MATH H1B – Final Exam Proofs Preview

***Part I (typical)*: Wednesday, December 14, 8am-9:40am**
(one sheet of notes allowed)

***Part II (proofs)*: Wednesday, December 14, 10am-11am**

At least one of the following proofs will appear on Part II of the exam (it will be a surprise which one/ones). The variable names and/or constants might be changed, but the content of the proof will be exactly as stated below. For all proofs, you may use any of our basic axioms and theorems about arithmetic without specifically quoting them. As usual, proofs should be in paragraph form.

RULES: You are allowed to consult your class notes, any handouts from the course website, your textbook, and each other to figure these out, but NO OTHER RESOURCES. Michael and I will only answer questions if it is unclear what the question is asking, and you are not permitted to talk to anyone outside the class about the problems, or to consult other books or anything on the internet other than our course website. At the exam, you will sign an honor code indicating that you have followed these rules.

Candidate 1. (a) State Rolle's Theorem.

(b) Prove Cauchy's Mean Value Theorem.

Candidate 2. (a) State Parts I and II of the Fundamental Theorem of Calculus.

(b) Summarize the proof of FTC I using a figure and 1-2 paragraphs. (A summary will skip lots of details and computations, but it says all the main ideas used.)

(c) Prove FTC II in your own words.

Candidate 3. Using the formal definition of definite integrals (i.e. the one involving limits of Riemann sums), prove the following.

Suppose f and g are integrable functions on the interval $[a, b]$. If $f(x) \geq g(x)$ for all $x \in [a, b]$, prove that $\displaystyle\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

Candidate 4. Suppose $y_1(x)$ and $y_2(x)$ are solutions of the homogeneous differential equation

$$f(x)y'' + g(x)y' + h(x)y = 0,$$

where f, g, and h are functions of x. If A and B are real numbers, prove (in your own words) that $Ay_1(x) + By_2(x)$ is also a solution of the given differential equation. (You may use theorems we know about derivatives.)

Candidate 5. Using the formal definition of definite integrals, prove that

$$\int_1^4 6x^2 = 126.$$

You may use the following formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \text{and} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

You should include the main formulas or theorems from Chapters 9 and 17 on your page of notes. If an obscure formula is needed, especially from older material, it will be provided.