Math 3010 Midterm 1 "Cheat Sheet"

Lincoln Sand

TABLE 2.1 ***Representation of a number system used by the Greeks as early as the sixth century BCE.***

Letter	Value	Letter	Value	Letter	Value
α	1	ι	10	ρ	100
β	2	κ	20	σ	200
γ	3	λ	30	τ	300
δ	4	μ	40	υ	400
ϵ	5	ν	50	ϕ	500
ς	6	ξ	60	χ	600
ζ	7	o	70	ψ	700
η	8	π	80	ω	800
θ	9	ϙ	90	ϡ	900

- Historical Data:

* Thales: 625 - 547 BC; Miletus; Advocated the deductive method. First man to have theorem named after him.

* Pythagoras: 580 - 497 BC; Croton; Explained musical harmony in terms of whole number ratios. Found some lengths are irrational.

* Zeno: 490 - 425 BC; Elia; Pupil of Parmenides. Proposed paradoxes involving infinity.

* Eudoxus: 400 - 347 BC; Cnidus; Devoloped theories of proportion and exhaustion.

* Aristotle: 384 - 322 BC; Athens; Advocated use of definitions/axioms/proofs in math and syllogism logic.

* Diophantus: 210 - 260 AD; Alexandria; Devloped algebraic notation and studied equations with integer unknowns.

* Archimedes: 287 - 212 BC; Syracuse; Discovered theorems using mechanical intuition with proofs.

* Euclid: 330 - 270 BC; Alexandria; His books set the standard for math rigor until 19th century.

* Plato: 427 - 346 BC; Athens; Theorems require sound definitions and proofs. The line and circle are pure.

* Hippocractes: 460 - 300 BC; Chios; Sophist philosopher, criticized fuzzy thinking. Squared the lune.

* Parmenides: 515 - 440 BC; Elia; Sophist, founded a school, "Whatever is is, and whatever is not cannot be".

* Chrysippus: 280 - 226 BC; Athens; Stoic philosopher, developed modern norations of evaluation of compound logic statements.

- Delian problems: Square the circle; Double the cube; Trisect an angle.

- Sum of geometric series:

$$S = \frac{a(1-r^n)}{1-r}$$

If, $|r| < 1$, then it converges to:

$$S = \frac{a}{1-r}$$

- Euclidean GCD algorithm/diophantine example:

Find $x, y \in \mathbb{Z}$ such that $gcd(198, 168) = 198x + 168y$.

$$198 = 1 \cdot 168 + 30; 168 = 5 \cdot 30 + 18; 30 = 1 \cdot 18 + 12;$$
$$18 = 1 \cdot 12 + 6; 12 = 2 \cdot 6 + 0$$

So, $gcd(198, 168) = 6$.

$$6 = 18 - 12 = 18 - (30 - 18) = 2 \cdot 18 - 30 = 2 \cdot (168 - 5 \cdot 30) - 30$$
$$= 2 \cdot 168 - 11 \cdot 30 = 2 \cdot 168 - 11 \cdot (198 - 168) = 13 \cdot 168 - 11 \cdot 198$$

So, $x = -11, y = 13$.

- Five platonic solids:

Triangles: Three at a vertex, four at a vertex, five at a vertex.

Squares: Three at a vertex.

Pentagons: Three at a vertex.

Higher polygons: Can't form a vertex because interior angle is too large.

- Pell's equation:

$$x^2 - Ny^2 = 1, N \in \mathbb{Z}, N > 0$$

1) Find continued fraction for $\sqrt{N}$

2) Approximate the expansion as the convergent $\frac{h_i}{q_i}$

3) (x, y) corresponds to a (h_i, q_i)

4) Check for fundamental solution. (x_1, y_1) is the smallest non-trivial solution and is called the fundamental solution.

$$x_{k+1} = x_1 x_k + N y_1 y_k$$
$$y_{k+1} = x_1 y_k + y_1 x_k$$

for $k \geq 1$.

- Example of method of false position:

We want to solve $x + \frac{x}{4} = 15$

Guess $x = 4$.

$x + \frac{4}{4} = 5$

This is off by 3, so guess $4 \cdot 3 = 12$.

$12 + \frac{12}{4} = 15$.

- Babylonian "divide and average" algorithm:

1) Start with initial guess x_0

2) Iterative step:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

3) Repeat the iterative step to desired accuracy.