Part 2: Numbers at Different Bases

Dr. Andi Toce
Lecture Notes for MAC 101 (Introduction to Computer Science)

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1. Computer Language and Storage

The smallest unit of information storage in a computer is a **bit**. A bit can only take values of 0 (zero) or 1 (one). All information in a computer is stored as a sequence of bits.

Bit Multiples:

1 Byte = 8 bits
1 Kilobyte = a thousand bytes = 10^3 bytes
1 Megabyte = a million bytes = 10^6 bytes
1 Gigabyte = a billion bytes = 10^9 bytes
1 Terabyte = a trillion bytes = 10^{12} bytes

2. Converting Binary, Octal and Hexadecimal to Decimal

Numbers at different bases (decimal, binary, octal, hexadecimal)

The base of a number system corresponds to the digits used by the system.

Base	Digits used	Name	Example	Decimal Equivalent
2	0,1	Binary	101_2	5
8	0,1,2,3,4,5,6,7	Octal	46_8	38
10	0,1,2,3,4,5,6,7,8,9	Decimal	109_{10}	109
16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	Hexadecimal	$2A3_{16}$	675

Understanding how numbers of different bases work.
We commonly use base-10 or decimal numbers.

Consider the decimal number 2365

Thousands	Hundreds	Tens	Ones
10^3	10^2	10^1	10^0
2	3	6	5

Finding the value

$2 * 10^3$	2000
$3 * 10^2$	300
$6 * 10^1$	60
$5 * 10^0$	5
Total	**2365**

Consider the binary number 1001101_2. Find the equivalent decimal number.

	1	0	0	1	1	0	1		
	2^6	2^5	2^4	2^3	2^2	2^1	2^0		
Product	64	0	0	8	4	0	1	**77**	**Sum**

Consider the octal number 427_8. Find the equivalent decimal number.

	4	2	7		
	8^2	8^1	8^0		
Product	256	16	7	**279**	**Sum**

Try now: Convert 111001_2 to a decimal number.

Try now: Convert 3771_8 to a decimal number.

Try now: Convert $2B5_{16}$ to a decimal number.

3. Converting Decimal Numbers to Other Bases

To convert a decimal to any other base we divide the number by the base repeatedly and keep record of the remainder. The remainder list is the new number. This process is better described via examples.

Convert 43_{10} to binary

Division by the base=2	Quotient	Remainder
43 / 2	21	1
21 / 2	10	1
10 / 2	5	0
5 / 2	2	1
2 / 2	1	0
1 / 2	0	1

Take the binary digits in reverse order: $\mathbf{43_{10} = 101011_2}$

Try Now: Convert 87_{10} to binary.

Convert 342_{10} to the octal equivalent.

Division by the base=8	Quotient	Remainder
342/8	42	6
42/8	5	2
5/8	0	5

342_{10} is equivalent to 526_8

Try Now: Convert 189_{10} to the octal equivalent.

Try Now: Convert 542_{10} to the corresponding Hexadecimal number.

4. Conversion Between Binary, Octal and Hexadecimal

Conversion between binary, octal and hexadecimal is easier. Can you tell why?

Convert 1011010_2 to octal and hexadecimal

Octal

1	011	010
1	3	2

Hexadecimal

101	1010
5	A

$1011010_2 = 132_8 = 5A_{16}$

Convert $E4A_{16}$ to binary

E	4	A
1110	0100	1010

$E4A_{16} = 111001001010_2$

Try Now: Convert 3617_8 to binary and hexadecimal

Try Now: Convert 1110001_2 to octal and hexadecimal

5. Addition and Subtraction of Binary Numbers

Binary Addition
Reflect on the addition of decimal numbers

Try Now: $3524_{10} + 692_{10}$

Now lets add $1011101_2 + 10110_2$

Carry		1	1	1			
First Number	1	0	1	1	1	0	1
Second Number			1	0	1	1	0
Answer	1	1	1	0	0	1	1

Try Now: Add 1100110_2 and 1111010_2

Binary Subtraction
First reflect on decimal subtraction. Do $1932_{10} - 281_{10}$

Now we subtract 11001_2 from 1110011_2

Borrowed			2	2			
First Number	1	~~1~~ 0	~~1~~ 0	0	0	1	1
Second Number			1	1	0	0	1
Answer	1	0	1	1	0	1	0

Now Try: $1101001_2 - 101110_2$

6. Addition and Subtraction – Other Bases

Try Now: $35A_{16} + E7_{16}$

Try Now: $35A_{16} - E7_{16}$

Try Now: $3527_8 + 726_8$

Try Now: $3527_8 - 726_8$