C1:
28.10:

$$x + \frac{2}{3}x = \frac{5}{3}x$$

$$\frac{5}{3}x - \frac{1}{3}\left(\frac{4}{3}x\right) = 10$$

$$\implies \frac{10}{9}x = 10$$

$$\implies x = 9$$

47.3:

$$\frac{200}{9} = 22 + \frac{2}{9}$$

$\frac{1}{5}$ is the largest unit fraction that is less than or equal to $\frac{2}{9}$.

$$\frac{2}{9} - \frac{1}{5} = \frac{1}{45}$$

Thus, $\frac{200}{9} = 22 + \frac{1}{5} + \frac{1}{45}$.
The greek representation is something like "22 and 1/5 and 1/45"??
47.7:

$$\frac{\text{Height of Stick}}{\text{Length of Stick's Shadow}} = \frac{\text{Height of Pyramid}}{\text{Length of Pyramid + Length of Shadow}}$$

Let $h =$ Height of Pyramid.
We then get:

$$\frac{6}{9} = \frac{h}{756 + 342}$$

This works out to $h = 732$ feet.

This measurement is most accurate at local noon when the sun is at the highest point in the sky and directly south.
47.8:
A triangular number is of the form:

$$T_n = 1 + 2 + \cdots + n$$

Now, write T_n both forwards and backwards:

$$1 + 2 + 3 + \cdots + (n-1) + n$$

$$n + (n-1) + (n-2) + \cdots + 2 + 1$$

Now, sum them together. You will note that each term becomes $n+1$. And since we have n of them, we get $n(n+1)$. But this is the same as $2 \cdot T_n$. Thus, $T_n = \frac{n(n+1)}{2}$.

An oblong number is of the form: $n(n+1)$.

$$n(n+1) = 2 \cdot \frac{n(n+1)}{2} = 2 \cdot T_n$$

Thus, an oblong number is twice that of a triangular number.

47.10:

Visual representation proof:

1) Triangular Number to Square

A triangular number T_n can be represented as a triangle of dots.

$8 \cdot T_n$ can be arranged as an octagon.

Adding 1 dot to it at the center turns the octagon into a square.

2) Square to Triangular Number

An odd square can be Visualized as a square grid of dots.

Removing 1 dot from the center of the square and rearranging the dots forms an octagon made up of 8 regular triangles.

Algebraic proof:

1) Triangular Number to Square

$$T_n = \frac{n(n+1)}{2}$$

$$\implies 8 \cdot T_n = 8 \cdot \frac{n(n+1)}{2} + 1 = 4n(n+1) + 1$$

Notice that $(2n+1)^2 = 4n^2 + 4n + 1 = 4n(n+1) + 1$.

So, $8 \cdot T_n + 1$ forms a square.

2) Square to Triangular Number

Consider the odd square: $(2n+1)^2$.

$$(2n+1)^2 - 1 = 4n^2 + 4n = 4n(n+1) = 8 \cdot \frac{n(n+1)}{2} = 8 \cdot T_n$$

47.12:

1) Using the formula $(n, \frac{n^2-1}{2}, \frac{n^2+1}{2})$ for an odd n:
- (3, 4, 5)
- (5, 12, 13)
- (7, 24, 25)
- (9, 40, 41)
- (11, 60, 61)

2) Using the formula $(m, \left(\frac{m}{2}\right)^2 - 1, \left(\frac{m}{2}\right)^2 + 1)$ for any even m:
- (4, 3, 5)
- (6, 8, 10)
- (8, 15, 17)
- (10, 24, 26)
- (12, 35, 37)

C2:

1. The ancient Egyptians used the formula:

$$\text{Area} = \left(\frac{8}{9} \cdot \text{Diameter}\right)^2$$

For the given problem:

$$\text{Area} = \left(\frac{8}{9} \cdot 12\right)^2$$

2. Modern formula:

$$\text{Area} = \pi \cdot \left(\frac{\text{Diameter}}{2}\right)^2$$

For the given problem:

$$\text{Area} = \pi \cdot 6^2$$

3. The formula for percentage error:

$$\text{Percentage Error} = \left|\frac{\text{Modern Value} - \text{Ancient Egyptian Value}}{ModernValue}\right| \cdot 100\%$$

The ancient Egyptian value is approximately 113.78 square units.
The modern value is approximately 113.10 square units.
The percentage error is approximately 0.60%.
C3:
Rectangle:

$$\frac{1}{2}(a+a) \cdot \frac{1}{2}(b+b) = a \cdot b$$

which is the correct formula.

Non-Rectanglular Parallelogram:

For a parallelogram with sides a and b and height h perpendicular to b, the actual equation is $b \cdot h$. The ancient Egyptian formula gives $\frac{1}{2}(a+a) \cdot \frac{1}{2}(b+b) = a \cdot b$, which overestimates the area since $a > h$ (because of the slant of the sides).

Trapezoid:

Let's say the parallel sides are a and b and the other two sides are c and d.

The true area formula is $\frac{1}{2}(a+b) \cdot h$ where h is the height. The ancient Egyptian formula is $\frac{1}{2}(a+b) \cdot \frac{1}{2}(c+d)$.

This does not generally equal the actual area formula since $\frac{1}{2}(c+d)$ is not necessarily the height.

Other quadralaterals:

The ancient Egyptian formula does not generally result in correct values. The formula assumes the shape can be approximated using a product of the average of the opposite sides, but this property only holds true for rectangles.

Proof for non-rectangular cases:

For non-rectangular parallelograms, we know that h is always less than a or b (whichever side is perpendicular to h). Since the ancient Egyptian formula uses the full side lengths instead of the height, it results in an overestimate.

For trapezoids, The height h is independent of the non-parallel side lenghts c and d. Thus, the ancient Egyptian formula, which approximates h using c and d does not reflect the actual geometry of the trapezoid.