In [1]:
```
import pandas as pd
import numpy as np

import matplotlib.pyplot as plt
import seaborn as sns

import statsmodels.formula.api as smf
from statsmodels.graphics.regressionplots import influence_plot
```

In [2]:
```
import warnings
warnings.filterwarnings('ignore')
```

In [3]:
```
#Read the data
cars = pd.read_csv("Cars.csv")
cars.head()
```

Out[3]:

	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149

Correlation Matrix

In [4]:
```
cars.corr()
```

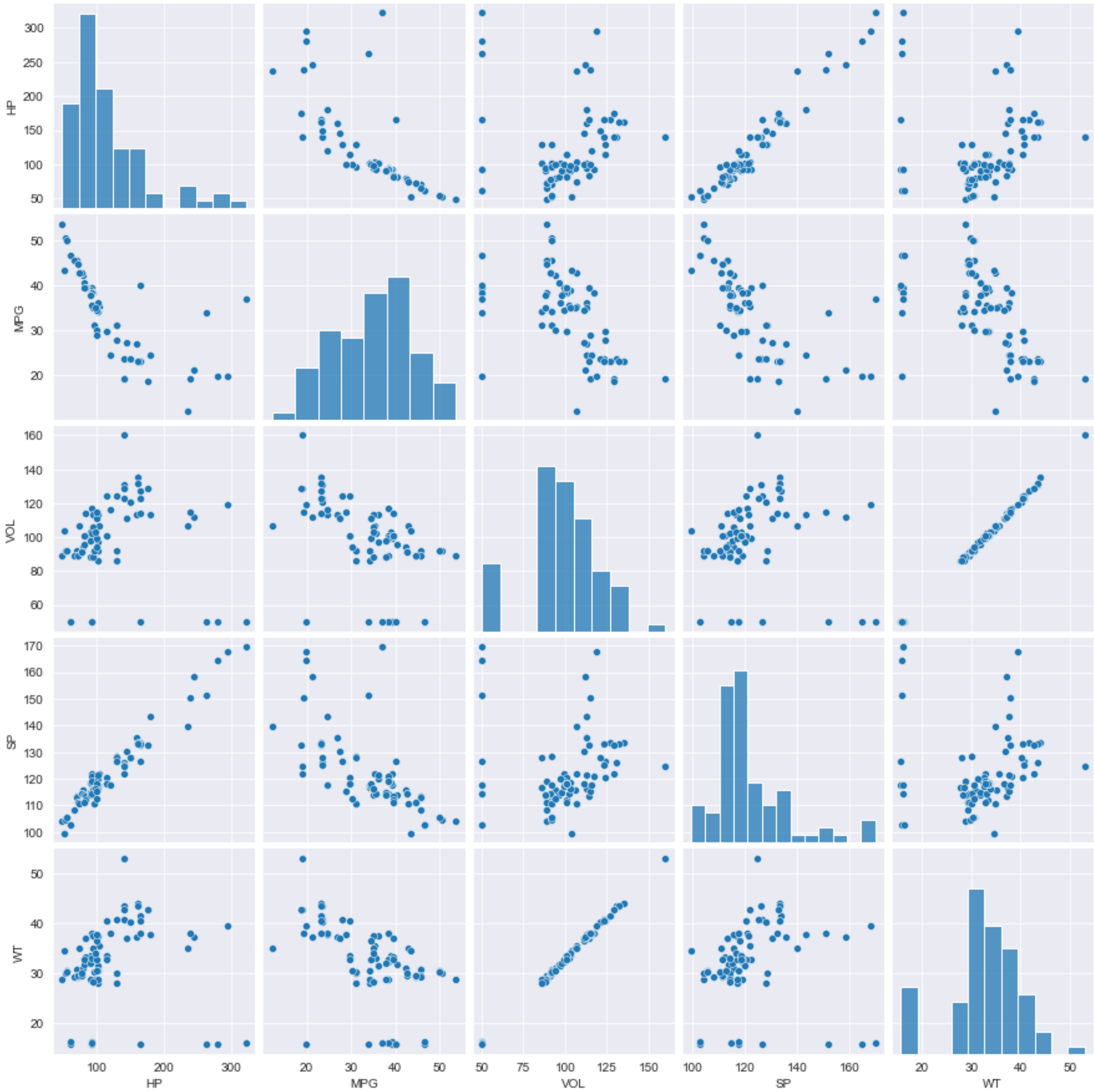
Out[4]:

	HP	MPG	VOL	SP	WT
HP	1.000000	-0.725038	0.077459	0.973848	0.076513
MPG	-0.725038	1.000000	-0.529057	-0.687125	-0.526759
VOL	0.077459	-0.529057	1.000000	0.102170	0.999203
SP	0.973848	-0.687125	0.102170	1.000000	0.102439
WT	0.076513	-0.526759	0.999203	0.102439	1.000000

Scatterplot between variables along with histograms

In [5]:
```
#Format the plot background and scatter plots for all the variables
sns.set_style(style='darkgrid')
sns.pairplot(cars)
```

Out[5]:
```
<seaborn.axisgrid.PairGrid at 0x7fe4193a8340>
```



Preparing a model

In [6]:
```
#Build model
import statsmodels.formula.api as smf
model = smf.ols('MPG~WT+VOL+SP+HP',data=cars).fit()
```

In [7]:
```
model.summary()
```

Out[7]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.758
Method:	Least Squares	**F-statistic:**	63.80
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	1.54e-23
Time:	20:29:08	**Log-Likelihood:**	-233.96
No. Observations:	81	**AIC:**	477.9
Df Residuals:	76	**BIC:**	489.9
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	30.6773	14.900	2.059	0.043	1.001	60.354
WT	0.4006	1.693	0.237	0.814	-2.972	3.773
VOL	-0.3361	0.569	-0.591	0.556	-1.469	0.796
SP	0.3956	0.158	2.500	0.015	0.080	0.711
HP	-0.2054	0.039	-5.239	0.000	-0.284	-0.127

Omnibus:	10.780	**Durbin-Watson:**	1.403
Prob(Omnibus):	0.005	**Jarque-Bera (JB):**	11.722
Skew:	0.707	**Prob(JB):**	0.00285
Kurtosis:	4.215	**Cond. No.**	6.09e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.09e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [8]:
```
#Coefficients
model.params
```

Out[8]:
```
Intercept    30.677336
WT            0.400574
VOL          -0.336051
SP            0.395627
HP           -0.205444
dtype: float64
```

In [9]:
```
#t and p-Values
print(model.tvalues, '\n', model.pvalues)
```

```
Intercept    2.058841
WT           0.236541
VOL         -0.590970
SP           2.499880
HP          -5.238735
dtype: float64
 Intercept    0.042936
WT           0.813649
VOL          0.556294
SP           0.014579
HP           0.000001
dtype: float64
```

In [10]:

```
#R squared values
(model.rsquared,model.rsquared_adj)
```

Out[10]:

```
(0.7705372737359842, 0.7584602881431413)
```

Simple Linear Regression Models

In [11]:

```
ml_v=smf.ols('MPG~VOL',data = cars).fit()
#t and p-Values
print(ml_v.tvalues, '\n', ml_v.pvalues)
ml_v.summary()
```

```
Intercept    14.106056
VOL          -5.541400
dtype: float64
 Intercept    2.753815e-23
VOL          3.822819e-07
dtype: float64
```

Out[11]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.280
Model:	OLS	**Adj. R-squared:**	0.271
Method:	Least Squares	**F-statistic:**	30.71
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	3.82e-07
Time:	20:29:08	**Log-Likelihood:**	-280.28
No. Observations:	81	**AIC:**	564.6
Df Residuals:	79	**BIC:**	569.4
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	55.8171	3.957	14.106	0.000	47.941	63.693
VOL	-0.2166	0.039	-5.541	0.000	-0.294	-0.139

Omnibus:	2.691	**Durbin-Watson:**	0.566
Prob(Omnibus):	0.260	**Jarque-Bera (JB):**	1.997
Skew:	-0.263	**Prob(JB):**	0.368
Kurtosis:	3.562	**Cond. No.**	462.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [12]:

```
ml_w=smf.ols('MPG~WT',data = cars).fit()
print(ml_w.tvalues, '\n', ml_w.pvalues)
ml_w.summary()
```

```
Intercept    14.248923
WT           -5.508067
dtype: float64
 Intercept    1.550788e-23
WT           4.383467e-07
dtype: float64
```

Out[12]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.277
Model:	OLS	**Adj. R-squared:**	0.268
Method:	Least Squares	**F-statistic:**	30.34
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	4.38e-07
Time:	20:29:08	**Log-Likelihood:**	-280.42
No. Observations:	81	**AIC:**	564.8
Df Residuals:	79	**BIC:**	569.6
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	55.2296	3.876	14.249	0.000	47.514	62.945
WT	-0.6420	0.117	-5.508	0.000	-0.874	-0.410

Omnibus:	2.735	**Durbin-Watson:**	0.555
Prob(Omnibus):	0.255	**Jarque-Bera (JB):**	2.045
Skew:	-0.263	**Prob(JB):**	0.360
Kurtosis:	3.573	**Cond. No.**	149.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [13]:

```
ml_wv=smf.ols('MPG~WT+VOL',data = cars).fit()
print(ml_wv.tvalues, '\n', ml_wv.pvalues)
ml_wv.summary()
```

```
Intercept    12.545736
WT            0.489876
VOL          -0.709604
dtype: float64
 Intercept    2.141975e-20
WT           6.255966e-01
VOL          4.800657e-01
dtype: float64
```

Out[13]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.282
Model:	OLS	**Adj. R-squared:**	0.264
Method:	Least Squares	**F-statistic:**	15.33
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	2.43e-06
Time:	20:29:08	**Log-Likelihood:**	-280.16
No. Observations:	81	**AIC:**	566.3
Df Residuals:	78	**BIC:**	573.5
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	56.8847	4.534	12.546	0.000	47.858	65.912
WT	1.4349	2.929	0.490	0.626	-4.397	7.266
VOL	-0.6983	0.984	-0.710	0.480	-2.658	1.261

Omnibus:	2.405	**Durbin-Watson:**	0.591
Prob(Omnibus):	0.300	**Jarque-Bera (JB):**	1.712
Skew:	-0.251	**Prob(JB):**	0.425
Kurtosis:	3.506	**Cond. No.**	597.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Calculating VIF

In [14]:

```
rsq_hp = smf.ols('HP~WT+VOL+SP',data=cars).fit().rsquared
vif_hp = 1/(1-rsq_hp) # 16.33

rsq_wt = smf.ols('WT~HP+VOL+SP',data=cars).fit().rsquared
vif_wt = 1/(1-rsq_wt) # 564.98

rsq_vol = smf.ols('VOL~WT+SP+HP',data=cars).fit().rsquared
vif_vol = 1/(1-rsq_vol) #  564.84

rsq_sp = smf.ols('SP~WT+VOL+HP',data=cars).fit().rsquared
vif_sp = 1/(1-rsq_sp) #  16.35

# Storing vif values in a data frame
d1 = {'Variables':['Hp','WT','VOL','SP'],'VIF':[vif_hp,vif_wt,vif_vol,vif_sp]}
Vif_frame = pd.DataFrame(d1)
Vif_frame
```

Out[14]:

	Variables	VIF
0	Hp	19.926589
1	WT	639.533818
2	VOL	638.806084
3	SP	20.007639

Residual Analysis

Test for Normality of Residuals (Q-Q Plot)

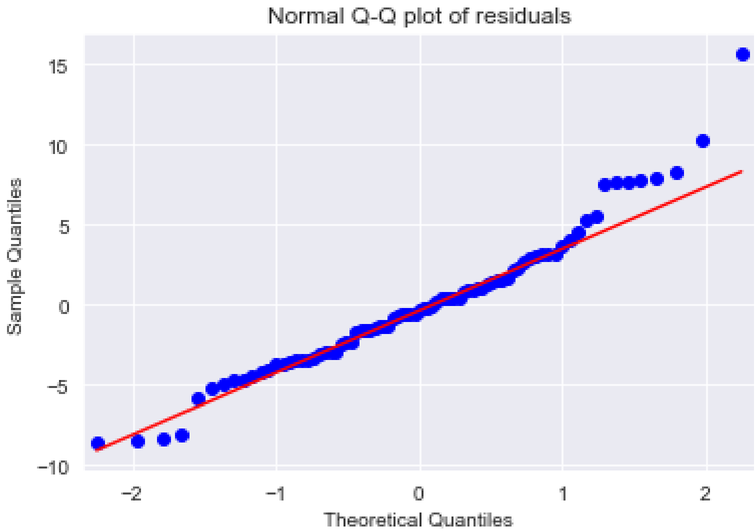
In [15]:
```
model.resid.min()
```

Out[15]:
```
-8.631961053868388
```

In [61]:
```
model.resid
```

Out[61]:
```
0     10.258747
1      7.624608
2      7.734060
3      3.157963
4      8.331584
        ...
76    15.617904
77     1.298838
78     7.863547
79     7.517122
80    -3.458218
Length: 81, dtype: float64
```

In [16]:
```
import statsmodels.api as sm
qqplot=sm.qqplot(model.resid,line='q') # line = 45 to draw the diagnoal line
plt.title("Normal Q-Q plot of residuals")
plt.show()
```



In [17]:
```
list(np.where(model.resid>10))
```

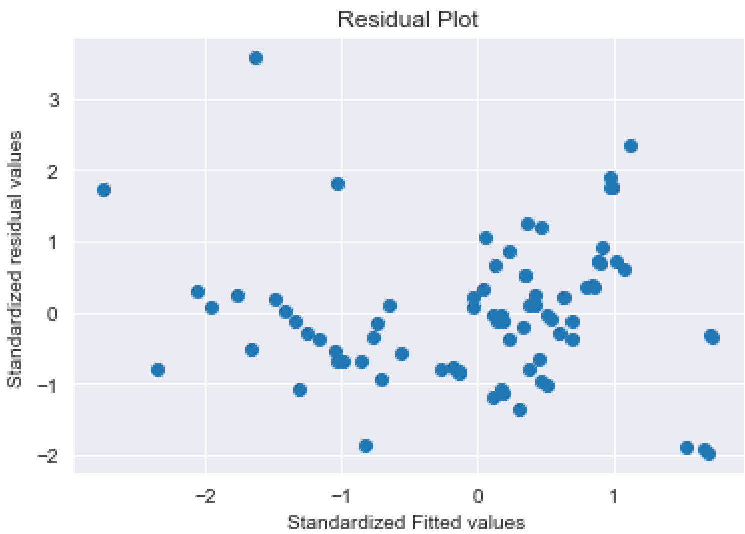
Out[17]:
```
[array([ 0, 76])]
```

Residual Plot for Homoscedasticity

In [18]:
```
def get_standardized_values( vals ):
    return (vals - vals.mean())/vals.std()
```

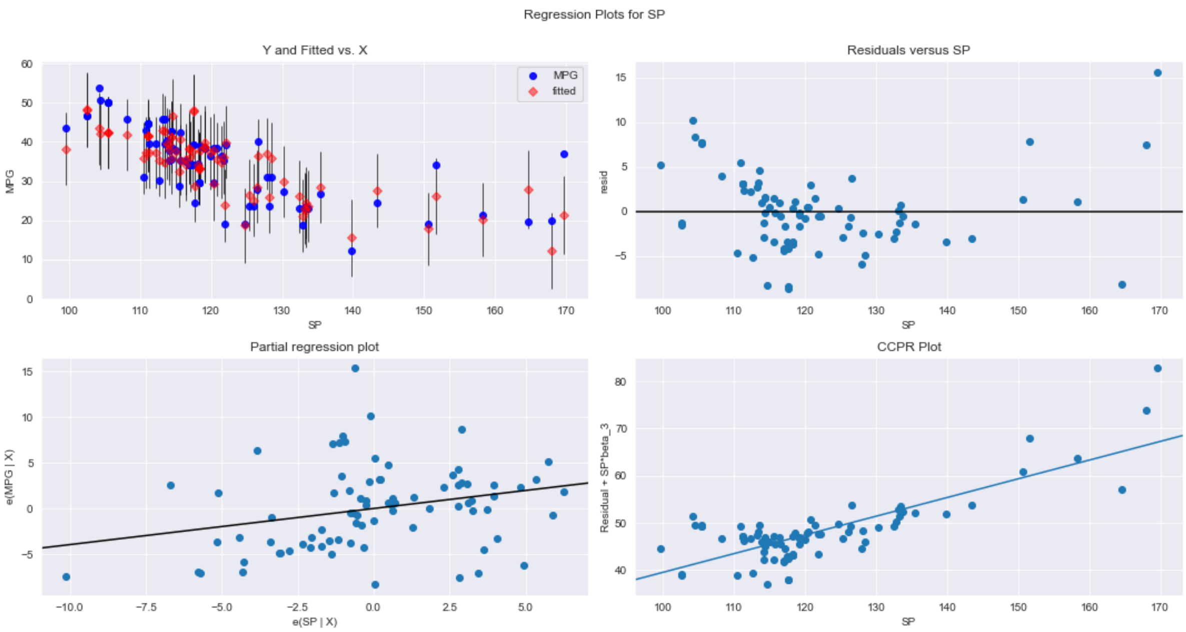
In [19]:
```
plt.scatter(get_standardized_values(model.fittedvalues),
            get_standardized_values(model.resid))

plt.title('Residual Plot')
plt.xlabel('Standardized Fitted values')
plt.ylabel('Standardized residual values')
plt.show()
```

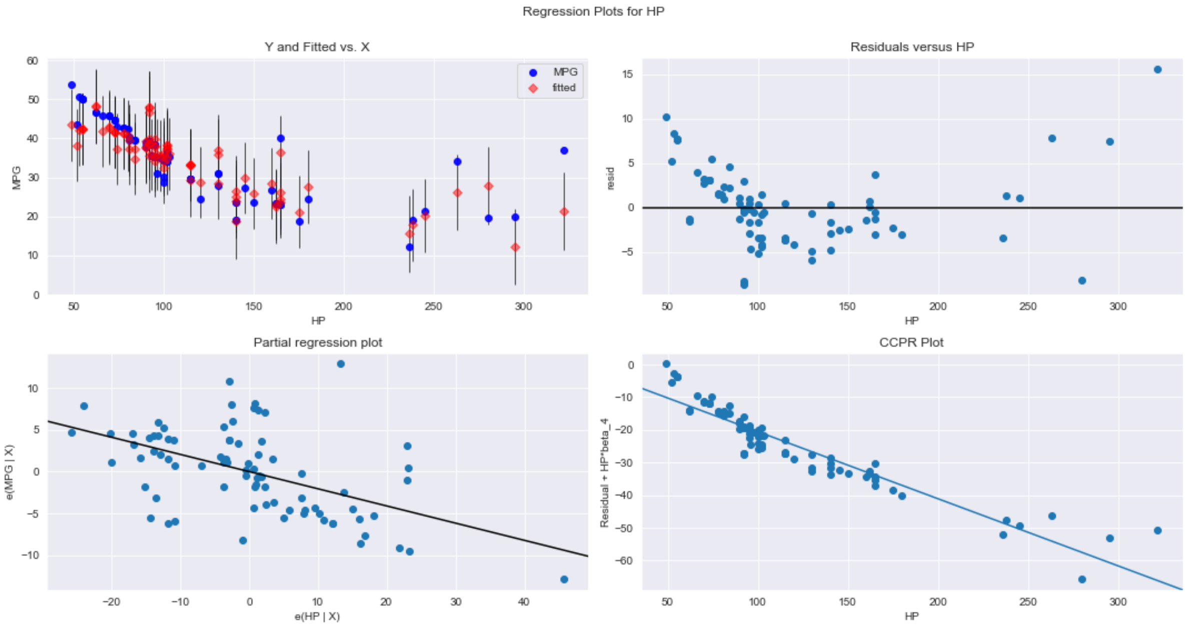


Residual Vs Regressors

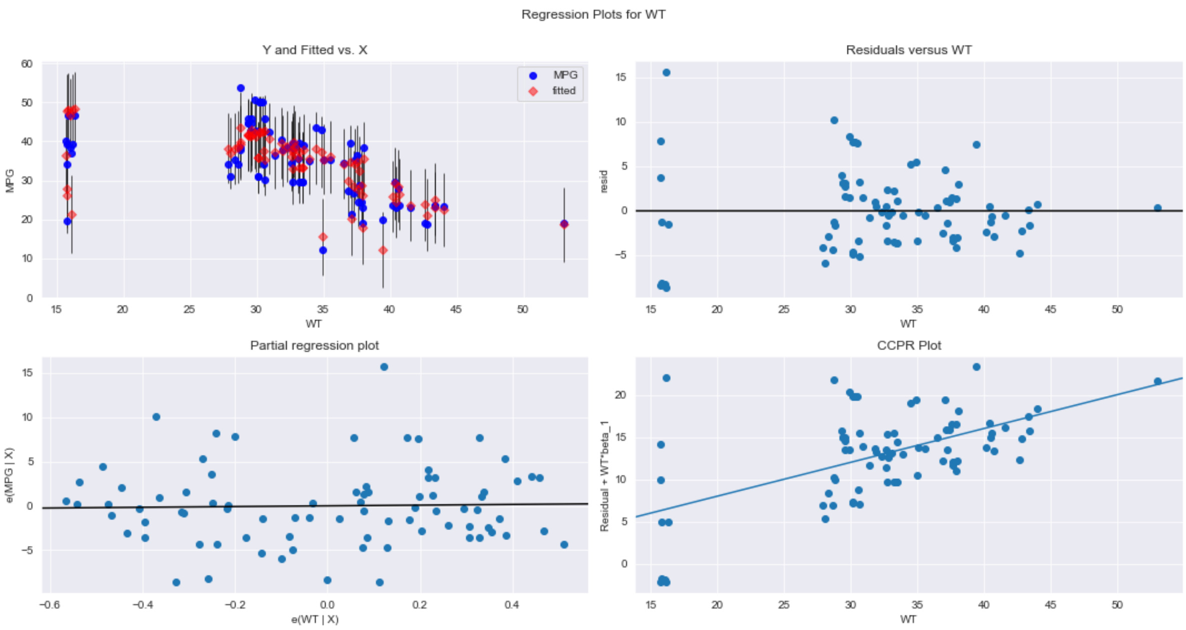
In [20]:
```
fig = plt.figure(figsize=(15,8))
fig = sm.graphics.plot_regress_exog(model, "SP", fig=fig)
plt.show()
```



In [21]:
```
fig = plt.figure(figsize=(15,8))
fig = sm.graphics.plot_regress_exog(model, "HP", fig=fig)
plt.show()
```



In [22]:
```python
fig = plt.figure(figsize=(15,8))
fig = sm.graphics.plot_regress_exog(model, "WT", fig=fig)
plt.show()
```



Model Deletion Diagnostics

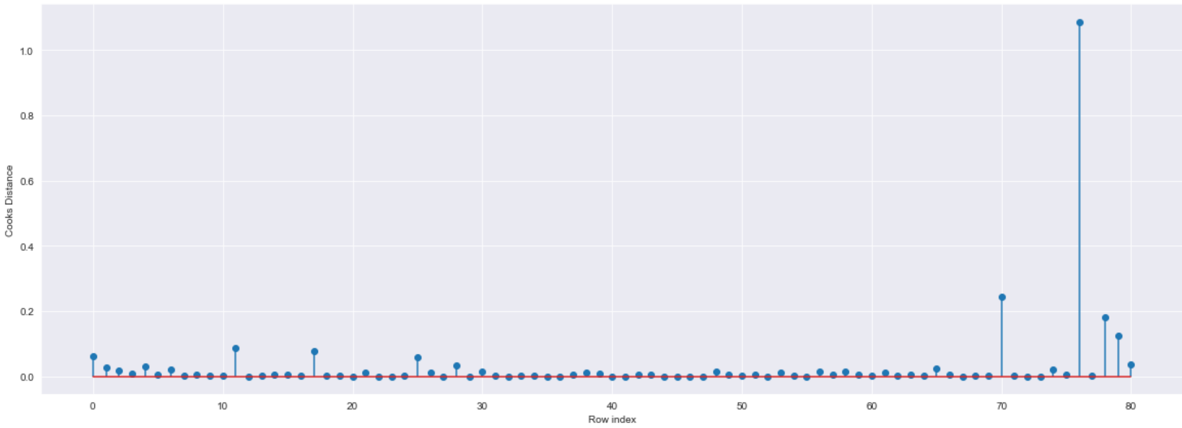
Detecting Influencers/Outliers

Cook’s Distance

In [23]:
```python
model_influence = model.get_influence()
(c, _) = model_influence.cooks_distance
```

In [24]:
```python
#Plot the influencers values using stem plot
fig = plt.subplots(figsize=(20, 7))
plt.stem(np.arange(len(cars)), np.round(c, 3))
plt.xlabel('Row index')
```

```
plt.ylabel('Cooks Distance')
plt.show()
```

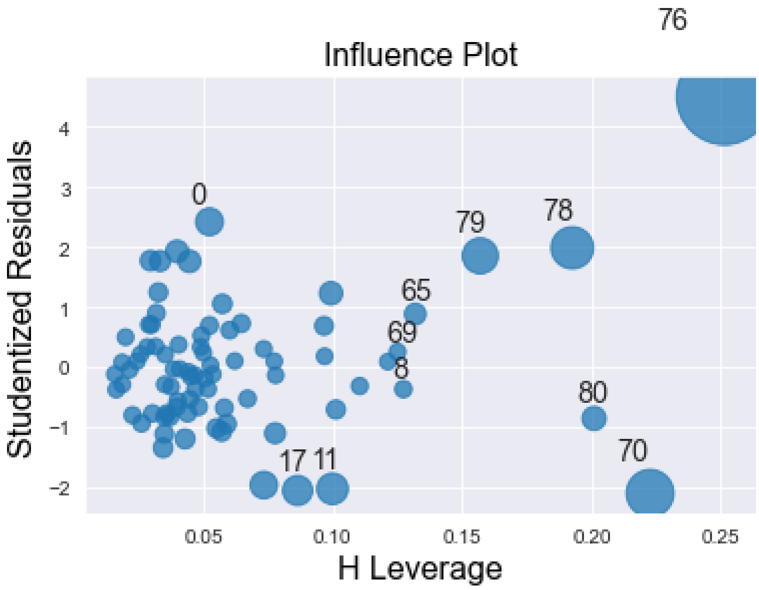


In [25]:
```
#index and value of influencer where c is more than .5
(np.argmax(c),np.max(c))
```

Out[25]: (76, 1.0865193998180052)

High Influence points

In [26]:
```
from statsmodels.graphics.regressionplots import influence_plot
influence_plot(model)
plt.show()
```



In [27]:
```
cars.shape
```

Out[27]: (81, 5)

In [28]:
```
k = cars.shape[1]
n = cars.shape[0]
leverage_cutoff = 3*((k + 1)/n)
leverage_cutoff
```

Out[28]: 0.2222222222222222

From the above plot, it is evident that data point 70 and 76 are the influencers

In [29]:
```
cars[cars.index.isin([70, 76])]
```

Out[29]:

	HP	MPG	VOL	SP	WT
70	280	19.678507	50	164.598513	15.823060
76	322	36.900000	50	169.598513	16.132947

In [30]:
```
#See the differences in HP and other variable values
cars.head()
```

Out[30]:

	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149

Improving the model

In [62]:
```
#Load the data
cars_new = pd.read_csv("Cars.csv")
```

In [63]:
```
#Discard the data points which are influencers and reasign the row number (reset_i
car1=cars_new.drop(cars_new.index[[70,76]],axis=0).reset_index()
car1
```

Out[63]:

	index	HP	MPG	VOL	SP	WT
0	0	49	53.700681	89	104.185353	28.762059
1	1	55	50.013401	92	105.461264	30.466833
2	2	55	50.013401	92	105.461264	30.193597
3	3	70	45.696322	92	113.461264	30.632114
4	4	53	50.504232	92	104.461264	29.889149
...	...	...	...	...	...	...
74	75	175	18.762837	129	132.864163	42.778219
75	77	238	19.197888	115	150.576579	37.923113
76	78	263	34.000000	50	151.598513	15.769625
77	79	295	19.833733	119	167.944460	39.423099
78	80	236	12.101263	107	139.840817	34.948615

79 rows × 6 columns

In [33]:
```
#Drop the original index
car1=car1.drop(['index'],axis=1)
```

In [34]:
```
car1
```

Out[34]:

	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149
...	...	...	...	...	...
74	175	18.762837	129	132.864163	42.778219
75	238	19.197888	115	150.576579	37.923113
76	263	34.000000	50	151.598513	15.769625
77	295	19.833733	119	167.944460	39.423099
78	236	12.101263	107	139.840817	34.948615

79 rows × 5 columns

Build Model

In [35]:
```
#Exclude variable "WT" and generate R-Squared and AIC values
final_ml_V= smf.ols('MPG~VOL+SP+HP',data = car1).fit()
final_ml_V.summary()
```

Out[35]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.816
Model:	OLS	**Adj. R-squared:**	0.809
Method:	Least Squares	**F-statistic:**	111.0
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	1.65e-27
Time:	20:29:10	**Log-Likelihood:**	-219.06
No. Observations:	79	**AIC:**	446.1
Df Residuals:	75	**BIC:**	455.6
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	25.5275	13.051	1.956	0.054	-0.471	51.526
VOL	-0.1825	0.023	-8.012	0.000	-0.228	-0.137
SP	0.4415	0.141	3.124	0.003	0.160	0.723
HP	-0.2291	0.035	-6.592	0.000	-0.298	-0.160

Omnibus:	6.541	**Durbin-Watson:**	1.130
Prob(Omnibus):	0.038	**Jarque-Bera (JB):**	5.833
Skew:	0.620	**Prob(JB):**	0.0541
Kurtosis:	3.485	**Cond. No.**	5.76e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.76e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [36]:
```
(final_ml_V.rsquared,final_ml_V.aic)
```

Out[36]:
```
(0.8161692010376005, 446.11722639447737)
```

In [37]:
```
#Exclude variable "VOL" and generate R-Squared and AIC values
final_ml_W= smf.ols('MPG~WT+SP+HP',data = car1).fit()
```

In [38]:
```
(final_ml_W.rsquared,final_ml_W.aic)
```

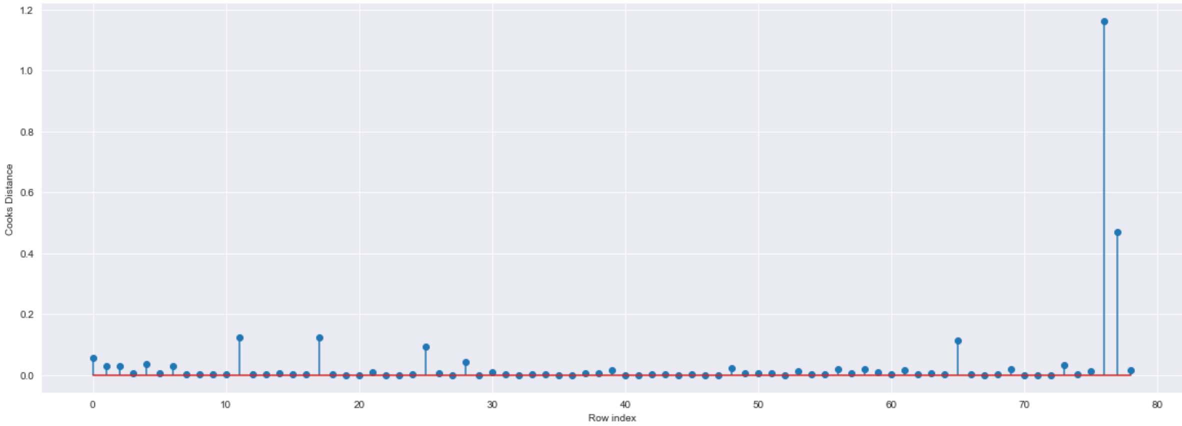
Out[38]:
```
(0.8160034320495304, 446.18843235750313)
```

Comparing above R-Square and AIC values, model 'final_ml_V' has high R- square and low AIC value hence include variable 'VOL' so that multi collinearity problem would be resolved.

Cook’s Distance

In [39]:
```
model_influence_V = final_ml_V.get_influence()
(c_V, _) = model_influence_V.cooks_distance
```

In [40]:
```
fig= plt.subplots(figsize=(20,7))
plt.stem(np.arange(len(car1)),np.round(c_V,3));
plt.xlabel('Row index')
plt.ylabel('Cooks Distance');
```



In [41]:
```
#index of the data points where c is more than .5
(np.argmax(c_V),np.max(c_V))
```

Out[41]: (76, 1.1629387469135215)

In [42]:
```
#Drop 76 and 77 observations
car2=car1.drop(car1.index[[76,77]],axis=0)
```

In [43]:
```
car2
```

Out[43]:

	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149
...	...	...	...	...	...
72	140	19.086341	160	124.715241	52.997752
73	140	19.086341	129	121.864163	42.618698
74	175	18.762837	129	132.864163	42.778219
75	238	19.197888	115	150.576579	37.923113
78	236	12.101263	107	139.840817	34.948615

77 rows × 5 columns

In [44]:
```
#Reset the index and re arrange the row values
car3=car2.reset_index()
```

In [45]:
```
car4=car3.drop(['index'],axis=1)
```

In [46]:
```
car4
```

Out[46]:

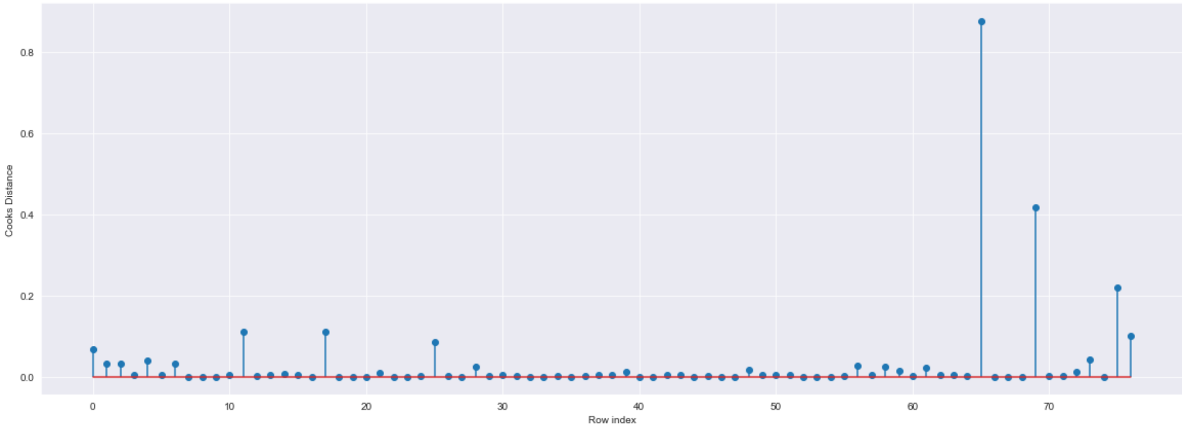
	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149
...	...	...	...	...	...
72	140	19.086341	160	124.715241	52.997752
73	140	19.086341	129	121.864163	42.618698
74	175	18.762837	129	132.864163	42.778219
75	238	19.197888	115	150.576579	37.923113
76	236	12.101263	107	139.840817	34.948615

77 rows × 5 columns

In [47]:
```
#Build the model on the new data
final_ml_V= smf.ols('MPG~VOL+SP+HP',data = car4).fit()
```

In [48]:
```
#Again check for influencers
model_influence_V = final_ml_V.get_influence()
(c_V, _) = model_influence_V.cooks_distance
```

In [49]:
```
fig= plt.subplots(figsize=(20,7))
plt.stem(np.arange(len(car4)),np.round(c_V,3));
plt.xlabel('Row index')
plt.ylabel('Cooks Distance');
```



In [50]:
```
#index of the data points where c is more than .5
(np.argmax(c_V),np.max(c_V))
```

Out[50]: (65, 0.8774556986296811)

Since the value is <1 , we can stop the diagnostic process and finalize the model

In [51]:
```
#Check the accuracy of the mode
```

```
final_ml_V= smf.ols('MPG~VOL+SP+HP',data = car4).fit()
```

In [52]:
```
final_ml_V.summary()
```

Out[52]:

OLS Regression Results

Dep. Variable:	MPG	**R-squared:**	0.867
Model:	OLS	**Adj. R-squared:**	0.861
Method:	Least Squares	**F-statistic:**	158.6
Date:	Wed, 02 Feb 2022	**Prob (F-statistic):**	6.81e-32
Time:	20:29:10	**Log-Likelihood:**	-200.71
No. Observations:	77	**AIC:**	409.4
Df Residuals:	73	**BIC:**	418.8
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	25.2974	11.336	2.232	0.029	2.706	47.889
VOL	-0.1362	0.021	-6.366	0.000	-0.179	-0.094
SP	0.4335	0.122	3.560	0.001	0.191	0.676
HP	-0.2635	0.031	-8.634	0.000	-0.324	-0.203

Omnibus:	9.478	**Durbin-Watson:**	1.195
Prob(Omnibus):	0.009	**Jarque-Bera (JB):**	9.184
Skew:	0.770	**Prob(JB):**	0.0101
Kurtosis:	3.703	**Cond. No.**	5.72e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.72e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [53]:
```
(final_ml_V.rsquared,final_ml_V.aic)
```

Out[53]:
```
(0.8669636111859063, 409.4153062719508)
```

Predicting for new data

In [54]:
```
#New data for prediction
new_data=pd.DataFrame({'HP':40,"VOL":95,"SP":102},index=[1])
new_data
```

Out[54]:

	HP	VOL	SP
1	40	95	102

In [55]:
```
final_ml_V.predict(new_data)
```

Out[55]:
```
1    46.035594
dtype: float64
```

In [56]:
```
final_ml_V.predict(cars_new.iloc[0:5,])
```

Out[56]:
```
0    45.428872
1    43.992392
2    43.992392
3    43.508150
4    44.085858
dtype: float64
```

In [57]:
```
cars_new.head()
```

Out[57]:

	HP	MPG	VOL	SP	WT
0	49	53.700681	89	104.185353	28.762059
1	55	50.013401	92	105.461264	30.466833
2	55	50.013401	92	105.461264	30.193597
3	70	45.696322	92	113.461264	30.632114
4	53	50.504232	92	104.461264	29.889149

In [58]:
```
pred_y = final_ml_V.predict(cars_new)
```

In [59]:
```
pred_y
```

Out[59]:
```
0      45.428872
1      43.992392
2      43.992392
3      43.508150
4      44.085858
         ...
76      7.165876
77     12.198598
78     14.908588
79      4.163958
80      9.161202
Length: 81, dtype: float64
```

In [ ]: