MATH H53 PRACTICE OF MIDTERM 2

The problems here may or may not reflect what we will have in the actual exam, these are just for practice.

1. If $z = xy + xe^{y/x}$, please show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$ holds.

2. Please find the absolute maximum and minimum values of $f(x, y) = (x^2 + 2y^2)e^{-2x^2-y^2}$ on $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

3. Please evaluate the following integration

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{-(x^2+y^2)} \, \mathrm{d}x \, \mathrm{d}y.$$

4. Let D be the region enclosed by the hyperbola $x^2 - y^2 = 1$ and $x = k$ with $k > 1$. Please find the mass of the lamina that occupies the region D with density function $\rho(x, y) = x$, then find its moment of inertia about the origin.