

Team number: _____
(Sort by first name)

Manager: _____

Scribe: _____

Sceptic: _____

Summarizer: _____

CSCI 4470 – Algorithms

Worksheet 24

Purpose: Find all-pairs shortest path graphs, and prove properties about them.

Student Learning Outcomes: 6. (partial) Analyze and prove graph algorithms for Minimum Spanning Tree and Shortest Path problems.

Answer the questions below. Please write your answers in the areas outlined with \Blue{ }. Turn in a compiled pdf document on eLC. Only one worksheet should be handed in from each group.

Please do not post the worksheets on the Web.

1. *13 points* Prove that EXTEND is associative. In other words, if you have 3 matrices A , B , and C , then

$$\text{EXTEND}(A, \text{EXTEND}(B, C)) = \text{EXTEND}((\text{EXTEND}(A, B), C)).$$

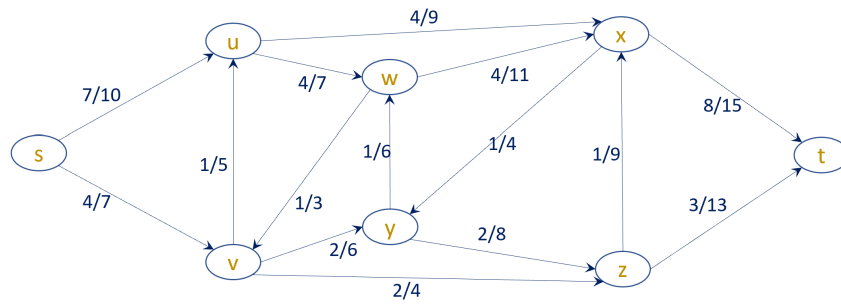
Specifically, if

- $D = \text{EXTEND}(A, B)$,
- $E = \text{EXTEND}(D, C)$,
- $F = \text{EXTEND}(B, C)$, and
- $G = \text{EXTEND}(A, F)$,

then $E = G$.

2. *14 points* Prove that if $h(v)$ is found using the method described in the book and in the text (Johnson's method), then $\hat{w}(u, v) \geq 0$ for all $(u, v) \in G.E$.

3. 13 points Consider the following flow network.



- Draw the residual network for this flow.
- Find an augmenting path that increases at the flow across at least one edge and decreases the flow across another edge. Make the flow be as large as possible for the chosen path.
- Describe the augmenting path as a ordered list of vertices and state how much flow goes through it.
- Finally, draw a new flow graph with the augmented flow.