Math 110. Sample Final Exam

1. Express $\det(\mathrm{adj}(A))$ in terms of $\det A$, where A is an $n \times n$-matrix.

2. Solve system of linear equations:

$$\begin{array}{rl} x_1 - 2x_2 + 3x_3 - 4x_4 &= 4 \\ x_2 - x_3 + x_4 &= -3 \\ x_1 + 3x_2 \qquad - 3x_4 &= 1 \\ -7x_2 + 3x_3 + x_4 &= -3 \end{array} .$$

3. Use Sylvester's rule to find inertia indices of quadratic form:

$$x_1x_2 - x_2^2 + x_3^2 + 2x_2x_4 + x_4^2.$$

4. Transform quadratic form $x_1x_2 + x_3x_4$ to the normal form by an orthogonal transformation.

5. Find the Jordan normal form of matrix:

$$\begin{bmatrix} 0 & 3 & 3 \\ -1 & 8 & 6 \\ 2 & -14 & -10 \end{bmatrix}.$$

6. Can a non-zero anti-symmetric matrix be nilpotent? If "yes" give an example, if "no" provide a proof.

7. Classify all linear operators in $\mathbb{R}^2$ up to linear changes of coordinates.

8. Find all those values of $a_1, \dots, a_n$ for which the following matrix is nilpotent:

$$\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & . & . \\ 0 & \dots & 0 & 0 & 1 \\ -a_n & -a_{n-1} & \dots & -a_2 & -a_1 \end{bmatrix}.$$

9. Find out if the following quadratic hypersurfaces in $\mathbb{C}^4$ can be transformed into each other by linear inhomogeneous changes of coordinates:

$$z_1z_2 + z_2z_3 + z_3z_4 = 1 \quad \text{and} \quad z_1^2 + z_2^2 + z_3^2 + z_4^2 = z_1 + z_2 + z_3 + z_4.$$

10. Prove that any orthogonal transformation in $\mathbb{R}^4$ with the determinant equal to -1 has an invariant 3-dimensional subspace.