

1a. $T(n) = T(n/5) + cn, T(1)=1$
 $T(n/5) = T(n/25) + c(n/5)$
 $T(n/25) = T(n/125) + c(n/25)$
 $T(n) = T(n/5^k) + c(n + (n/5) + (n/25) + \dots + (n/5^{k-1}))$
 $c(n + (n/5) + (n/25) + \dots + (n/5^{k-1})) = cn(1 + 1/5 + 1/25 + \dots + 1/5^{k-1})$
 $T(n) = T(1) + cn * 5/4 = 1 + 5/4 cn$
 $T(n) = O(n)$

1b. $T(n) = 4T(n/3) + cn^2, T(3)=1$
 $T(n/3) = 4T(n/9) + c(n/3)^2$
 $T(n/9) = 4T(n/27) + c(n/9)^2$
 $T(n) = 4^k T(n/3^k) + cn^2(1 + 4/9 + 16/81 + \dots + ((4^{k-1})/(9^{k-1})))$
 $\sum_{i=0}^{k-1} (4/9)^i = 9/5(1 - (4/9)^k)$
 $T(n) = 4^k T(n/3^k) + cn^2 * 9/5$
 $T(n) = 4^k = 4^{\log_3 n}$
 $T(n) = O(n^{\log_3 4})$

1c. $T(n) = 16T(n/2) + n^3, T(1)=1$
 $T(n/2) = 16T(n/4) + (n/2)^3$
 $T(n/4) = 16T(n/8) + (n/4)^3$
 $T(n) = 16^k T(n/2^k) + n^3(1 + 1/8 + 1/64 + \dots + 1/8^{k-1})$
 $\sum_{i=0}^{k-1} (1/8)^i = 8/7(1 - (1/8)^k)$
 $T(n) = 16^k T(n/2^k) + n^3 * 8/7$
 $T(n) = 16^k = n^4$
 $T(n) = O(n^4)$

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2. def find_fixed_point(A, low, high):
    if low > high:
        return None

    mid = (low + high) // 2

    if A[mid] == mid:
        return mid
    elif A[mid] > mid:
        return find_fixed_point(A, low, mid - 1)
    else:
        return find_fixed_point(A, mid + 1, high)

def find_fixed_point_main(A):
    return find_fixed_point(A, 0, len(A) - 1)
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Recurrence relation: $T(n) = T(n/2) + O(1)$

This equals out to $O(\log n)$

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3. def merge(S1_prime, S2_prime):
    S_prime = S1_prime + S2_prime
    maximal_set = []

    for point in S_prime:
        dominated = False
        for other in maximal_set:
            if dominates(point, other) or dominates(other, point):
                dominated = True
                break
        if not dominated:
            maximal_set.append(point)

    return maximal_set

def dominates(A, B):
    return A[0] < B[0] and A[1] < B[1]

def maximal_subset(S):
    if len(S) <= 1:
        return S

    S.sort(key=lambda point: point[0])

    mid = len(S) // 2
    S1 = S[:mid]
    S2 = S[mid:]

    S1_prime = maximal_subset(S1)
    S2_prime = maximal_subset(S2)

    return merge(S1_prime, S2_prime)
```

Recurrence Relation: $T(n) = 2T(n/2) + O(n^2)$

This solves out to $O(n^2)$