MATH H1B EXAM #2, PART I
Monday, October 31, 2011

No calculators are permitted on this portion of the exam. You may use a notecard or half a sheet of paper filled with notes. A list of familiar Maclaurin series and a few other formulas are provided on the next page.

You must show your work for all problems, unless specifically indicated otherwise. Answers with insufficient or incorrect work will be given little (if any) credit. In particular, if you are using a test for convergence, you must explicitly show that your series meets the necessary conditions to use the test.

You are not required to write all your explanations in complete sentences, but your work should be organized and easy to read. You may use the back of any page to do scratchwork that you don't want graded. If you make a mistake, you can cleanly erase or put an X or a single line through anything we should ignore. If you need more space, draw an arrow to indicate that you will continue on the back.

Some advice: don't stay stuck on a problem for long unless you have already solved the other problems. I arranged the problems roughly in order of increasing difficulty, and I don't expect that everyone will finish everything.

This exam has 6 problems on 8 pages, including this cover sheet and the list of formulas on the next page.

Problem	Max	Score
1	16	
2	12	
3	5	
4	10	
5	10	
6	12	
Total	65	

Some useful formulas

1. (4 points each) Determine whether each of the statements below is true or false. Choose TRUE only if the statement is always true under the given circumstances. If the statement is false, give a counterexample. (You do not need to carefully justify your counterexample.)

   (a) If $\{a_n\}$ and $\{b_n\}$ are divergent sequences, then the sequence $\{a_n+b_n\}$ also diverges.

   TRUE *FALSE*

   (b) If $\{c_n\}$ and $\{d_n\}$ are divergent sequences, then the sequence $\{c_n d_n\}$ also diverges.

   TRUE *FALSE*

   (c) If $x_n \leq y_n \leq 0$ and $\sum_{n=4}^{\infty} x_n$ diverges, then $\sum_{n=4}^{\infty} y_n$ also diverges.

   TRUE *FALSE*

   (d) If $\sum_{n=0}^{\infty} a_n = A$ and $\sum_{n=0}^{\infty} b_n = B$, where $A, B \in \mathbb{R}$, then $\sum_{n=0}^{\infty} a_n b_n = AB$.

   TRUE *FALSE*

2. (6 points each) Show whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n + cos^3(\pi n)}$

(b) $\displaystyle\sum_{n=5}^{\infty} ne^{-2n^2}$

3. (5 points) Use familiar Taylor series to evaluate the following limit.

$$\lim_{x \to 0} \frac{x - \tan^{-1}(x)}{x^3}$$

4. (10 points) Let $f(x) = e^{-x^2}$. Find T_3, the degree 3 Taylor polynomial approximating f near $x = 2$. Estimate the maximum possible error for T_3 on the interval $(1, 3)$ using Taylor's Inequality. (No need to convert your answer to decimal form.)

5. (10 points) Give an example of a power series whose interval of convergence is precisely $(-1, 4]$.

6. (10 points) Express $\int \sqrt[5]{17+2y+y^2}\,dy$ as a Taylor series, and find its radius of convergence. (Hint: complete the square, and use a binomial series to start out.)