = x^{21} - x + a$. Max zeros de g?

$g'(x) = 21x^{20} - 1$: g' tem 2 zeros $\rightarrow$ g tem no max 3

e) $\lim_{x \to 0} (x^4 + 1)^{1/x^2} = ?$

$$\lim_{x\to 0} (x^4+1)^{1/x^2} = \lim_{x\to 0} e^{\ln (x^4+1)^{1/x^2}} = \lim_{x \to 0} e^{\frac{1}{x^2} \ln(x^4+1)} =$$

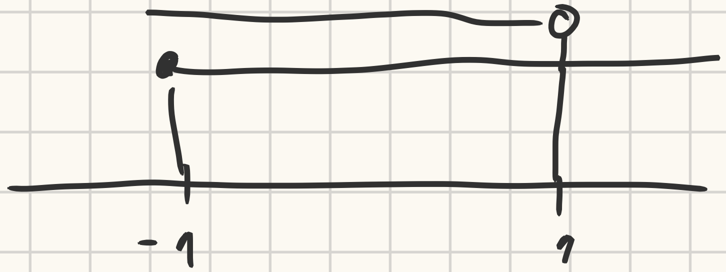
$$= e^{\lim_{x\to 0} \frac{\ln(x^4+1)}{x}} = e^{\lim_{x\to 0}\left(\frac{\frac{4x^3}{x^4+1}}{1}\right)} = e^{\lim_{x\to 0}\left(\frac{4x^3}{x^4+1}\right)} = e^{\frac{0}{1}} = e^0 = \underline{1}$$

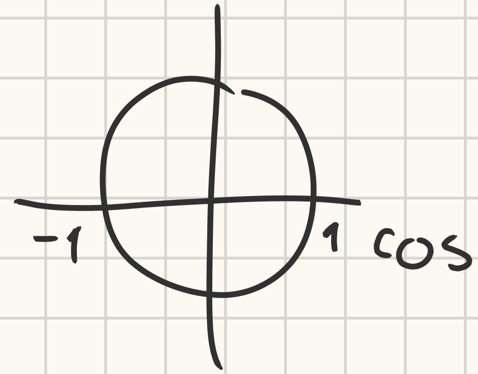
f) $x \in [2,4]$ $\quad G(x) = \int_{2x+1}^{2} \ln t \, dt$. $G'(x) = ?$

$$G'(x) = \ln(2).(2)' - \ln(2x+1)(2x+1)' = 0 - \ln(2x+1).2 = -2\ln(2x+1)$$

② $f(x) = \ln(\arccos(x))$, $f^{-1} = ?$, $D_{f^{-1}} = ?$, $CD_{f^{-1}} = ?$

$D_f = \{ x \in \mathbb{R} : -1 \leq x \leq 1 \wedge \arccos(x) > 0 \}$





C.A

- $x \geq -1$
- $x \leq 1$
- $\arccos(x) > 0 \Leftrightarrow x < \cos(0) \Leftrightarrow x < 1$

$D_f = [-1, 1[= CD_{f^{-1}}$

CD_f)

$]-\infty, 0[< \ln(x) <]0, +\infty[$

$0 \leq \arccos(x) \leq \pi$

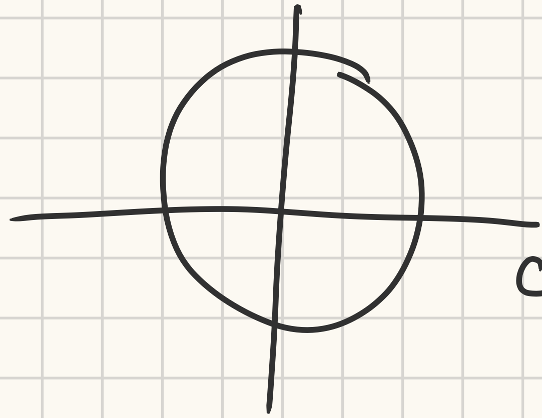
$0 < \ln(\arccos(x)) \leq \ln(\pi)$

$\ln(0)$ até $\ln(\pi)$

$CD_f =]0, \ln(\pi)] = D_{f^{-1}}$

f^{-1}) $y = \ln(\arccos(x)) \Leftrightarrow e^y = \arccos(x) \Leftrightarrow \cos(e^y) = x$

$$f^{-1}(x) = \cos(e^{x})$$



b) reta tangente a f^{-1} em $x = \ln\left(\frac{\pi}{2}\right)$

$$(f^{-1})' = (\cos(e^{x}))' = -e^{x}\operatorname{sen}(e^{x})$$

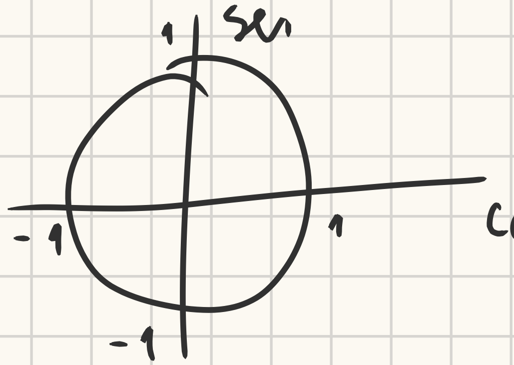
$$(f^{-1})'\left(\ln\frac{\pi}{2}\right) = -e^{\ln\pi/2}\operatorname{sen}\left(e^{\ln\pi/2}\right) = -\frac{\pi}{2}\cdot\operatorname{sen}\left(\frac{\pi}{2}\right) = 1\cdot -\frac{\pi}{2} = -\frac{\pi}{2} = m$$

$$f^{-1}\left(\ln\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$y = mx + b \Leftrightarrow 0 = -\frac{\pi}{2}\ln\left(\frac{\pi}{2}\right) + b$$

$$\Leftrightarrow \frac{\pi}{2}\ln\left(\frac{\pi}{2}\right) = b$$

$$y = \frac{\pi}{2}\left(-x + \ln\left(\frac{\pi}{2}\right)\right) \Leftrightarrow y = \frac{\pi}{2}\left(\ln\left(\frac{\pi}{2}\right) - x\right)$$



③ $g(x) = x - \cos(x)$. $T^3_\pi g = ?$

$$g(x) = x - \cos(x) \qquad g(\pi) = \pi - \cos(\pi) = \pi + 1$$
$$g'(x) = 1 + \operatorname{sen}(x) \qquad g'(\pi) = 1 + \operatorname{sen}(\pi) = 1$$
$$g''(x) = \cos(x) \qquad g''(\pi) = \cos(\pi) = -1$$
$$g'''(x) = -\operatorname{sen}(x) \qquad g'''(\pi) = 0$$

$$T^3_\pi g = \frac{\pi+1}{0!}(x-\pi)^0 + \frac{1}{1!}(x-\pi)^1 + \frac{-1}{2!}(x-\pi)^2 + \frac{0}{3!}(x-\pi)^3$$
$$= \pi + 1 + x - \pi - \frac{1}{2}(x-\pi)^2 = 1 + x - \frac{1}{2}(x-\pi)^2$$

$$g^{IV}(x) = -\cos(x)$$

$$R^3_\pi = \frac{-\cos(\theta)}{4!}(x-\pi)^4$$

$\cos(\pi) = -1$

$x = 3, \quad c = \pi \rightarrow \theta \in \,]3, \pi[$

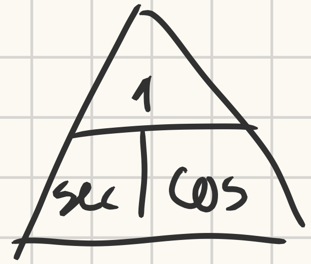
$$\left| R^3_\pi \right| = \left| \frac{\cos(\theta)}{8} \right| \underbrace{\left|(3-\pi)^4\right|}_{>0} = \left| \frac{\cos(\theta)}{8} \right| (3-\pi)^4 < \left| \frac{\cos(\pi)}{8} \right| (3-\pi)^4$$

$$< \left| -\frac{1}{8} \right| (3-\pi)^4$$

$$< \frac{(3-\pi)^4}{8}$$

4. $\displaystyle\int \frac{1}{x^2\sqrt{x^2-3}}\, dx$ (usando $x = \sqrt{3}\sec t$)

$x = \sqrt{3}\sec(t) \Leftrightarrow dx = \sqrt{3}\sec(t).tg(t) \Leftrightarrow \frac{x}{\sqrt{3}} = \sec t \Leftrightarrow$



$$\int \frac{1}{3\sec^2 t\sqrt{3\sec^2 t - 3}} \sqrt{3}\sec(t).tg(t)\, dt =$$

$$= \int \frac{\sqrt{3}\sec(t).tg(t)}{3\sec^2 t\sqrt{3 tg^2 t}}\, dt = \int \frac{\cancel{\sqrt{3}}\,\cancel{\sec(t)}.\cancel{tg(t)}}{3\sec^{\cancel{2}}(t).\cancel{tg(t)}.\cancel{\sqrt{3}}}\, dt = \frac{1}{3}\int \cos(t)\, dt$$

$$= \frac{1}{3}\operatorname{sen}(t) + C = *$$

$$\sec(t) = \frac{x}{\sqrt{3}} \qquad \operatorname{sen}(t) = ?$$

$$\frac{x}{\sqrt{3}} = \frac{1}{\cos t} \Leftrightarrow \cos t = \frac{\sqrt{3}}{x}$$

$$\cos^2 t + \operatorname{sen}^2 t = 1$$

$$\Leftrightarrow \operatorname{sen}^2 t = 1 - \cos^2 t$$

$$\Leftrightarrow \operatorname{sen}^2 t = 1 - \frac{3}{x^2}$$

$$\Leftrightarrow \operatorname{sen} t = \sqrt{\frac{x^2-3}{x^2}} \Leftrightarrow \operatorname{sen} t = \frac{\sqrt{x^2-3}}{x}$$

$$* \frac{1}{3}\operatorname{sen} t + C = \frac{1}{3}\frac{\sqrt{x^2-3}}{x} + C, \quad C \in \mathbb{R}$$

b) $$\int \frac{x-2}{(1+x^2)(x+3)}\, dx$$

$$\frac{x-2}{(1+x^2)(x+3)} = \frac{Ax+B}{(1+x^2)} + \frac{C}{(x+3)} = \frac{A(x^2+3x)+B(x+3)+C(x^2+1)}{(1+x^2)(x+3)}$$

$$= \underline{A+C(x^2)+(B+3A)(x)+(3B+C)(1)}$$

$$\parallel$$

$$\begin{cases} A+C=0 \\ B+3A=1 \\ 3B+C=-2 \end{cases} \quad \begin{cases} A=-C \\ \\ C=-2-3B \end{cases} \quad \begin{cases} A=2+3B \Leftrightarrow A-2=3B \\ \frac{A-2}{3}+3A=1 \\ \end{cases} \quad \begin{cases} 1/2-2=3B \\ A=1/2 \\ \end{cases}$$

$$\frac{A-2}{3}=1-3A \Leftrightarrow A-2=3-9A \Leftrightarrow$$

$$\Leftrightarrow A=3-9A+2 \Leftrightarrow A+9A=5$$

$$\Leftrightarrow 10A=5 \Leftrightarrow A=\frac{1}{2}$$

$$\frac{1}{2}-\frac{4}{2}=3B \Leftrightarrow -\frac{3}{2}=3B$$

$$\Leftrightarrow -\frac{3}{2}\cdot\frac{1}{3}=B \Leftrightarrow B=-\frac{1}{2}$$

$$\begin{cases} B=-1/2 \\ A=1/2 \\ C=-2+\frac{3}{2} \end{cases} \quad \begin{cases} B=-1/2 \\ A=1/2 \\ C=-1/2 \end{cases}$$

$$\int \frac{x-2}{(1+x^2)(x+3)} = \int \frac{\frac{1}{2}(x) + \left(-\frac{1}{2}\right)}{(1+x^2)} + \frac{-\frac{1}{2}}{(x+3)}\, dx$$

$$= \int \frac{x-1}{2} \cdot \frac{1}{(1+x^2)} + \left(-\frac{1}{2}\right) \cdot \frac{1}{x+3}\, dx = \frac{1}{2} \int \frac{x-1}{x^2+1}\, dx - \frac{1}{2} \int \frac{1}{x+3}\, dx$$

$$= \frac{1}{2}\left(\int \frac{x}{x^2+1}\, dx - \int \frac{1}{x^2+1}\, dx - \int \frac{1}{x+3}\, dx \right) = *$$

C.A $\displaystyle\int \frac{x}{x^2+1}\, dx \longrightarrow \int \frac{\sqrt{t-1}}{t} \cdot \frac{1}{2\sqrt{t-1}}\, dt$

$t = x^2+1 \Leftrightarrow \sqrt{t-1} = x$

$\frac{1}{2}(t-1)^{-\frac{1}{2}} = dx$

$\Leftrightarrow \frac{1}{2\sqrt{t-1}}\, dt = dx$

$$= \int \frac{1}{2t}\, dt = \frac{1}{2} \int \frac{1}{t}\, dt = \ln|t| = \frac{1}{2}\ln|x^2+1|$$

$$* \ \frac{1}{2}\left(\frac{1}{2}\ln|x^2+1| - \text{arctg}(x) - \ln|x+3|\right) = -\frac{\ln|x+3|}{2} + \frac{\ln|x^2+1|}{4} - \frac{\text{arctg}(x)}{2}$$

(5) $f: \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = \begin{cases} |\text{arctg}(x)|, & x \leq 0 \\ \text{sen}(2x), & x > 0 \end{cases}$

a) $\int \text{arctg}(x)\, dx = ?$

$$\int \text{arctg}(x)\, dx = \int 1 \cdot \text{arctg}(x) =$$

Substituição

$u = \text{arctg}(x) \qquad dv = 1$

$du = \frac{1}{1+x^2} \qquad v = x$

$$= \text{arctg}(x) \cdot x - \int x \frac{1}{1+x^2}\, dx = \text{arctg}(x) \cdot x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$$

$= \operatorname{arctg}(x) \cdot x - \frac{1}{2} \ln|1+x^2| + C, \quad C \in \mathbb{R}$

b) f é integravel em $[-1, 1]$? $\int_{-1}^{1} f(x) = ?$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} |\operatorname{arctg}(x)| = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \operatorname{sen}(2x) = 0$

f é contínua em $[-1, 1]$, logo é integrável em $[-1, 1]$

$f(0) = |\operatorname{arctg}(0)| = 0$

$$\int_{-1}^{1} f(x) = \int_{-1}^{0} |\operatorname{arctg}(x)| \, dx + \int_{0}^{1} \operatorname{sen}(2x) \, dx$$

$$= \left[\left| \operatorname{arctg}(x) \cdot x - \frac{\ln|1+x^2|}{2} \right| \right]_{-1}^{0} + \frac{1}{2} \left[-\cos(2x) \right]_{0}^{1}$$

$$= \left[\operatorname{arctg}(|x|) \cdot |x| - \frac{\ln(1+x^2)}{2} \right]_{-1}^{0} + \frac{1}{2} \left[-\cos(2x) \right]_{0}^{1}$$

$$= \operatorname{arctg}(0) \cdot 0 - \frac{\ln(1)}{2} - \left(\operatorname{arctg}(|-1|) \cdot |-1| - \frac{\ln(1+(-1)^2)}{2} \right) + \frac{1}{2}\left(-\cos(2) - (-\cos(0)) \right)$$

$$= 0 - \left(\operatorname{arctg}(1) \cdot 1 - \frac{\ln(2)}{2} \right) + \frac{1}{2}\left(-\cos(2) + 1 \right) = -\left(\frac{\pi}{4} - \frac{\ln(2)}{2} \right) - \frac{\cos(2)+1}{2}$$

$$= \frac{\pi}{4} + \frac{\ln(2)}{2} - \frac{\cos(2)+1}{2} = \frac{\pi}{4} + \frac{2\ln(2)}{4} - \frac{2\cos(2)+2}{4} = \frac{\pi + \ln(4) - 2\cos(2)+2}{4} + c$$

$, c \in \mathbb{R}$

(6) $f(x) = e^{2x}$, $g(x) = e^{-x}$. Esboce região delimitada por $f, g, x = -\frac{1}{2}, x = 1$, 0

$f(1) = e^2$ $\quad g(1) = e^{-1} = \frac{1}{e} > \frac{1}{3}$

$f(1/2) = e^{2/2} = e^1 = e$ $\quad g(1/2) = e^{-1/2} = \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{3}} > \frac{1}{3}$

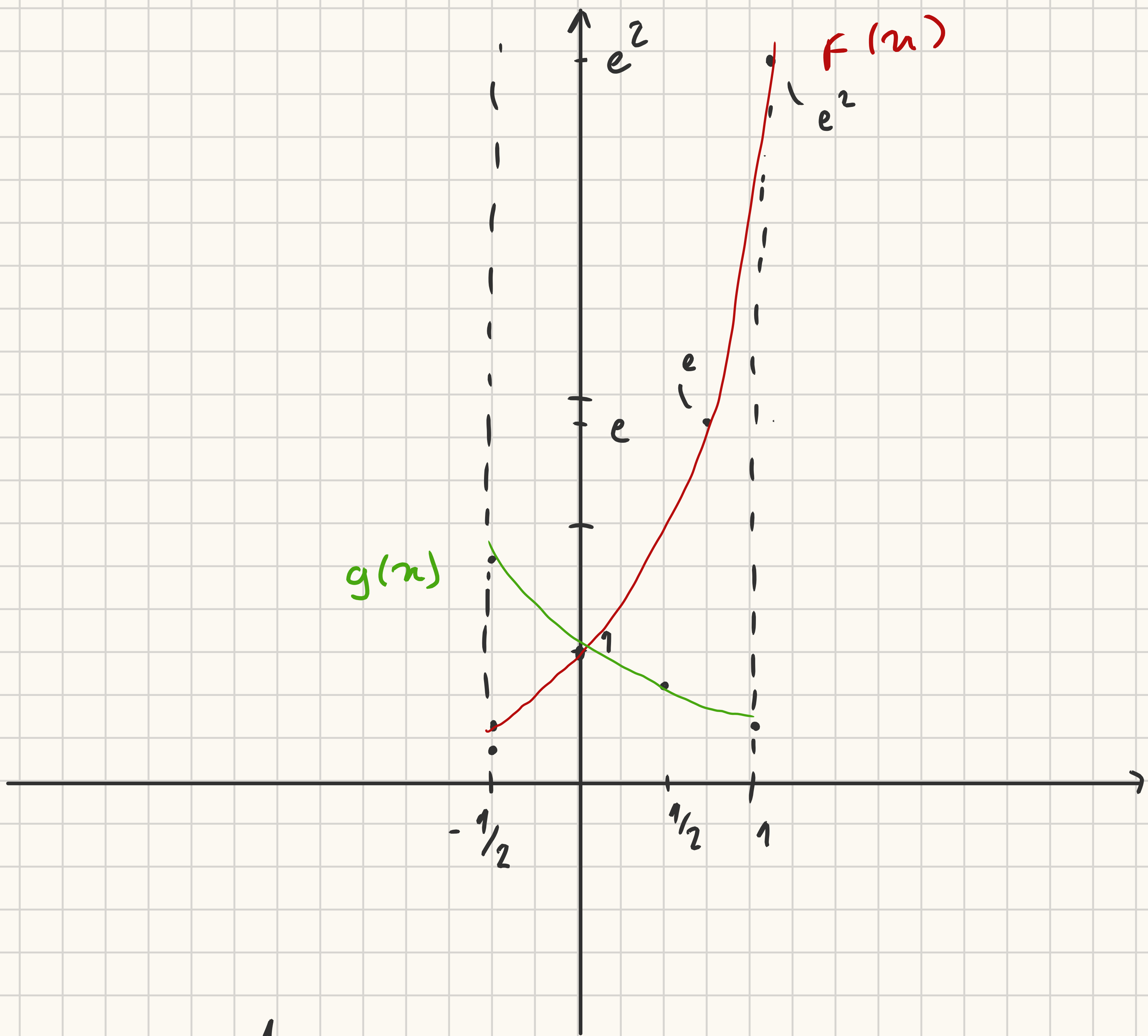
$f(0) = e^0 = 1$ $\quad g(0) = e^{-0} = 1$

$f(-1/2) = e^{-1/2 \cdot 2} = e^{-1} = \frac{1}{e}$ $\quad g(-1/2) = e^{1/2} = \sqrt{e}$

$e^{2x} = e^{-x} \Leftrightarrow 2x = -x$

$\Leftrightarrow 2x + x = 0 \Leftrightarrow 3x = 0$

$\Leftrightarrow x = 0$



$$\int_{-1/2}^{0} g(x) - f(x) \; + \int_{0}^{1} f(x) - g(x)$$

$$= \int_{-1/2}^{0} e^{-x} - e^{2x}\,dx + \int_{0}^{1} e^{2x} - e^{-x}\,dx =$$

$$- \int_{-1/2}^{0} -e^{-x}\,dx - \frac{1}{2}\int_{-1/2}^{0} 2e^{2x}\,dx + \frac{1}{2}\int_{0}^{1} 2e^{2x}\,dx - \int_{0}^{1} -e^{-x}\,dx$$

$$= -\left[e^{-x}\right]_{-1/2}^{0} - \frac{1}{2}\left[e^{2x}\right]_{-1/2}^{0} + \frac{1}{2}\left[e^{2x}\right]_{0}^{1} - \left[e^{-x}\right]_{0}^{1}$$

$$-\left(e^{-0} - e^{+1/2}\right) - \frac{1}{2}\left(e^{2\cdot 0} - e^{2\cdot -\frac{1}{2}}\right) + \frac{1}{2}\left(e^{2\cdot 1} - e^{2\cdot 0}\right) - \left(e^{-1} - e^{-0}\right)$$

$$= -\cancel{1} + \sqrt{e} - \frac{1}{2} + \cancel{\frac{1}{e}} + \frac{1}{2}e^{-1} + \frac{1}{2}e^{2} - 1 - e^{-1} + \cancel{1}$$

$$= e^{1/2} + \frac{e^{-1}}{2} + \frac{e^{2}}{2} - e^{-1} - \frac{1}{2} - 1 = \frac{2\sqrt{e} + e^{-1} + e^{2} - 2e^{-1} - 3}{2} = \frac{2\sqrt{e} + e^{2} - 2e^{-1} - 3}{2e}$$

$= \quad 2$

$$\lim (\ldots) = \lim e^{\ln(\ldots)} = e^{\lim \ln(\ldots)}$$

o) $\displaystyle\int \frac{1}{x^3+8}\,dx = \int \frac{1}{(x-(-2))(x^2-2x+4)}$

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & 8 \\ -2 & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$$\frac{1}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2-2x+4)} = \frac{A(x^2-2x+4)+B(x^2+2x)+C(x+2)}{(x+2)(x^2-2x+4)}$$

$$\begin{cases} A + B = 0 \\ -2A + 2B + C = 0 \\ 4A + 2C = 1 \end{cases} \qquad \begin{cases} A = -B \\ -2A - 2A = -C \\ \end{cases} \qquad \begin{cases} -4A = -C \\ 3C = 1 \end{cases} \qquad \begin{cases} -4A = -1/3 \\ C = 1/3 \end{cases} \qquad \begin{cases} B = -1/12 \\ A = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12} \\ C = 1/3 \end{cases}$$

$$\int \frac{1}{(x+2)(x^2-2x+4)}\,dx = \int \frac{1/12}{x+2}\,dx + \int \frac{(-1/12)x + 1/3}{(x^2-2x+4)}\,dx$$

$$= \frac{1}{12}\int \frac{1}{x+2}\,dx \quad - \frac{1}{12}\int \frac{x - 4}{x^2-2x+4}\,dx$$

$$= \frac{1}{12}\ln|x+2| \quad - \frac{1}{12}\left(\int \frac{x}{x^2-2x+4}\,dx - \int \frac{4}{x^2-2x+4}\,dx \right)$$

$$= \quad // \quad -$$