Nuclear Physics - Summary - Nuclear Properties

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This summary is based on the book Chapter 3 - 5 from Krane, Kenneth: Introductory Nuclear Physics.

Static nuclear properties are the following:
- radius
- mass
- binding energy
- angular momentum (orbital + spin)
- parity (symmetry of the wave function)
- magnetic moments (magnetic dipole moment)
- electric moments (electric quandrupole moment)
- energies of excited states

1 Nuclear Radius

The nuclear radius is measured with two different parameters:
- **mean radius**: where the density falls to half its central value. $R = R_0 A^{1/3}$
- **skin thickness**: the region where the density falls from 90% to 10%

There are two main ways to measure the nuclear radius: trough the electromagnetic force (charge distribution) or trough the strong force (matter distribution).

The **radius based on the charge distribution**, probed trough the electromagnetic (or Coulomb) force.
- **low energy scattering experiments:** the radius can be calculated from the distribution of the scattering angles.
- **K X-ray energy differences (also called isotope shift)**: comparing the energy differences between isotopes, when the electrons shift from the L shell to the K shell. The energy difference is related to the slight difference in the Coulomb energy, which depends on the nuclear radius.
- **muonic X-rays**: similar to the regular isotope shift, but with captured muons (μ^-) instead of electrons. This is more accurate since the higher mass of the μ compared to the e^- means that the μ orbits are closer to the nucleus and the different shells have larger energy differences.
- **direct measurements of the Coulomb energy differences** between **mirror nuclei**. Mirror nuclei have the same number of protons as the number of neutrons in the pair and the same number of neutrons as the number of protons in the other nucleus. Examples: ${}^3_1H_2$ and ${}^3_2He_1$; ${}^{13}_7N_6$ and ${}^{13}_6C_7$; ${}^{39}_{20}Ca_{19}$ and ${}^{39}_{19}K_{20}$. The energy difference is the measure of the Coulomb energy of the extra proton compared to the extra neutron in the mirror nuclei. The energy difference can be measured using
  - nuclear β decay where one of the protons changes to a neutron and emits a positron. The max energy of the e^+ is the Coulomb energy difference.
  - nuclear reactions: if an element is bombarded with protons occasionally the proton gets captured and knocks out a neutron from the nucleus. The minimum p^+ energy needed for this is the Coulomb energy difference.

An exotic atom is an otherwise normal atom in which one or more sub-atomic particles have been replaced by other particles of the same charge. For example, electrons may be replaced by other negatively charged particles such as muons (**muonic atoms**) or pions (**pionic atoms**). Because these substitute particles are usually unstable, exotic atoms typically have very short lifetimes.

Measuring the **nuclear density distribution** (the distribution of n^0 and p$^+$), trough the strong force.
- **high energy scattering experiments**. To probe the nuclear force, we need to overcome the Coulomb force first, we need higher energies for the scattering experiment.
- **radioactive decay**: the α decay probability depends on the radius of the nucleon. The α particle needs to escape a Coulomb potential barrier, which depends on the nuclear radius.
- **π mesonic X-rays**: The π^- interacts trough the electromagnetic and the strong force. The π^- mesons gets captured like and electron by the nucleus. First the π^- cascades down the "electron" shells to the

lowest energy state. During this photons get emitted in the X-ray energy range. Then the π^- can get absorbed into the nucleus and this "disappearance" rate can be used to measure the radius.

All methods to measure the nuclear radius give consistent results, with $R_0 = 1.2 - 1.25$ fm.

2 Nuclear Mass

Ways to **measure the nuclear mass**:

- **mass spectrometer:** has the following components:
  - ion source
  - velocity selector (B and E field) $v = \frac{E}{B}$, only a certain velocity passes after this step
  - momentum selector (B field) $mv = qBr$, separates particles based on their momentum, after the velocity selection step this effectively separates ions with different mass.
  - $m = \frac{qrB^2}{E}$
  - for high accuracy measurements we measure the mass difference between two similar sized atoms or molecules → **mass dublet method**
  - Suitable for most isotopes, except fast decaying radioactive materials.
- **nuclear reactions:**
  - Suitable for fast decaying radioactive isotopes. e.g. ^{12}N (half life of 0.01s)
  - Example: nuclear fission (x + X → $y + Y$)
  - ^{1}H + ^{14}N → ^{3}H + ^{12}N
  - By measuring the kinetic energy of the reacting particles, which is equal to the released energy in the reaction (Q)
  - Q depends on the mass difference between the initial particles (x, X) and the final particles (y, Y)
  - The kinetic energy gets distributed between the final particles based on their mass ratio. The reason for this is momentum conservation.
  - If we measure Q based in the kinetic energy of the final particles we can calculate the mass of the particle that is unknown.

Nuclear abundances: All natural materials are composed of a certain mixture of isotopes. The ratio of the different isotopes compared to each other is the nuclear abundance.

Isotopes can be separated with the following methods:

- mass spectrometer: collects the isotopes with different masses instead of measuring the mass
- **laser isotope separation:** works with two lasers.
  - Laser 1 excites the target isotope,
  - laser 2 ionises the target isotope.
  - The ionised isotopes can be deflected with a B field and collected.

Nuclear binding energy: the difference in mass energy between ${}^{A}_{Z}X_N$ and its constituents Zp^+ and Nn^0. This can also be expressed as the **mass defect** $\Delta = (m - A)c^2$

Neutron separation energy: the amount of energy that needs to be supplied to a nucleus to remove a neutron. This is the difference in binding energies between ${}^{A}_{Z}X_N$ and ${}^{A-1}_{Z}X_{N-1}$. Indicates shell structure of the nucleus.

Proton separation energy: the amount of energy that needs to be supplied to a nucleus to remove a proton. This is the difference in binding energies between ${}^{A}_{Z}X_N$ and ${}^{A-1}_{Z-1}X_N$.

The **Mass defect** is the difference between the combined mass of the protons and neutrons in a nucleus and the mass of the nucleus.

Semi empirical mass formula: A formula that can be used to approximately calculate the nuclear mass and the binding energy for various nuclei (based on the liquid drop model, also known as the 'Weizsaecker formula' or 'Bethe-Weizsaecker formula'). The binding energy component of the formula was constructed to fit the binding energy per nucleon plot. The nucleus with the highest binding energy per nucleon is iron (Fe). Fe is the most stable nucleus. We can produce energy from lighter than Fe nuclei trough nuclear fusion and from heavier elements trough nuclear fission.

The main components of the binding energy are:
- a term for the average nuclear density
- a term to correct for the surface, where nucleons interact with fewer nucleons compared to the centre
- a Coulomb term, because the p^+ are charged
- a symmetry term, which accounts for the approximately equal number of p^+ and n^0 in light nuclei
- pairing force, which depends on if the nucleons are all paired or not

$$M(A, Z) = Zm_p + Nm_n - B(A, Z)/c^2$$

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

, where $\delta = a_p A^{-3/4}$ for Z and N even and $\delta = -a_p A^{-3/4}$ for Z and N odd, and $\delta = 0$ for odd A. $a_v, a_s, a_c, a_{sym}, a_p$ are constant.

3 Nuclear Angular Momentum and Parity:

The **total angular momentum** (I) is a combination of the orbital angular momentum (l) and the spin (s).

The **parity** depends on the symmetry of the wave function. It can be even (+) or odd (-). It is denoted as I^π, for example: $0^+, 2^+$ etc.

4 Nuclear Electromagnetic Moments:

Because of the nuclear size scale, only the lowest order, non vanishing moments matter.

The nuclear dipole moment (μ)

$$\mu = \frac{e\hbar}{2m} l$$

, where $\frac{e\hbar}{2m}$ is called a magneton. μ_B is the Bohr magneton, and μ_N is the nuclear magneton.

$$\mu = g_l l \mu_N$$

, where g_l is the g factor associated with the orbital angular momentum.

$$\mu = g_s s \mu_N$$

, where g_s is the g factor associated with the spin. For a spin=1/2 point like particle (like the e^-) the $g_s = 2$. However, for the p and n, we measure very different numbers. Which means that they are not point like particles. And the neutron has an internal charge distribution.

Because of the pairing force, usually only the valance nucleon contributes to the overall magnetic moment. μ is generally small.

The electric quadrupole moment (Q) Usually only the valance nucleon contributes to the overall quadrupole moment. Q is generally small.

A unit of $10^{-28} m^2$ is called a barn (b). Barn is often used in nuclear physics.

There is a discrepancy between the calculations of Q and μ which indicates:
- Neither the p^+ or the n^0 are point like particles $\rightarrow$ both of them are made up of 3 quarks.

- The n^0 has an internal charge distribution, which can explain the non 0 magnetic moment.

The nucleons like to pair up into n-n and p-p pairs. The nuclear pairing force favours nucleons so that the μ and spin add to 0 for the pairs.

Nucleons have **excited states**:
- vibrational
- rotational (+ combination of vibration and rotation)
- pair breaking - one pair of nucleons gets broken apart and one of them goes to a higher energy level

5 Nuclear Force:

The **strong force** is one of the basic forces. It is interacting between quarks and the force carrier particles are the gluons.

The **strong nuclear force** is the force between nucleons. It is derived from the strong force, but it is not the same. It is the interaction between the nucleons (p, n) and the force carriers are mesons (e.g. π^0, π^+, π^-). One model for the strong nuclear force is the **force exchange model** where the interaction between nucleons is trough mesons. Depending on the range of the interaction, different mesons can act as the force carriers.

We can learn many things about the nucleon - nucleon force based on the **Deuteron** and nucleon - nucleon (n-p, n-n, p-p) scattering experiments.

The scattering **cross section** (σ) describes the likely hood of scattering.

The **scattering length** (a) describes the strength of the scattering.

Properties of the nuclear force are:
- **Attractive central potential** - nucleons stay together
- **Spin dependence** (spin orbit potential). The cross section of interactions depends on the spin alignment. E.g. the parahydrogen and the orthohydrogen have different scattering cross sections.
- **Non central or tensor term** - observed mixed l states in the quadrupole moment
- **Charge symmetry:** interactions between p - p are the same as between n - n
- **Near charge independence:** difference between cross sections between n - n, p - p and n - p interactions. This can be explained with the exchange force model, where the interaction is trough a meson exchange. The same particles would exchange a neutral meson (e.g. π^0) and the different particles would exchange a charged meson (e.g. π^+ or π^-). The charged and the uncharged mesons have a slightly different mass, which can influence the cross section of the interactions.
- **Repulsive at short distances.** The nucleons are not smashed into each other in the centre of the nuclei. The density distribution in the core of the nuclei is constant.
- **Depends on the relative velocity (momentum)** of the nucleons. This is based on spin polarisation observed in high energy scattering experiments.

6 Nuclear Models:

There is no single unified model to describe nuclei. Instead we use a combination of models. The two main categories for nuclear models are the Shell model and the Liquid drop model.

More realistic models combine features of these two basic models.

6.1 Shell Model

This model is analogues to the atomic model of electron shells. Similarly **nucleons are occupying shells with certain energy levels**. This model is supported trough various observations, such as the jumps in the measured nuclear radius as a function of atomic mass number (A). Observations seem to indicate a set of **magic**

numbers for the cumulative number of nucleons that occupy individual nuclear shells.

Problem: it can not explain the observed collective properties of the nucleons.

6.2 Liquid drop model

Nucleons, especially even N and even Z nucleons, have some collective properties. To explain the collective properties, the liquid drop model treats **nucleons as one single entity**. There are two sets of collective properties for nucleons:

- one for A $<$ 150 $\rightarrow$ spherical shape $\rightarrow$ excited states trough **vibration**
- 150 $<$ A $<$ 190 $\rightarrow$ **deformed ellipsoid shape** $\rightarrow$ excited states trough **rotation**

Rotational and vibrational modes can be combined. A quantum of vibration energy is a **phonon** (similar to a quantum of electromagnetic energy is a photon).

Problem: it can not explain the observed shell structure in nucleons.