

Teste 1 - 2024

1

a) $]a, c[$, $f(a) < 0$, $f(c) > 0$

$\exists n_0 \in]a, c[$ tal que $f(n_0) = 0$

2

$]c, b[$, $f(c) > 0$, $f(b) < 0$

$\exists y_0 \in]c, b[: f(y_0) = 0$

b) $g: [0, +\infty[\rightarrow \mathbb{R}$ e $\varphi: D\varphi \rightarrow \mathbb{R}$ $\varphi(u) = g(1-u^2)$

$$f(u) = 1-u^2 \quad \varphi(u) = (g \circ f)(u)$$

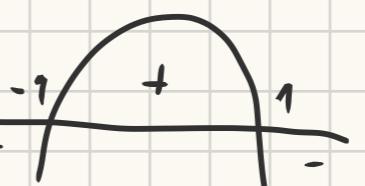
$$D\varphi = \{ u \in \mathbb{R} : 1-u^2 \in [0, +\infty[\}$$

$$= \{ u \in \mathbb{R} : 1-u^2 \geq 0 \} = [-1, 1] \quad \text{Pelo T. Weierstrass } \varphi \text{ tem máx e min globais no Domínio}$$

C.A

$$1-u^2 \geq 0 \Leftrightarrow u^2 \leq 1$$

$$u = \pm 1$$

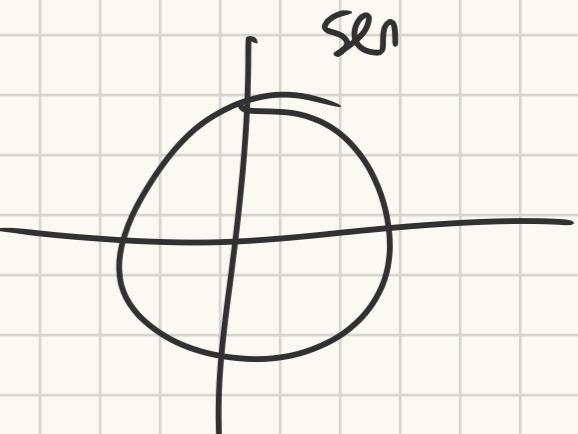


(por ser fechado e limitado)

c) $h(n) = (n+1) \arcsen(\sqrt{n} - 1)$, $n \in [0, 4]$ T Lagnrange existe um c $\in]0, 4[$ tal que:

$$h'(n) = \frac{h(4) - h(0)}{4 - 0} = \frac{5 \arcsen(1) - \arcsen(-1)}{4} =$$

$$= \frac{5 \frac{\pi}{2} + \frac{\pi}{2}}{4} = \frac{6 \frac{\pi}{2}}{4} = \frac{3\pi}{4}$$



d) $g(n) = n^{21} - n + a$, $a \in \mathbb{R}$. Número máx de zeros?

$$g'(n) = 21n^{20} - 1$$

$21n^{20} - 1 = 0 \Rightarrow n^{20} = \frac{1}{21}$ tem 2 soluções \rightarrow Pelo T. Rolle entre dois zeros de g' há no max um zero de g

No máx g tem 3 zeros

e) $\lim_{n \rightarrow 0} (n^4 + 1)^{1/n^2} = ?$

$$\lim_{n \rightarrow 0} (n^4 + 1)^{1/n^2} = \lim_{n \rightarrow 0} e^{\ln((n^4 + 1)^{1/n^2})} = \lim_{n \rightarrow 0} e^{\frac{1}{n^2} \ln(n^4 + 1)} = e^{\lim_{n \rightarrow 0} \frac{\ln(n^4 + 1)}{n^2}}$$

R. Cauchy

$$= e^{\lim_{n \rightarrow 0} \left(\frac{\frac{4n^3}{n^4 + 1}}{2n} \right)} = e^{\lim_{n \rightarrow 0} \left(\frac{2n^2}{n^4 + 1} \right)} = e^0 = 1$$

$\lim_{n \rightarrow 0} (\cos(2n))^{1/n} = \lim_{n \rightarrow 0} e^{\ln(\cos(2n))^{1/n}} = \lim_{n \rightarrow 0} e^{\frac{\ln(\cos(2n))}{n}}$

$$= e^{\lim_{n \rightarrow 0} \frac{\ln(\cos(2n))}{n}} \quad \text{R. Cauchy} \quad = e^{\lim_{n \rightarrow 0} \left(\frac{\frac{-2\sin(2n)}{\cos(2n)}}{1} \right)} = e^{\lim_{n \rightarrow 0} (-2\tg(2n))} = e^0 = 1$$

Deve aparecer em escolha multipla

f) $x \in [2, 4]$, $g(x) = \int_{2n+1}^x \ln t \, dt$, $g'(x) = ?$

f. Fundamental do Cálculo.

$$g'(x) = g(2) \cdot (2)' - g(2n+1) \cdot (2n+1)' =$$

$$= \ln(2) \cdot 0 - \ln(2n+1) \cdot 2 = \cancel{-2\ln(2n+1)} = g'(u)$$

T. Fermat

f diferenciável

- se c é extremante (maximizante ou minimizante) então $f'(c) = 0$
- Mas $f'(c) = 0 \not\Rightarrow c$ é extremante

Ex: $f(n) = n^3$

$f'(n) = 0 \Leftrightarrow 3n^2 = 0 \Leftrightarrow n = 0$ Mas 0 não é extremante de $f(n)$

$$(\sin^2(n))' = 2\sin(n) \cdot \cos(n) = \sin(2n)$$

Exs de revisão

① $f(n) = \arccos(1 - e^n) + \pi$

a) $D_f = ?$ $D_f = \{ n \in \mathbb{R} : -1 \leq 1 - e^n \leq 1 \}$ $D_f =]-\infty, \ln(2)]$

CA

- $1 - e^n \geq -1 \Leftrightarrow -e^n \geq -2 \Leftrightarrow e^n \leq 2 \Leftrightarrow n \leq \ln(2)$

- $1 - e^n \leq 1 \Leftrightarrow -e^n \leq 0 \Leftrightarrow e^n \geq 0$ cond. universal

b) Monotonia e extremos?

$$f'(n) = -\frac{(1-e^n)'}{\sqrt{1-(1-e^n)^2}} = \frac{e^n}{\sqrt{1-(1-e^n)^2}} > 0 \quad \forall n \in D_f \rightarrow \begin{array}{l} f \text{ estrita} \\ \text{crescente} \end{array}$$

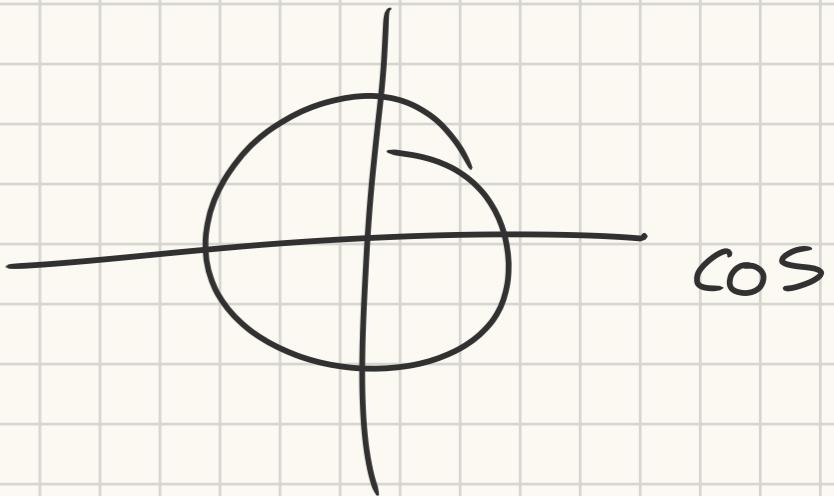
$$\begin{aligned} f(\ln(2)) &= \arccos(1 - e^{\ln(2)}) + \pi = \arccos(1 - 2) + \pi = \arccos(-1) + \pi \\ &= \pi + \pi = 2\pi \end{aligned}$$

($\ln(2)$ é máximo, 2π é o máximo absoluto)

c) Characterizar $f^{-1}(u)$

$$D_f = CD_f^{-1} =]-\infty, \ln(2)]$$

$$CD_f = D_f^{-1}$$



CD_f)

$$0 \leq \arccos(u) \leq \pi$$

$$0 < \arccos(1-e^x) \leq \pi$$

$$\pi < \arccos(1-e^x) + \pi \leq 2\pi$$

$$CD_f = [\pi, 2\pi] = D_f^{-1}$$

$$y = \arccos(1-e^x) + \pi \Leftrightarrow y - \pi = \arccos(1-e^x) \Leftrightarrow$$

$$\Leftrightarrow \cos(y - \pi) = 1 - e^x \Leftrightarrow \cos(y - \pi) + 1 = e^x \Leftrightarrow (\ln(\cos(y - \pi) + 1)) = x$$

$$f^{-1}(u) = \ln(\cos(u - \pi) + 1)$$

② Determinar f tal que $f'(u) = \frac{3\cos(\ln u)}{u}$ e $f(1) = 2$

$$\int \frac{3\cos(\ln(u))}{u} du = 3 \int \frac{1}{u} \cos(\ln(u)) du = 3\sin(\ln(u)) + C$$

$$f(1) = 2 \Rightarrow 3\sin(\ln(1)) + C = 2 \Rightarrow 3\sin(0) + C = 2$$

$$\Rightarrow C = 2$$

$$f(u) = 3\sin(\ln(u)) + 2$$

③

$$a) \int \arcsin(2u) du = \arcsin(2u) \cdot u - \int u \cdot \frac{2}{\sqrt{1-(2u)^2}} du$$

$$v = \arcsin(2u) \quad dv = 1$$

$$du = \frac{2}{\sqrt{1-(2u)^2}} \quad v = u$$

$$= u \arcsin(2u) - 2 \int \frac{u}{\sqrt{1-4u^2}} du$$

$$\begin{aligned}
 &= u \arcsin(2u) - 2 \int u (1-4u^2)^{-\frac{1}{2}} = u \arcsin(2u) + \frac{2}{8} \int -8u (1-4u^2)^{-\frac{1}{2}} \\
 &= u \arcsin(2u) + \frac{1}{4} \left(\frac{(1-4u^2)^{\frac{1}{2}}}{\frac{1}{2}} \right) = u \arcsin(2u) + \frac{\sqrt{1-4u^2}}{2} + C, C \in \mathbb{R}
 \end{aligned}$$

b) $\int \frac{1}{u^2 \sqrt{u^2+4}} du$, usar $u = 2\tan(t)$

$$\begin{aligned}
 &\int \frac{1}{u^2 \sqrt{u^2+4}} du \\
 &u = 2\tan(t) \Leftrightarrow \arctan\left(\frac{u}{2}\right) = t, t \in]0, \frac{\pi}{2}[\\
 &du = 2\sec^2(t) dt
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{2\sec^2(t)}{4\tan^2 t \sqrt{4\tan^2 t + 4}} dt = \int \frac{2\sec^2(t)}{4\tan^2 t \sqrt{4\sec^2 t}} dt = \int \frac{2\sec^2 t}{4\tan^2 t \cdot 2\sec t} dt
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{2\sec^2 t}{8\tan^2 t \cdot \sec t} = \frac{1}{4} \int \frac{\sec t}{\tan^2 t} dt =
 \end{aligned}$$

$$= \frac{1}{4} \int \frac{1}{\cos t} \cdot \frac{\cos^2 t}{\sin^2 t} dt = \frac{1}{4} \int \frac{\cos t}{\sin^2 t} dt =$$

$$= \frac{1}{4} \int \cot g t \cosec t dt = -\frac{1}{4} \cosec t + C$$

$$= -\frac{1}{4} \cosec(\arctg(\frac{n}{2})) + C, C \in \mathbb{R}$$

CA OU *

$$\cdot \tg^2 t + 1 = \sec^2 t$$

$$\frac{u}{2} = \tg t \Leftrightarrow \cot g t = \frac{2}{u}$$

$$\cdot \cot g^2 t + 1 = \cosec^2 t$$

$$\cot g^2 t + 1 = \cosec^2 t$$

$$\left(\frac{2}{u}\right)^2 + 1 = \cosec^2 t \Leftrightarrow \frac{4}{u^2} + 1 = \cosec^2 t \Leftrightarrow \cosec^2 t = \frac{4+u^2}{u^2}$$

$$\Leftrightarrow \cosec t = \sqrt{\frac{4+n^2}{n^2}}$$

$$* -\frac{1}{4} \cdot \sqrt{\frac{4+n^2}{n^2}} + C, C \in \mathbb{R}$$

④

$f(n) = e^{2n}$. Polinomio de MacLaurin de orden 2 para determinar uma aproximação de \sqrt{e}

$$f(0) = 1$$

$$f'(n) = 2e^{2n}$$

$$f''(n) = 4e^{2n}$$

$$f'(0) = 2$$

$$f''(0) = 4$$

$$\begin{aligned} T_0^2 f(n) &= \frac{1}{0!} (n-0)^0 + \frac{2}{1!} (n-0)^1 + \frac{4}{2!} (n-0)^2 \\ &= 1 + 2n + 2n^2 \end{aligned}$$

$$e^{2n} = \sqrt{e} \Leftrightarrow 2n = \frac{1}{2} \Leftrightarrow n = \frac{1}{4}$$

$$\text{Se } f(n) = e^{2n} \rightarrow \sqrt{e} = f\left(\frac{1}{4}\right)$$

$$\begin{aligned} T_0^2 f\left(\frac{1}{4}\right) &= 1 + 2 \cdot \frac{1}{4} + 2 \cdot \left(\frac{1}{4}\right)^2 = \frac{3}{2} + \frac{2}{16} = \frac{3}{2} + \frac{1}{8} = \frac{12}{8} + \frac{1}{8} \\ &= \frac{13}{8} \end{aligned}$$