

Asymptotics

Definition:

- Asymptotic Runtime: bound of execution time of an algorithm when the size of the problem goes to infinity
 - o For all cases (both best case and worst case), we are assuming the input size is **infinitely large**.
 - o When looking at best-case runtime and worst-case runtime, we want to think about how values in this infinitely large sized input may create different runtime.

2 Types of Asymptotic Questions:

- Time Complexity: How long does it take to run a program if we give it a certain input?
- Space Complexity: How much space does this program take when we run it on our computers?

3 Types of Bounds:

1. Omega Bound (Ω)
 - a. Lower bound
 - b. Describes the **fastest** a program can run as the size of the input approaches infinity
2. Big O Bound (O)
 - a. Upper bound
 - b. Describes the **slowest** a program can run as the size of the input approaches infinity
3. Theta Bound (Θ)
 - a. Tight bound
 - b. Describes how a function behaves on a given set of inputs
 - c. If we are describing how a function behaved on just 1 input, we can always use Θ bound to denote it because the outcome of running a function on a specific input is deterministic.
 - d. If we are analyzing how the function behaves on various inputs and some inputs performed better than others, then we won't be able to give an overall bound (aka. a Θ bound) for the function. → would have to use omega and big O bounds instead
 - e. However, we can use Θ bound when the **upper (big O)** and **lower (omega)** bounds are the **same**!

Rules in Analyzing Asymptotics

- Ignore lower degree terms
- Ignore constant scaling factors
- For exponential runtime, the base matters.
- For polynomial runtime, the exponent matters.
- For logarithmic runtime, the base does not matter.

Useful Formula

- Arithmetic Series
 - $1 + 2 + 3 + \dots + N = \Theta(N^2)$
- Geometric Series
 - In increasing order, proportional to the last term.
 - $1 + 2 + 4 + 8 + \dots + N = \theta(N)$
 - $1 + 2 + 4 + 8 + \dots + N^2 = \theta(N^2)$
- Log Rules
 - Power rule
 - Product rule
 - Quotient rule
 - Change of base rule

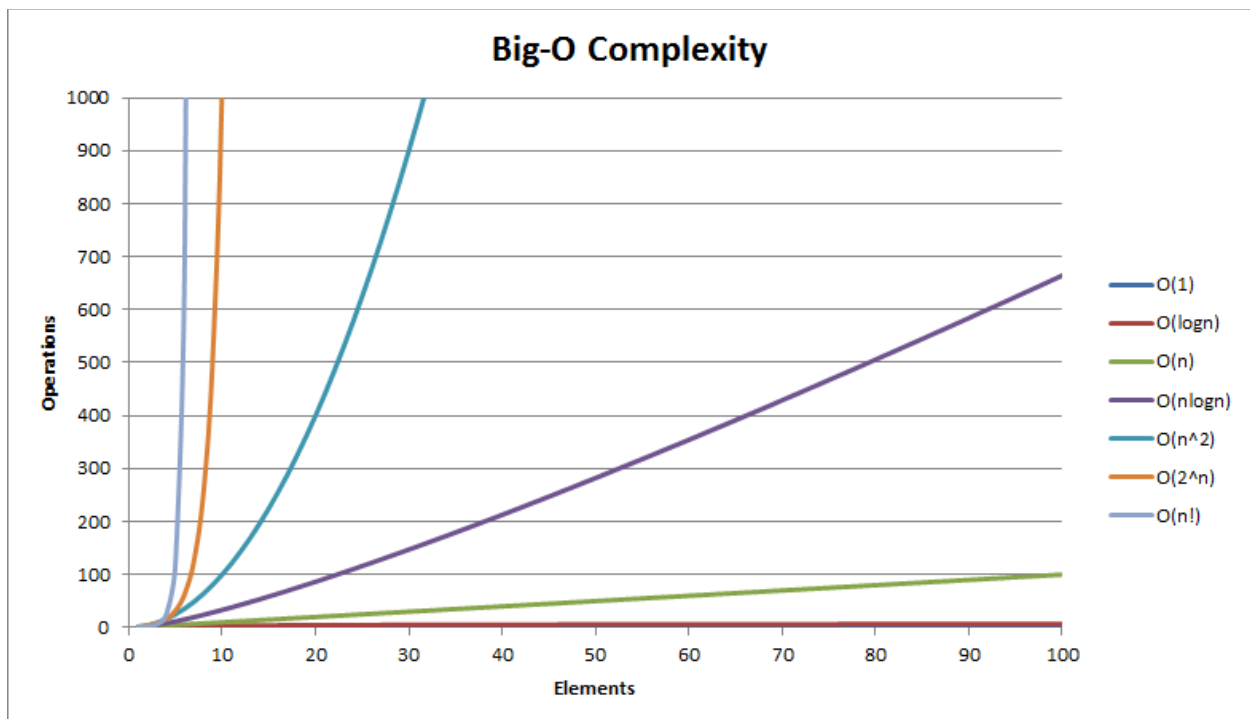


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