

1 Performance Equations and Power

1.1 Performance

- $\text{Performance} = \frac{1}{\text{Execution Time}}$
- $\text{Speedup, B over A} = \frac{\text{Performance}_B}{\text{Performance}_A}$
- $\text{Performance Improvement} = \text{Speedup} - 1$
- $\text{Execution Time} = \text{Cycle Time} \times \text{Cycles}$
- $\text{Cycles} = \text{Instructions} \times \text{CPI}$
- $\text{Clock Speed} = \frac{1}{\text{Cycle Time}}$

1.2 Power

- $\text{Total Power} = \text{Dynamic Power} + \text{Leakage Power}$
- $\text{Dynamic Power} \propto \text{Activity} \times \text{Capacitance} \times \text{Voltage}^2 \times \text{Frequency}$

```
.data
arraybuf: .align 2
          .space 20
```

Allocate array of five 32-bit integers

2 Memory

2.1 Memory Organization

1. **Scratchpad:** holds a number of values for extremely fast access by the processor, made up of 32 **registers**
2. **Register:** element of a scratchpad, holds 32 bits - can either be a primitive value or an address in memory

2.2 Procedures

1. **Activation Record:** area on the stack allocated by a procedure, \$fp points to the start of the activation record and \$sp points to the end.
2. **Arguments:** \$a0 to \$a4 are populated with arguments Procedure A wants to call Procedure B with.
3. **Return:** when Procedure B returns, it needs to put the return value into \$v0 or \$v1 and hand control back to Procedure A
4. **Jump and Link:** jal Bstart jumps and hands control to procedure Bstart
5. **Jump Register:** jr \$ra jumps to the register \$ra, which holds the return address.
6. **Scratchpad:**
 - (a) Caller should save \$ra, \$a0..., \$t0... \$fp (if required).
 - (b) Callee should save \$s0...

(d) Write the line(s) of code that print this count. (3 points)

Solution:

```
li    $v0, 1
add   $a0, $t3, $zero
syscall
```

3 Numeric Representations

3.1 Unsigned Integers

1. **Decimal Numbers:** $3512 = 3 \times 10^3 + 5 \times 10^2 + 1 \times 10^1 + 2 \times 10^0$
2. **Binary Numbers:** $10101 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
3. **Hexadecimal:** $0x19af = 1 \times 16^3 + 9 \times 16^2 + 10 \times 16^1 + 15 \times 16^0$

3.2 Signed Integers

1. **Binary Numbers - Two's Complement:** first bit represents sign; the rest counts up from 0 (when positive) and up from -2^{31} (when negative)
 - $0000...0000 = 0$
 - $0111...1111 = 2^{31} - 1$
 - $1000...0000 = -2^{31}$
 - $1111...1111 = -1$
 - $x_{31}x_{30}...x_0 = -2^{31}x_{31} + 2^{30}x_{30} + 2^{29}x_{29} + ... + 2^0x_0$
 - Addition can be done just like decimal
 - Let \bar{x} be x with bits inverted.
 - $-x = \bar{x} + 1$

3.3 Floating Point Numbers

1. **Single-Precision Floating Point:**
 - (a) Put the number into scientific notation in binary, of the form $\pm 1.\text{fraction} \times 2^{\text{exponent}}$. Bits go from 31 to 0, left to right.
 - (b) Bit 31 (S) stores the sign, 0 for positive and 1 for negative
 - (c) Bits 30-23 (E) stores (exponent + bias) as an unsigned int, bias is 127
 - (d) Bits 22-0 (F) stores the fraction section (right of the point) of the number, trailing 0s are added to the right as needed.
 - (e) **Addition:** normalize the smaller exponent to match the larger exponent, add the fractions
2. **Double-Precision Floating Point:** same as single-precision, but 11 exponent bits, 52 fraction bits, bias is 1023.

4 Digital Design

4.1 Boolean Algebra

- **OR:** $A + B \iff A \text{ or } B$
- **AND:** $A.B \iff A \text{ and } B$
- **NOT:** $\bar{A} \iff \text{not } A$
- $\overline{A+B} = \bar{A}.\bar{B}$
- $\overline{A.B} = \bar{A} + \bar{B}$
- $A.(B+C) = (A.B) + (A.C)$
- $A + (B.C) = (A+B).(A+C)$
- **Sum of Products:** using a truth table, pick all true conditions and OR them together

4.2 Ripple-Carry Adder

1. A 1-bit adder takes in a carry (carry-in), puts out a carry (carry-out), and adds two digits.
2. Ripple-Carry connects the adders together; the carry-out of one adder serves as the carry-in of the next adder. This is similar to how we do addition, one pair of digits at a time and carrying when needed.

4.3 Carry-Lookahead Adder

1. Instead of waiting for the previous adder to put in a carry, we calculate whether there exists a carry ahead of time.
2. $c_{i+1} = a_i.b_i + (a_i.b_i).c_i$: a_i is i^{th} bit of a, b_i is i^{th} bit of b, c_i is whether i^{th} bit has a carry
3. Can chunk bits into 4 - do those 4 bits generate a carry?

5 Finite State Machines

1. **Finite State Machine:** the machine takes **input** and stores a **state**, using that information to move to a new state
2. **Finite State Table:** enumerate all possible permutations of current state and inputs, record the next state that the machine stores (output)
3. **Finite State Diagram:**
 - (a) What are possible output states? Draw a bubble for each.
 - (b) What are inputs? What values can those inputs take?
 - (c) For each state, what do I do for each possible input value? Draw an arc out of every bubble for every input value to represent a transition to a different state.