Math 16A, Solutions to Midterm Exam # 1

(1) In each case we perform the multiplication and the composition of the two functions.

(a) The product of the two functions equals
$f(x)g(x) = (x^2+4)(\sqrt{x}-1) = (x^2+4)(x^{1/2}-1) = x^{5/2} - x^2 + 4x^{1/2} - 4.$

(b) Substituting the function $g(x)$ for z into the function $f(z)$, we get
$f(g(x)) = (\sqrt{x}-1)^2 + 4 = x - 2\sqrt{x} + 5.$

(c) Multiplication of functions is commutative, so the answer is the same as in part (a), namely, $g(x)f(x) = f(x)g(x) = x^{5/2} - x^2 + 4x^{1/2} - 4.$

(d) Substituting the function $f(x)$ for z into $g(z)$, we get $g(f(x)) = \sqrt{x^2+4} - 1.$

(2) The slope of the tangent line is the value of the derivative $g'(x) = 10(x-1)^9$. So, the slope at $x = 2$ is $g'(2) = 10$. The tangent line is the line of slope 10 which passes through the point $(2, f(2)) = (2, 1)$. The equation of that line is $y - 1 = 10(x-2)$, or, equivalently, $y = 10x - 19$.

(3) This is problem 67 on page 135 in the text book. The time t at which the binoculars hit the ground is a solution of the equation $s(t) = 0$. Using the quadratic formula, or try-and-error, we find $s(t) = -16(t+2)(t-4)$. The only positive solution to the equation $s(t) = 0$ is $t = 4$, which means the binoculars hit the ground after four seconds. The velocity at time t is the derivative $v(t) = s'(t) = -32t + 32$. So, the velocity after four seconds equals $v(4) = -96$. We conclude that the speed of the binoculars when they hit the ground is 96 feet per second.

(4)

(5) This is problem 51 on page 115. We first determine $g(1)$ by simply plugging in:

$$g(1) = 5 \cdot \sqrt{f(1)} = 5 \cdot \sqrt{4} = 5 \cdot 2 = 10.$$

Using the Constant Coefficient Rule and the General Power Rule, we express the derivative of $g(x)$ in terms of the derivative of $f(x)$:

$$g'(x) = 5 \cdot \frac{d}{dx}\sqrt{f(x)} = 5 \cdot \frac{d}{dx}(f(x))^{1/2} = 5 \cdot \frac{1}{2} \cdot f(x)^{-1/2} \cdot f'(x) = \frac{5f'(x)}{2\sqrt{f(x)}}$$

Substituting $x = 1$ into this expression we get

$$g'(1) = \frac{5f'(1)}{2\sqrt{f(1)}} = \frac{5 \cdot 3}{2 \cdot \sqrt{4}} = 15/4.$$