Textbook Section 7.1:
1) For $f(t) = t$, the Laplace transform becomes:

$$F(s) = \int_0^\infty e^{-st} t dt$$

which computes to:

$$F(s) = \frac{1}{s^2}$$

The result is valid for $Re(s) > 0$.
6) For $f(t) = \sin^2(t)$, the Laplace transform becomes:

$$F(S) = \int_0^\infty e^{-st} \sin^2(t) dt$$

We can simplify this using the trig identity:

$$\sin^2(t) = \frac{1 - \cos(2t)}{2}$$

So, the Laplace transform becomes:

$$F(s) = \frac{1}{2} \int_0^\infty e^{-st} [1 - \cos(2t)] dt$$

which computes to:

$$F(s) = \frac{2}{s(s^2 + 4)}$$

This result is valid for $Re(s) > 0$.

20) The Laplace transform of $f(t) = e^{at}$ is $\frac{1}{s-a}$ for $s > a$. For our case, $a = 1$.

For integration by parts, $u(t) = t$, $v'(t) = e^t$.

We know that the Laplace transform of t^n is $\frac{n!}{s^{n+1}}$ for $n \geq 0$ and $s > 0$. For $n = 1$, the Laplace transform is $\frac{1}{s^2}$. We can use the linearity of the Laplace transform to multiply this by the transform of e^t, which is $\frac{1}{s-1}$ for $s > 1$.

$$L\{te^t\} = L\{t\} \cdot L\{e^t\}$$

$$L\{te^t\} = \frac{1}{s^2} \cdot \frac{1}{s-1}$$

So, $F(s) = \frac{1}{s^2(s-1)}$.

30) Let's factor the denominator $\frac{9+s}{4-s^2}$ using partial fractions. The partial fraction decomposition is $\frac{7}{4(s+2)} - \frac{11}{4(s-2)}$.

So, the inverse Laplace transform is:

$$L^{-1}\{\frac{7}{4(s+2)}\} - L^{-1}\{\frac{11}{4(s-2)}\}$$

Using the table, this translates to:

$$\frac{7}{4}e^{-2t} - \frac{11}{4}e^{2t}$$

Hence the inverse Laplace transform is:

$$f(t) = \frac{7}{4}e^{-2t} - \frac{11}{4}e^{2t}$$

Textbook Section 7.2:

1) We start by taking the Laplace transform of each term in the differential equation.

$$L\{x''\} = s^2X(s) - sx(0) - x'(0)$$

Plugging in initial conditions, we get:

$$L\{x''\} = s^2X(s) - 5s$$

By applying the Laplace transform to both sides of the equation, we get:

$$s^2X(s) - 5s + 4X(s) = 0$$

Now, solve for $X(s)$.

$$X(s) = \frac{5s}{s^2 + 4}$$

To get $x(t)$, we have to compute the inverse Laplace transform of $X(s)$. $X(s)$ corresponds to $5\cos(2t)$ in the time domain.

So, the initial value problem is:

$$x(t) = 5\cos(2t)$$

4) Doing the same process as above gives us the Laplace transform of the differential equation as:

$$s^2X(s) - 2s + 3 + 8(sX(s) - 2) + 15X(s) = 0$$

Then we solve for $X(s)$.

$$X(s) = \frac{2s - 13}{(s+3)(s+5)}$$

Then we have to take the partial fraction decomposition of $X(s)$.

$$X(s) = \frac{23}{2(s+5)} - \frac{19}{2(s+3)}$$

Then we take the inverse Laplace transform:

$$x(t) = L^{-1}\{\frac{23}{2(s+5)}\} - L^{-1}\{\frac{19}{2(s+3)}\}$$

$$x(t) = \frac{23}{2}e^{-5t} - \frac{19}{2}e^{-3t}$$

20) The partial fraction decomposition of $\frac{2s+1}{s(s^2+9)}$ is $-\frac{s-18}{9(s^2+9)} + \frac{1}{9s}$

$$L^{-1}\{\frac{1}{s}\} = 1$$

$$L^{-1}\{\frac{s}{s^2+9}\} = \cos(3t)$$

$$L^{-1}\{\frac{2}{s^2+9}\} = \frac{2}{3}\sin(3t)$$

Putting this together using the properties of Theorem 2 gives:

$$f(t) = -\frac{1}{9}\int_0^t \cos(3\tau)d\tau + \frac{2}{3}\int_0^t \sin(3\tau)d\tau + \frac{1}{9}$$

So, we get that:

$$f(t) = -\frac{1}{27}\sin(3t) - \frac{2}{9}\cos(3t) + \frac{1}{9}$$

Textbook Section 7.3:

3) The Laplace transform of $\sin(bt)$ is $\frac{b}{s^2+b^2}$. So the Laplace transform of $\sin(3\pi t)$ is $\frac{3\pi}{s^2+(3\pi)^2}$. Using the translation theorem, we replace s with $s+2$.

So,

$$L\{e^{-2t}\sin(3\pi t)\} = \frac{3\pi}{(s+2)^2+(3\pi)^2}$$

8) We will first complete the square in the denominator. Completing the square for s^2+4s+5 gives s^2+4s+4.

We can rewrite $F(s)$ as:

$$F(s) = \frac{s+2}{(s+2)^2+1}$$

Using the translation theorem,

$$f(t) = L^{-1}\{\frac{s+2}{(s+2)^2+1}\} = e^{2t}\cos(t)$$

19) Let $s^2 = x$. Then the denominator factors into $(s^2+1)(s^2+4)$. The partial fraction decomposition of $F(s)$ is:

$$F(s) = \frac{2(s+2)}{3(s^2+4)} - \frac{2s+1}{3(s^2+1)}$$

Using a table, we find that:

$$\frac{2}{3}\cos(2t) + \frac{2}{3}\sin(2t) - \frac{2}{3}\cos(t) - \frac{1}{3}\sin(t)$$

30)

$$L\{x''\} = s^2X(s)$$

$$L\{x'\} = sX(s)$$

Applying the Laplace transform to the entire differential equation gives:

$$(s^2 + 4s + 8)X(s) = \frac{1}{s+1}$$

Solve for $X(s)$.

$$X(s) = \frac{1}{(s+1)(s^2+4s+8)}$$

Taking the partial fraction, we get:

$$X(s) = -\frac{s+3}{5(s^2+4s+8)} + \frac{1}{5(s+1)}$$

$$s^2 + 4s + 8 = (s+2)^2 + 4$$

We eventually get that:

$$x(t) = \frac{1}{5}e^{-t} - \frac{1}{10}e^{-2t}\sin(2t) - \frac{1}{5}e^{-2t}\cos(2t)$$

Textbook Section 7.4:
5)

$$(f * g)(t) = \int_0^t e^{a\tau}e^{a(t-\tau)}d\tau$$

Which is equal to:

$$(f * g)(t) = e^{at}t$$

So, $(f * f)(t) = e^{at}t$.
10)

$$F(s) = \frac{1}{s^2} \cdot \frac{1}{s^2+k^2}$$

$$f(t) * g(t) = t * \frac{1}{k}\sin(kt)$$

Doing some algebra and computation (of the convolution) gives:

$$f(t) = \frac{kt - \sin(kt)}{K^3}$$

19) The given function can be considered the integral of $\sin(t)$ from 0 to t divided by t. We know the Laplace transform of $\sin(t)$ is $\frac{1}{s^2+1}$.

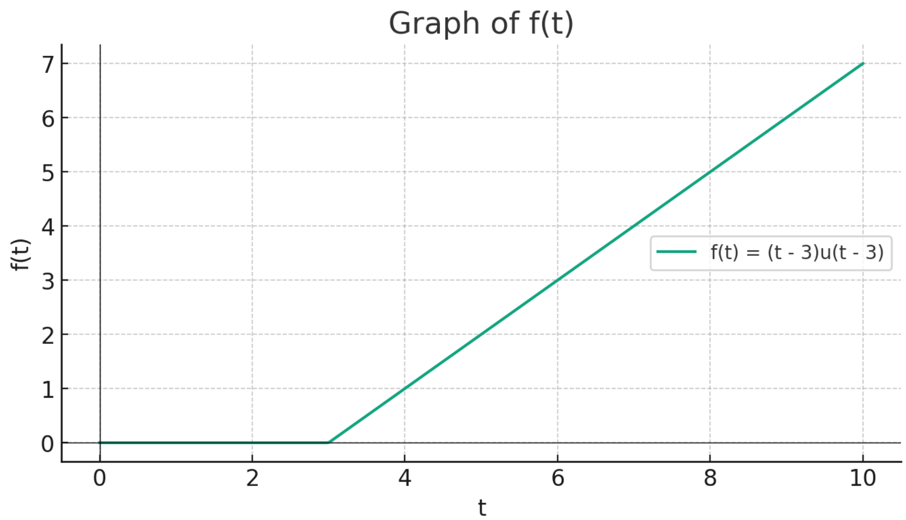
According to Theorem 3, the Laplace transform of $\frac{\sin(t)}{t}$ is the integral of $\frac{1}{s^2+1}$ with respect to s from 0 to ∞.

Calculating this gives us:

$$F(s) = -arctan(s) + \frac{\pi}{2}$$

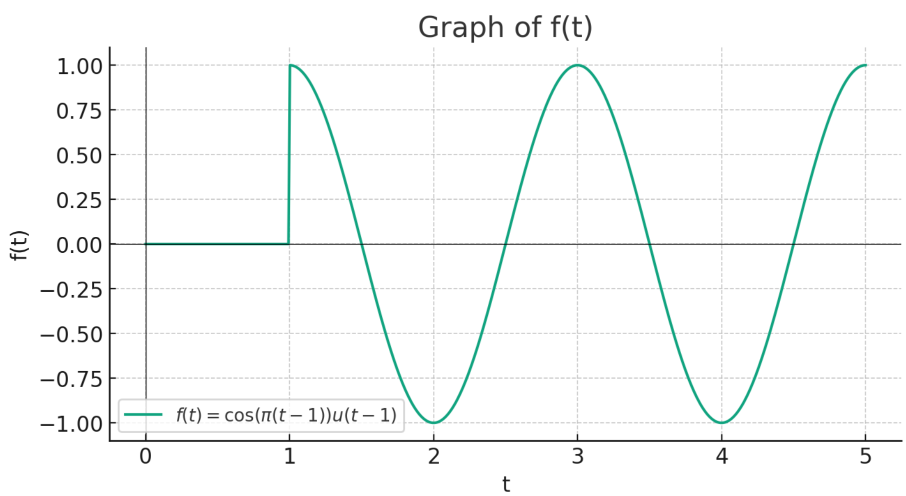
Textbook Section 7.5:
1) Using the second shifting (delay theorem), we get that:

$$f(t) = (t-3)u(t-3)$$



6) Applying the first shifting theorem, we get that:

$$f(t) = \cos(\pi(t-1))u(t-1)$$



21) We can write:

$$L\{f(t)\} = L\{tu(t) - tu(t-1)\} + L\{2u(t-1) - tu(t-1) - 2u(t-2) + tu(t-2)\}$$

Using the first shifting theorem, we get that:

$$L\{f(t)\} = \frac{e^{2s} - 2e^{s} + 1}{s^2} e^{-2s}$$

Textbook Section 7.6:
1)

$$L\{x'\} = sX(s)$$
$$L\{x''\} = s^2 X(s)$$

Applying the Laplace transform to the entire differential equation, we get:

$$(s^2 + 4)X(s) = 1$$

$$X(s) = \frac{1}{s^2 + 4}$$

$$x(t) = \frac{1}{2}\sin(2t)$$