Algorithm Design - Exam

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Exercise 1. A certain string-processing language offers a primitive operation which splits a string into two pieces. Since this operation involves copying the original string, it takes n units of time for a string of length n, regardless of the location of the cut. Suppose, now, that you want to break a string into many pieces. The order in which the breaks are made can affect the total running time. For example, if you want to cut a 20-character string at positions 3 and 10, then making the first cut at position 3 incurs a total cost of 20+17=37, while doing position 10 first has a better cost of 20+10=30.

Design a dynamic programming algorithm that given m cuts, finds the minimum cost of cutting a string into $m + 1$ pieces.

Exercise 2. Design an assignment of a group of n students to m classes. Student i should take a minimum of l_i and a maximum of u_i within a set C_i of classes. Class j should be attended by a minimum of m_j and a maximum of M_j students in order to be activated.

1. Design an algorithm that solves the problem of the existence of a feasible schedule in polynomial time.

2. Study the time complexity of the algorithm.

Exercise 3. In the Multiway Cut problem we are given a graph $G = (V, E)$ with positive costs on the edges and a set of k terminals $\{t_1, \ldots, t_k\}$. The goal is to find a minimum cost set of edges whose removal disconnects each pair of terminals. Denote by F_i the minimum cut that disconnects terminal t_i from the other $k-1$ terminals. Prove that $\cup_{i=1}^{n} F_i$ is a 2-approximation of the minimum multiway cut.

Exercise 4. You are given a graph $G = (V, E)$. We would like to see if we can partition V into two sets V_1 and V_2, such that there exist a Hamiltonian cycles for V_1 and V_2. Show that this problem is NP-complete.

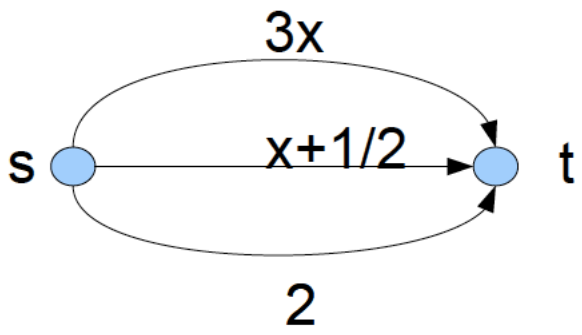
1. First, argue why the problem is in NP.

2. Second, give a reduction from any NP-hard problem of your choice.

Exercise 5.

i. Define the concepts of equilibrium flow, optimal flow and Price of Anarchy in a network congestion game.

ii. Find the equilibrium flow, the optimal flow and the price of anarchy in the following network congestion game with a total demand of 3 to be delivered from s to t:



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