MATH H1B – Exam 1 Proofs Preview

Exam 1, here we come!
***Review*: Sunday, September 25, 2-4pm**
***Part I (proofs)*: Monday, September 26, 10am-11am**
***Part II (typical calc)*: Tuesday, September 27, 11am-12:30pm**

As was mentioned in the handout about the exam, the first proof on Part I will be one of the following proofs (it will be a surprise which one). The variable names and/or constants might be changed, but the content of the proof will be exactly as stated below.

For all proofs below, you may use any of our basic axioms and theorems about arithmetic without specifically quoting them. The proof should be in paragraph form. If you are doing a nonstandard proof (like contradiction or induction), the first thing you should say is "Proof by contradiction/induction". For induction proofs, make sure you clearly show and label all parts – first the base case, and then the induction step (i.e., showing that $P(n)$ is true implies $P(n+1)$ is true).

RULES: You are allowed to consult your class notes, any handouts from the course website, your textbook, and each other to figure these out, but NO OTHER RESOURCES. Michael and I will only answer questions if it is unclear what the question is asking, and you are not permitted to talk to anyone outside the class about the problems, or to consult other books or anything on the internet other than our course website. On Monday, you will sign an honor code indicating that you have followed these rules.

Candidate 1. Prove that for every $\epsilon > 0$, there exists some $\delta > 0$ such that $|x-3| < \delta$ implies $|5x-15| < \epsilon$. *Hint: to prove this, find such a δ. It's allowed to depend on ϵ.*

In addition to basic arithmetic, you may use any of the following theorems about absolute values. Given any real numbers a and b, we have:

(i) $|a+b| \leq |a| + |b|$,

(ii) $|a-b| \geq |a| - |b|$, and

(iii) $|ab| = |a| \cdot |b|$.

(Note: what you are doing here is a rigorous proof that $\lim_{x\to 3} 5x = 15$.)

Candidate 2. Prove that $2^n < n!$ for all $n \in \mathbb{Z}^+$ such that $n \geq 4$. For a positive integer A, the symbol $A!$, read "A factorial", is defined as $A! = 1 \cdot 2 \cdot 3 \cdots (A-1) \cdot A$.

Candidate 3. Suppose $x \in \mathbb{R}$ and $0 < x < 1$. Let $S = \{x^n : n \in \mathbb{Z}^+\}$.

(a) Is S is bounded above? Prove your answer.

(b) Is S bounded below? Prove your answer.

Candidate 4. Suppose A and B are nonempty subsets of $\mathbb{R}$ with $A \subset B$. Assume that B is bounded above.

(a) Prove that $\sup(A)$ and $\sup(B)$ both exist.

(b) Prove that $\sup(A) \leq \sup(B)$.

(c) If A is also bounded below, show that $\inf(A)$ exists and that $\inf(A) \leq \sup(A)$.

Recall that in homework, we proved the "infimum version" of the supremum axiom. You may use that theorem for this problem, as long as you carefully state the theorem.

NOTE: THERE ARE ONLY FOUR POTENTIAL PROOFS HERE, NOT THE FIVE I INDICATED BEFORE. I ASSUME NOBODY IS GOING TO COMPLAIN TOO MUCH.