Algorithm Design - Exam

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120 minutes

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Exercise 1. Consider the problem of finding a max s-t flow in a directed graph with real capacities on the edges.

1. Demonstrate that a greedy algorithm will eventually converge to the optimal solution.
2. Analyze the time complexity of the Ford and Fulkerson algorithm.
3. Show how the algorithm can be made to run in strong polynomial time.

Exercise 2. You are given an array A of positive and negative integers. Our objective is to compute the sum of all elements $\sum_{i=1}^{n} A_i$. Unfortunately, the numbers in A can have very large magnitude and we would like to minimize the risk of having a numerical overflow. To this end, we would like to bracket the sum such that the maximal value throughout the computation is minimized. For example, given the array $[30, 10, -9, 20, -5, -15, -12, 11]$, the optimal order is

$$30 + ((10 + (-9 + 20) - 5)) - 15) + (-21 + 20))$$

as it stores a maximal value of 20, which is also the final solution. The trivial sequence from left to right attains a maximal value of 51.

1. Give the recursive form used in the dynamic programming formulation. Specifically consider how the value of the optimal sequence from position $OPT[i, j]$ can be computed from $OPT[i, k]$ and $OPT[k, j]$, $k \in \{i, \dots j\}$.
2. Prove correctness of your recursive form.
3. Argue why your recursive form can be implemented in $O(n^2)$ time.

Exercise 3. You are tasked with designing a randomized algorithm for estimating π. Specifically: we are able to place points in the Hamming cube ranging from $[-1, -1]$ to $[1, 1]$ uniformly at random. We want to use this random generation to estimate π.

1. Find an estimator X such that $\mathbb{E}[X] = \pi$.
2. Give the variance of the estimator.

Exercise 4. You are given a graph $G(V, E)$. We would like to see if we can partition V into two sets V_1 and V_2, such that there exist a Hamiltonian cycles for V_1 and V_2. Show that this problem is NP-complete.

1. First, argue why the problem is in NP.
2. Second, give a reduction from any NP-hard problem of your choice.

Exercise 5. 1. Define the concepts of Dominant Strategy (DS), Pure Nash Equilibrium (PNE) and Mixed Nash Equilibrium (MNE) of a Game in Normal Form. Do they always exist?

2. Discuss the complexity of the problem of finding a MNE in a zero sum game.
3. Find all DS, PNE and MNE of the following 2-player game:

$$\begin{pmatrix} 4 & 3 \\ -3 & 6 \end{pmatrix}$$