Nuclear Physics - Summary - Nuclear Decays

by Dr. Helga Dénes (hdenes@yachaytech.edu.ec)

This summary is based on the book Chapters 6.1, 8-10 from Kenneth Krane: Introductory Nuclear Physics.

1 Nuclear Decays

parent radio nucleid → spontaneous decay → daughter nucleid

Types of decays:
- α decay → α particle (^4He) gets ejected or cluster decay (heavier particle gets ejected)
- β decay → β particle (e^-, e^+) + ν gets ejected
- γ decay → nuclear energy transition → γ gets ejected, this is usually following and α or β decay

Decays follow an **exponential decay law**:

$$\lambda = \frac{dN/dt}{N}$$

where λ is the **decay or disintegration constant**, N is the number of nuclei at t time, and dN nuclei decay in dt time

$$N(t) = N_0 e^{\lambda t}$$

where N_0 is the number of nuclei at $t = 0$.
The **half life** ($t_{1/2}$) is the time it takes for half of the initial nuclei to decay.

$$t_{1/2} = \frac{0.693}{\lambda}$$

The **average lifetime** (τ) is the average time a nucleus is likely tu survive befor it decays.

$$\tau = \frac{1}{\lambda}$$

The **activity (A)** is the rate at which decays occur in a sample:

$$A(t) = \lambda N(t) = A_0 e^{-\lambda t}$$

where A_0 is the initial activity in a sample at $t = 0$, $A_0 = \lambda N_0$. The units of activity are:
- Curie Ci (1Ci = 3.7^{10} decays/second)
- Bequerel Bq (1Bq = 1 decay/second)

2 α decay

- The Coulomb repulsion ($\sim Z^2$) effect becomes important for heavy nuclei compared to the strong force ($\sim A$).
- energetically the α particle is the best to eject, since it results in the largest energy release for a light particle.
- other nuclei that can be ejected: ^{12}C but these are very rare, they have a much longer half-life compared to the decay trough ^4He

The basic reaction:

$$^A_Z X_N \rightarrow^{A-4}_{Z-2} X'_{N-2} +^4 He$$

The energy release can be calculated from the mass difference:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha$$

$$T_\alpha = \frac{Q}{1 + m_\alpha/m_{X'}}$$

since the mass of the alpha particle is relatively small to the mass of the daughter particle, we can also use the following approximation for calculating the kinetic energy of the α particle:

$$T_\alpha = Q(1 - 4/A)$$

- The released energy gets distributed as kinetic energy between the daughter and the α particle. The α particle gets most of the energy (~98%).
- The small energy for the daughter is effectively a small recoil.
- Q must be > 0 for decays and λ must be not too small.
- If we can measure T_α $\rightarrow$ we can calculate Q $\rightarrow$ and we can measure the atomic mass for X' if the mass of X is known.
- **large Q $\rightarrow$ short half life**
- even Z and N $\rightarrow$ short half life
- odd-even and odd-odd nuclei have relatively longer half lives
- **adding n^0 to a nucleus increases the half life** $\rightarrow$ The strong nuclear force from the extra n^0 helps to balance the effects of the Coulomb force.

2.1 Theory of α decay

The one body model:

- The α particle is already formed inside the nucleus and is orbiting the daughter particle $\rightarrow$ at some point the α particle tunnels out of the potential $\rightarrow$ α decay
- we can model the α decay with a simple potential well + Coulomb potential barrier
- from the simple model we can calculate the tunneling probability $\rightarrow$ the λ $\rightarrow$ the half life

The desintegration probability:

$$\lambda = fP$$

where f is the frequency at which the α particle approaches the edge of the potential and P is the tunnelling probability. f can be estimated from the kinetic energy of the α particle.

$$P = e^{-2G}$$

where G is the Gamow factor:

$$G = \sqrt{\frac{2m}{\hbar^2}} \int_a^b [V(r) - Q]^{1/2} dr$$

$$G = \sqrt{\frac{2m}{\hbar^2 Q}} \frac{zZ'e^2}{4\pi\epsilon_0} [arccos\sqrt{x} - \sqrt{x(1-x)}]$$

where $x = \frac{a}{b} = \frac{Q}{B}$
From this the half-life can be calculated as:

$$t_{1/2} = 0.693 \frac{a}{c} \sqrt{\frac{mc^2}{2(V_0 + Q)}} e^{2\sqrt{\frac{2mc^2}{(\hbar c)^2 Q}} \frac{zZ'e^2}{4\pi\epsilon_0} (\frac{\pi}{2} - 2\sqrt{\frac{Q}{B}})}$$

This is a simplified model, which is a relatively good approach and gives reasonable estimates for $t_{1/2}$. However, there are some discrepancies due between calculations and measurements. These are due to:

- this simple model does not use the exact wave function
- it does not consider the momentum of the α particle
- the shape of the nucleus is not symmetrical (heavy nuclei are deformed) $\rightarrow$ the radius is not the same over the nucleus $\rightarrow$ **the tunnelling probability depends strongly on the radius (Coulomb barrier) of the nucleus.**

Calculating half lives shows which decays are more likely or less likely to happen. For a decay to happen the half life can not be too long. Example: Some elements can decay trough the emission of ^{12}C, however the half-life associated with this decay is much longer than the half-life associated with the 4He decay for the same element $\rightarrow$ the ^{12}C decay is very rare.

Nucleon emission - Proton decay (decay trough ejecting a p^+) for most nuclei energetically forbidden, but can happen for some nuclei with negative proton separation energies (S_p). Example: $^{151}Lu \rightarrow {}^{150}Yb + p^+$

2.2 Angular momentum and parity

- ^{4}He spin = 0 (both n and p are paired), only the orbital component (l) matters.
- parity change: $(-1)^{l_\alpha}$
- parity selection rule → which transition can occur:
  - if initial and final are the same: l_α even
  - if initial and final are not the same: l_α odd
- a given initial state can decay into a number of final states (excited states - rotation and vibration) in the daughter → "fine structure" of α decay
- decays to different energy levels:
  - centrifugal force raises the barrier (rotational excitation)
  - excitation energy lowers the barrier
- relative decay probability: decaying into the 0^+ state is the most likely compared to the $2^+, 4^+$ or 6^+ states → the wave function is more different for the later states, it is easier to go from 0^+ to 0^+ because the wave function is very similar.
- parity: $0 \to 3$ $l_\alpha = 3$ odd → $0^+ \to 3^-$ possible, $0^+ \to 3^+$ not possible
- if the initial and final spin are not 0 more combinations are possible
- the angular distribution of the ejected α particles indicates the shape of the nucleus and the angular momentum of the nucleus.

2.3 Applications:

- power source → thermoelectric generator (space crafts (e.g. ^{238}Pu), pacemaker)
- cancer treatment (e.g. ^{223}Ra)
- smoke detectors (e.g. ^{241}Am)

Health effects: Not dangerous unless α decaying materials get inhaled or ingested.

3 β decay

Types:

- $\beta^- : n^0 \to p^+ + e^- + \bar{\nu}_e$
- $\beta^+ : p^+ \to n^0 + e^+ + \nu_e$
- e^- capture: $e^- + p^+ \to n^0 + \nu_e$
- inverse β decay (n or p + ν_e or $\bar{\nu}_e$ → p or n + e$^-$ or e$^+$)
- double β decay (two simultaneous β^- or β^+ decays)

- β^+ and e^- capture only occur for bound p^+ in nuclei.
- A stays the same, Z and N can change
- unstable nuclei with too many protons or neutrons decay until they reach a stable state
- β decay needs to create new particles ($e^-, e^+, \nu, \bar{\nu}$)

3.1 Energy release:

The energy spectrum of the e^- is continuous → not a 2 body interaction → additional particle: the ν (neutral, spin = 1/2). In contrast α decay is a 2 body interaction and the α particle has a discrete energy spectrum.

3.1.1 β decay of free n ($t_{1/2} \sim$ 10 minutes):

$$n \to P^+ + e^- + \bar{\nu}$$

$$Q = (m_n - m_p - m_e - m_\nu)c^2$$

$$Q = T_p + T_e + T_\nu$$

- T_p is a small recoil
- the e^- and the $\bar{\nu}$ share most of the energy → continuous spectrum
- from Q and the measured masses we know that ν has a very small mass
- because of the small mass of the e^- and the ν → relativistic motion

3.1.2 β^- decay in nuclei:

$$ {}^A_Z X_N \rightarrow {}^A_{Z+1} X_{N-1} + e^- + \bar{\nu} $$

$$ Q_{\beta^-} = [m(^A X) - m(^A X')]c^2 $$

This can be used for measuring the mass of the daughter nucleus.

$$ Q_{\beta^-} = T_e + E_{\bar{\nu}} $$

3.1.3 β^+ decay in nuclei:

$$ {}^A_Z X_N \rightarrow {}^A_{Z-1} X_{N+1} + e^+ + \nu $$

$$ Q_{\beta^+} = [m(^A X) - m(^A X') - 2m_e]c^2 $$

3.1.4 e^- capture in nuclei:

$$ {}^A_Z X_N + e^- \rightarrow {}^A_{Z-1} X_{N+1} + \nu $$

- if an e^- gets captured from an e^- orbit $\rightarrow \gamma$ emission from other e^- filling the hole of the captured e^-
- β^+ and e^- capture lead to the same final nucleus, but both are not always energetically possible. β^+ needs a larger energy difference between the initial and final states. $\rightarrow$ If β^+ is possible e^- capture is possible too, but not the other way around.
- e^- capture happens for p^+ rich nuclei that need to decay, but the energy difference between the parent and the daughter is not high enough to cover the production of a e^+. In this case, capturing an e^- is energetically more favourable. e.g. ^{83}Ra + $e^- \rightarrow$ ^{83}Kr + ν_e

3.2 Fermi theory of β decay:

- β decay is interpreted as a weak interaction
- If we treat the decay rate as a result of a weak perturbation than this is also called **Fermi's Golden Rule**
- **The probability of the transition depends on 3 factors**:
  1. The availability of the final states $p^2(Q - T_e)^2$
  2. The Fermi function, which accounts for the nuclear Coulomb field $F(Z', p)$
  3. The nuclear matrix element $|M_{fi}|^2$ which accounts for the exact wave function of the initial and final states, and the additional momentum from the forbidden terms $S(p, q)$
- there are **allowed decays** and **forbidden decays depending on the nuclear matrix element**. Forbidden decays are not actually forbidden, they are just less likely to happen compared to the allowed decays.
- Actually forbidden decays are restricted by the angular momentum and the parity.

The momentum spectrum:

$$ N(p) \propto p^2(Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q) $$

- Experimental test: Kurie plot (plotting $\sqrt{\frac{N(p)}{p^2 F(Z',p)}}$ against T_e) $\rightarrow$ the intercept with the x-axis gives Q
- large range of β decay half lives (differences are due to differences in M_{fi}, which corresponds to differences in the nuclear wave functions)
- the shortest $t_{1/2}$ decays are called superallowed decays
- β decay can have $t_{1/2}$ between milliseconds and 10^6 years, decay probabilities are very small for $l > 0$
- The full theory of β decay: Fermi model + particle exchange (W, Z bosons) $\rightarrow$ interaction trough the **weak force**

3.3 Angular momentum and parity rules

3.3.1 No orbital angular momentum

- $s_e = 1/2, s_\nu = 1/2 \rightarrow$ parallel or antiparallel spin $\rightarrow$ two types of decays:
  - parallel spins: **Gamow-Teller decay** S=1
  - anti-parallel spins: **Fermi decay** S=0
- $\Delta I = |I_i - I_f|$
- parity: $(-1)^l$
- allowed decay selection rules:
  - $l = 0, \Delta I = 0, 1; \Delta\pi = \text{no}$
  - first forbidden decay: $l = 1, \Delta I = 0, 1, 2; \Delta\pi = \text{yes}$
  - second forbidden decay: $l = 2, \Delta I = 2, 3; \Delta\pi = \text{no}$
  - third forbidden decay: $l = 3, \Delta I = 3, 4; \Delta\pi = \text{yes}$
- decay probabilities are very small for $l > 0 \rightarrow$ they are called forbidden decays
- the probability of the higher forbidden decays is smaller compared to the lower forbidden decays.
- It is also possible to have mixed Fermi-GT decays. The ratio of the mixing is determined by the initial and final wave functions.

3.4 Additional types of β decay

3.4.1 Inverse β decay

- Capture of ν or $\bar{\nu}$
- Important for detecting neutrinos.
- Note the **lepton conservation** rules! If we have a lepton/anti-lepton on one side then we need to have the same amount of leptons/anti-leptons on the other side.

$\bar{\nu}_e + p^+ \rightarrow n^0 + e^+$
$\nu_e + n^0 \rightarrow p^+ + e^-$

3.4.2 Double β decay ($\beta\beta$ decay)

${}^A_Z N_N \rightarrow^A_Z -2X'_N + 2 + 2e^+ + 2\nu$ (similar for β^-)

- two simultaneous β decays
- this decay happens when a *beta* decay would only be possible from the ground state to a higher energy state in the daughter. Since these decays have very low probability, energetically it is much better to do a double β decay. e.g. ^{48}Ca $\rightarrow$ ^{48}Ti + $2e^-$ + 2 $\bar{\nu}_e$
- Sometimes one β decay would have a negative Q value, but the double β decay has a positive Q value.
- very difficult to detect because $\beta\beta$ decay has very long half lives
- Interesting because detecting a neutrino less double β decay would prove that the ν is its own anti particle. If this is true, then the neutrino would be a Majorana particle.
- **A Majorana particle, is a fermion that is its own antiparticle. Regular fermions with distinct anti particles are called Dirac fermions.**

3.4.3 β delayed nucleon emission

- Precursor: β decay into an excited state $\rightarrow$ nucleon emission (n^0, p^+)
- The energy of the excited states must be high. $\rightarrow$ It needs to cover the separation energy of the nucleon + the kinetic energy of the emitted particle + a small recoil energy
- Very important for the operation of fission based nuclear reactors
- Can be used to map the energies and the populations of exited states
- Example: ^{17}N $\rightarrow$ ^{17}O* + e^- + $\bar{\nu}_e$ $\rightarrow$ ^{16}O + n^0

3.5 Applications:

- β light (contains tricium and phosphor) e.g. emergency lights
- monitoring thickness of materials (e.g. paper)
- cancer treatment (RNT radio nucleid treatment)

- medical imaging, e.g. PET scans (Positron emission tomography)
- radio carbon dating (ratio of ^{14}C and ^{12}C)

Health effects: moderately penetrating radiation, relatively easy to shield

4 γ decay

- γ decay is an **energy transition** of the nucleus trough a photon emission
- Typically very high energy: 0.1 - 10 MeV $\rightarrow$ in the γ wavelength regime (atomic transitions are in the eV range)
- very usefull to study excited states
- Most α and β decays are followed by a γ decay

Energetics:

$$E_i = E_f + E_\gamma + T_R$$

where E_i is the initial state, E_f is the final state, T_R is the recoil energy and E_γ is the energy of the emitted γ.

$$\Delta E = E_i + E_f$$

$$\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2}$$

$$E_\gamma \approx \Delta E + \frac{\Delta E^2}{2Mc^2}$$

$$\Delta E \ll Mc^2$$

$$\Delta E \sim E_\gamma$$

- low energy γ rays $\rightarrow$ small recoil ($\sim$ 1 eV)
- high energy γ rays $\rightarrow$ large recoil ($\sim$ 100 eV)$\rightarrow$ radiation damage (can remove atom from material lattice)

4.1 Angular momentum and parity

- orbital angular momentum $\rightarrow$ L determines the moment: dipole, quadrupole, etc.
  - L=0 $\rightarrow$ monopole $\rightarrow$ no γ emission
  - L=1 $\rightarrow$ dipole
  - L=2 $\rightarrow$ quadrupole
- parity determines if the radiation is electric or magnetic in nature:
  - $\Delta\pi$ = no $\rightarrow$ even parity $\rightarrow$ electric
  - $\Delta\pi$ = yes $\rightarrow$ odd parity $\rightarrow$ magnetic
- generally several multipoles can be emitted $\rightarrow$ mixture of emissions
- if $I_i = I_f = 0$ (possible for a few even-even nuclei) the first excited state is 0^+ and the ground state is 0^+ $\rightarrow$ no γ emission $\rightarrow$ internal conversion

4.2 Internal conversion

- instead of a γ an e^- from the atom gets the ΔE and gets ejected from the atom
- this is then followed by a cascade of γ emission from other e^--s filling the hole left by the decay (much lower energy compared to the γ decay photon)
- internal conversion electrons have **discrete energies** which depend on the excitation energy of the nucleus and the binding energy of the e^- (e.g. which electron shell it was on: K, L, M):

$$T_e = \Delta E - B$$

where T_e is the kinetic energy of the e^- and B is the binding energy of the electron.

- generally both γ emission and internal conversion can happen, they both have a decay probability ($\lambda = \lambda_\gamma + \lambda_e$), the probabilities depend on the individual isotopes

4.3 Applications

- γ ray spectroscopy $\rightarrow$ map excited states
- cancer treatment
- scanning containers
- sterilisation of medical equipment and food
- treating art objects (e.g. killing insects and bacteria)

Health effects: γ rays are the worst for our health. γ radiation can only be shielded with thick layers of lead and concrete. Small amount of γ ray radiation can lead to cancer, large amount of γ radiation can lead to radiation sickness and death.