Nuclear Physics - Summary - Nuclear Properties

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This summary is based on the book Chapter 3 - 5 from Krane, Kenneth: Introductory Nuclear Physics.

Static nuclear properties are the following:
- radius
- mass
- binding energy
- angular momentum (orbital + spin)
- parity (symmetry of the wave function)
- magnetic moments (magnetic dipole moment)
- electric moments (electric quandrupole moment)
- energies of excited states

1 Nuclear Radius

The nuclear radius is measured with two different parameters:
- **mean radius**: where the density falls to half its central value. $R = R_0 A^{1/3}$
- **skin thickness**: the region where the density falls from 90% to 10%

There are two main ways to measure the nuclear radius: trough the electromagnetic force (charge distribution) or trough the strong force (matter distribution).

The **radius based on the charge distribution**, probed trough the electromagnetic (or Coulomb) force.
- **low energy scattering experiments:** the radius can be calculated from the distribution of the scattering angles.
- **K X-ray energy differences (also called isotope shift)**: comparing the energy differences between isotopes, when the electrons shift from the L shell to the K shell. The energy difference is related to the slight difference in the Coulomb energy, which depends on the nuclear radius.
- **muonic X-rays**: similar to the regular isotope shift, but with captured muons (μ^-) instead of electrons. This is more accurate since the higher mass of the μ compared to the e^- means that the μ orbits are closer to the nucleus and the different shells have larger energy differences.
- **direct measurements of the Coulomb energy differences** between **mirror nuclei**. Mirror nuclei have the same number of protons as the number of neutrons in the pair and the same number of neutrons as the number of protons in the other nucleus. Examples: ${}^3_1H_2$ and ${}^3_2He_1$; ${}^{13}_7N_6$ and ${}^{13}_6C_7$; ${}^{39}_{20}Ca_{19}$ and ${}^{39}_{19}K_{20}$. The energy difference is the measure of the Coulomb energy of the extra proton compared to the extra neutron in the mirror nuclei. The energy difference can be measured using
  - nuclear β decay where one of the protons changes to a neutron and emits a positron. The max energy of the e^+ is the Coulomb energy difference.
  - nuclear reactions: if an element is bombarded with protons occasionally the proton gets captured and knocks out a neutron from the nucleus. The minimum p^+ energy needed for this is the Coulomb energy difference.

An exotic atom is an otherwise normal atom in which one or more sub-atomic particles have been replaced by other particles of the same charge. For example, electrons may be replaced by other negatively charged particles such as muons (**muonic atoms**) or pions (**pionic atoms**). Because these substitute particles are usually unstable, exotic atoms typically have very short lifetimes.

Measuring the **nuclear density distribution** (the distribution of n^0 and p^+), trough the strong force.
- **high energy scattering experiments**. To probe the nuclear force, we need to overcome the Coulomb force first, we need higher energies for the scattering experiment.
- **radioactive decay**: the α decay probability depends on the radius of the nucleon. The α particle needs to escape a Coulomb potential barrier, which depends on the nuclear radius.
- **π mesonic X-rays**: The π^- interacts trough the electromagnetic and the strong force. The π^- mesons gets captured like and electron by the nucleus. First the π^- cascades down the "electron" shells to the

lowest energy state. During this photons get emitted in the X-ray energy range. Then the π^- can get absorbed into the nucleus and this "disappearance" rate can be used to measure the radius.

All methods to measure the nuclear radius give consistent results, with $R_0 = 1.2 - 1.25$ fm.

2 Nuclear Mass

Ways to **measure the nuclear mass**:
- **mass spectrometer:** has the following components:
  - ion source
  - velocity selector (B and E field) $v = \frac{E}{B}$, only a certain velocity passes after this step
  - momentum selector (B field) $mv = qBr$, separates particles based on their momentum, after the velocity selection step this effectively separates ions with different mass.
  - $m = \frac{qrB^2}{E}$
  - for high accuracy measurements we measure the mass difference between two similar sized atoms or molecules $\rightarrow$ **mass dublet method**
  - Suitable for most isotopes, except fast decaying radioactive materials.
- **nuclear reactions:**
  - Suitable for fast decaying radioactive isotopes. e.g. ^{12}N (half life of 0.01s)
  - Example: nuclear fission (x + X $\rightarrow y + Y$)
  - ^{1}H + ^{14}N $\rightarrow$ ^{3}H + ^{12}N
  - By measuring the kinetic energy of the reacting particles, which is equal to the released energy in the reaction (Q)
  - Q depends on the mass difference between the initial particles (x, X) and the final particles (y, Y)
  - The kinetic energy gets distributed between the final particles based on their mass ratio. The reason for this is momentum conservation.
  - If we measure Q based in the kinetic energy of the final particles we can calculate the mass of the particle that is unknown.

Nuclear abundances: All natural materials are composed of a certain mixture of isotopes. The ratio of the different isotopes compared to each other is the nuclear abundance.

Isotopes can be separated with the following methods:
- mass spectrometer: collects the isotopes with different masses instead of measuring the mass
- **laser isotope separation:** works with two lasers.
  - Laser 1 excites the target isotope,
  - laser 2 ionises the target isotope.
  - The ionised isotopes can be deflected with a B field and collected.

Nuclear binding energy: the difference in mass energy between $^A_Z X_N$ and its constituents Zp^+ and Nn^0. This can also be expressed as the **mass defect** $\Delta = (m - A)c^2$

Neutron separation energy: the amount of energy that needs to be supplied to a nucleus to remove a neutron. This is the difference in binding energies between $^A_Z X_N$ and $^{A-1}_{Z} X_{N-1}$. Indicates shell structure of the nucleus.

Proton separation energy: the amount of energy that needs to be supplied to a nucleus to remove a proton. This is the difference in binding energies between $^A_Z X_N$ and $^{A-1}_{Z-1} X_N$.

The **Mass defect** is the difference between the combined mass of the protons and neutrons in a nucleus and the mass of the nucleus.

Semi empirical mass formula: A formula that can be used to approximately calculate the nuclear mass and the binding energy for various nuclei (based on the liquid drop model, also known as the 'Weizsaecker formula' or 'Bethe-Weizsaecker formula'). The binding energy component of the formula was constructed to