Math 1B Practice Midterm 2 Solution
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1. **Solution:**
$\int \frac{x^2}{(x+1)^3}dx = \int \frac{(x+1)^2-2(x+1)+1}{(x+1)^3}dx = \int \frac{1}{x+1}dx - 2\int \frac{1}{(x+1)^2}dx + \int \frac{1}{(x+1)^3}dx = ln \mid x+1 \mid + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C.$

2. **Solution:**
$\int \frac{x}{\sqrt{1+x}+\sqrt{x}}dx = \int x(\sqrt{1+x} - \sqrt{x})dx = \int x\sqrt{1+x}dx - \int x^{\frac{3}{2}}dx.$
For the first integral, we make the substitution $u = x+1$, then $\int x\sqrt{1+x}dx = \int (u-1)\sqrt{u}du = \int (u^{\frac{3}{2}} - \sqrt{u})du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C.$
For the second, we have $\int x^{\frac{3}{2}}dx = \frac{2}{5}x^{\frac{5}{2}} + C.$
Combining these two results gives $\int \frac{x}{\sqrt{1+x}+\sqrt{x}}dx = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C.$

3. **Solution:**
a) $\int_0^1 \frac{x+1}{x^{\frac{4}{3}}}dx = \int_0^1 (x^{-\frac{1}{3}} + x^{-\frac{4}{3}})dx = lim_{t\to 0^+} \int_t^1 (x^{-\frac{1}{3}} + x^{-\frac{4}{3}})dx = lim_{t\to 0^+}(\frac{3}{2}x^{\frac{2}{3}} - 3x^{-\frac{1}{3}}) \mid_t^1 = lim_{t\to 0^+}(\frac{3}{2} - \frac{3}{2}t^{\frac{2}{3}} - 3 + \frac{3}{\sqrt[3]{t}}) = \frac{3}{2} - 0 - 3 + lim_{t\to 0^+}\frac{3}{\sqrt[3]{t}} = +\infty.$
Hence the improper integral is **divergent.**
b) Given the fact $lim_{x\to\infty}\frac{\frac{x}{x^2-1}}{\frac{1}{x}} = 1$, and $\int_2^\infty \frac{1}{x}dx$ is divergent, we can apply Limit Comparison Test to get that the original improper integral is **divergent.**
Remark: Another approach is to notice that $\frac{x}{x^2-1} < \frac{1}{x}$, and then use Comparison test to get the conclusion.

4. **Solution:**
The given integrand is $f(x) = ln(cosx)$, then the derivative is $f'(x) = -tanx$, the second derivative should be $f^{(2)}(x) = -sec^2(x)$. In this sense, we can take $K = sec^2(\frac{3}{4})$. One can find the value of n by solving the inequality $\frac{sec^2(\frac{3}{4})(\frac{3}{4})^3}{24n^2} < 4 \cdot 10^{-4}$.

5. **Solution:**
The arc length is $\int_1^2 \sqrt{1+(\frac{dy}{dx})^2}dx = \int_1^2 \sqrt{1+(\frac{x^2}{2} - \frac{1}{2x^2})^2}dx = \int_1^2 \sqrt{1+\frac{1}{4}(x^2 - \frac{1}{x^2})^2}dx = \int_1^2 \sqrt{\frac{1}{4}(x^2 + \frac{1}{x^2})^2}dx = \int_1^2 \frac{1}{2}(x^2 + \frac{1}{x^2})dx = (\frac{1}{6}x^3 - \frac{1}{2x}) \mid_1^2 = \frac{17}{12}.$