Textbook Section 2.3:

13) Separate variables in Eq. (12) and substitute $u = v\sqrt{\frac{p}{g}}$ to obtain the upward-motion velocity function given in Eq. (13) with initial condition $v(0) = v_0$.

Equation 12:

$$\frac{dv}{dt} = -g - pv^2 = -g\left(1 + \frac{p}{g}v^2\right).$$

Remember:

$$\int \frac{1}{1+u^2}du = tan^{-1}(u) + C$$

Equation 13:

$$v(t) = \sqrt{\frac{g}{p}}tan(C_1 - t\sqrt{pg})$$

with

$$C_1 = tan^{-1}\left(v_0\sqrt{\frac{p}{g}}\right).$$

Work:

$$-g\left(1 + \frac{p}{g}v^2\right)$$

with

$$u = v\sqrt{\frac{p}{g}}$$

$$\implies du = \sqrt{\frac{p}{g}}dv$$

yields:

$$\frac{du}{dt} = -g(1+u^2)$$

$$\implies \sqrt{\frac{p}{g}}\int \frac{dv}{1+u^2} = \int -gdt$$

$$\implies \sqrt{\frac{p}{g}}tan^{-1}(u) = -gt + u_0$$

$$\implies tan^{-1}(u) = \sqrt{\frac{p}{g}} \cdot (-gt + u_0)$$

$$\implies tan^{-1}(u) = -\sqrt{pg}t + \sqrt{\frac{p}{g}}u_0$$

$$\implies u = tan\left(\sqrt{\frac{p}{g}}u_0 - \sqrt{pg}t\right)$$

$$v\sqrt{\frac{p}{g}} = tan\left(\sqrt{\frac{p}{g}}u_0 - \sqrt{pg}t\right)$$

$$v = \sqrt{\frac{g}{p}}tan\left(tan^{-1}\left(\sqrt{\frac{p}{g}}v_0\right) - \sqrt{pg}t\right)$$

14) Integrate the velocity function in Eq. (13) to obtain the upward-motion function in Eq. (14) with initial condition $y(0) = y_0$.

Recall:

$$\int tan(u)du = -ln\left|cos(u)\right| + C$$

Equation 14:

$$y(t) = y_0 + \frac{1}{p}ln\left|\frac{cos(C_1 - t\sqrt{pg})}{cos(C_1)}\right|$$

Work:

$$\frac{dy}{dt} = \sqrt{\frac{g}{p}}tan\left(C1 - \sqrt{pg}t\right)$$

Let $u = C1 - \sqrt{pg}t. \implies du = -\sqrt{pg}dt.$

$$\implies \frac{dy}{du} = -\sqrt{\frac{g}{p}}tan(u)$$

$$\implies dy = -\sqrt{\frac{g}{p}}tan(u)du$$

$$-\sqrt{\frac{p}{g}}dy = tan(u)du$$

$$-\sqrt{\frac{p}{g}}\int dy = \int tan(u)du$$

$$-\sqrt{\frac{p}{g}}y = -ln\left|cos(u)\right| + u_0$$

$$y = \frac{g}{p}(-u_0 + ln\left|cos(u)\right|)$$

Since $y(0) = y_0$, then the inside of ln must be 1. And since $cos(C1 - \sqrt{pg}t)$ at $t = 0$ is $C1$, we have to divide it out.

$$y = y_0 + \frac{g}{p}\left(ln\left|\frac{cos(C1 - \sqrt{pg}t)}{cos(C_1)}\right|\right)$$

Not sure how $\frac{g}{p}$ is supposed to become $\frac{1}{p}$.

Textbook Section 2.4:

4) $y' = x - y, y(0) = 1; y(x) = 2e^{-x} + x - 1$

For euler's method with step size of 0.25: We get an approximation of y at 0.5 of 0.625.

For euler's method with step size of 0.1: We get an approximation of y at 0.5 of 0.681.

The actual value of y at 0.5 is 0.713.

Textbook Section 2.5:

4) $y' = x - y, y(0) = 1; y(x) = 2e^{-x} + x - 1$

x	approximation	actual
0.1	0.9100	0.9097
0.2	0.8381	0.8375
0.3	0.7824	0.7816
0.4	0.7416	0.7406
0.5	0.7142	0.7131

Textbook Section 2.6:

4) $y' = x - y, y(0) = 1; y(x) = 2e^{-x} + x - 1$

x	approximation	actual
0.25	0.80762	0.80762
0.5	0.71309	0.71306