MATH H1B EXAM #2, PART II
Tuesday, November 1, 2011

Sign below to indicate that you have followed the rules in preparing for Problem #1:

You are allowed to consult your class notes, any handouts from the course website, your textbook, and each other to figure these out, but NO OTHER RESOURCES. You are not permitted to talk to anyone outside the class about the problems, or to consult other books or anything on the internet other than our course website.

Signature: ____

No calculators or notes are permitted on this portion of the exam. If you make a mistake while working, you can either erase cleanly or draw a single line (no crazy scribbling, please) through any parts we should ignore. You may use the back of any page for scratch work.

For the proofs, you may use any of our basic axioms and theorems about arithmetic without specifically quoting them. All proofs should be in paragraph form.

This exam has 6 problems on 7 pages, including this cover sheet.

Problem	Max	Score
1	5	
2	5	
3	10	
4	15	
5	10	
6	10	
Total	55	

1. (5 points) Prove that the sum of a convergent series and a divergent series is divergent. (You may use any of the basic laws about limits.)

2. (5 points) Suppose that $\{a_n\}$ is a sequence with positive terms. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ is a divergent series.

3. This problem deals with the Mean Value Theorem.

   (a) (5 points) Carefully state the Mean Value Theorem for a function named f.

   (b) (5 points) The proof of the Mean Value Theorem involves applying Rolle's Theorem to a function g which is related to f. Give a formula for the function g. (No justification is necessary for this part of the question.)

4. This problem deals with a special case of the Limit Comparison Theorem.

   (a) (10 points) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. Prove that if $\sum b_n$ is divergent and $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, then $\sum a_n$ is divergent.

   (b) (5 points) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. Give a counterexample to the statement "If $\sum b_n$ is divergent and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ is convergent."

5. (10 points) Give an ϵ-δ proof that

$$\lim_{x \to -1.5} \frac{9 - 4x^2}{3 + 2x} = 6.$$

6. (10 points) Prove that if a power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = b$ (where $b \neq 0$), then it converges whenever $|x| < |b|$. You may use any appropriate test for series convergence.