

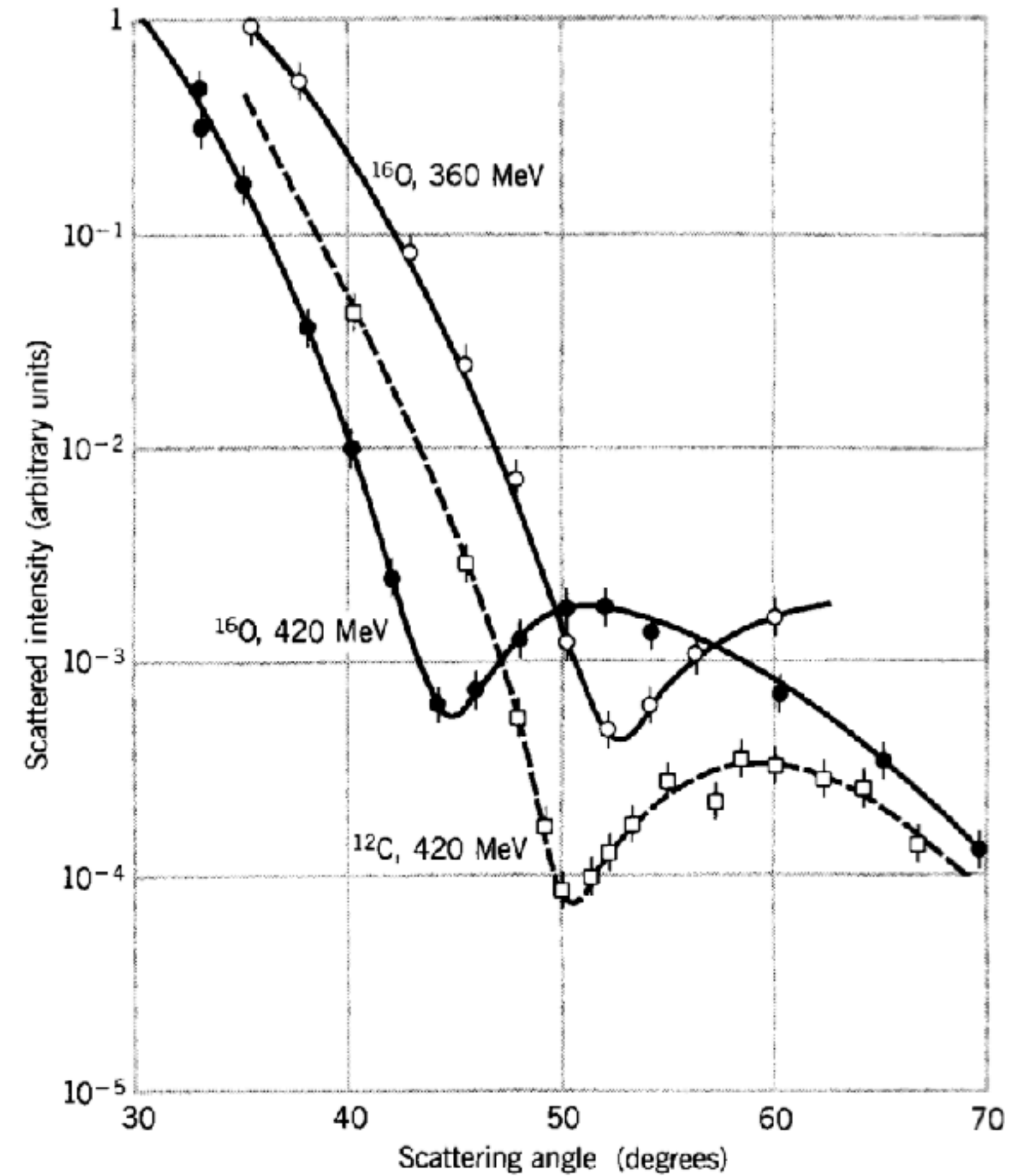
# Introduction to Nuclear and Particle Physics

**Nuclear structure**

**Helga Dénes 2023 S2 Yachay Tech**

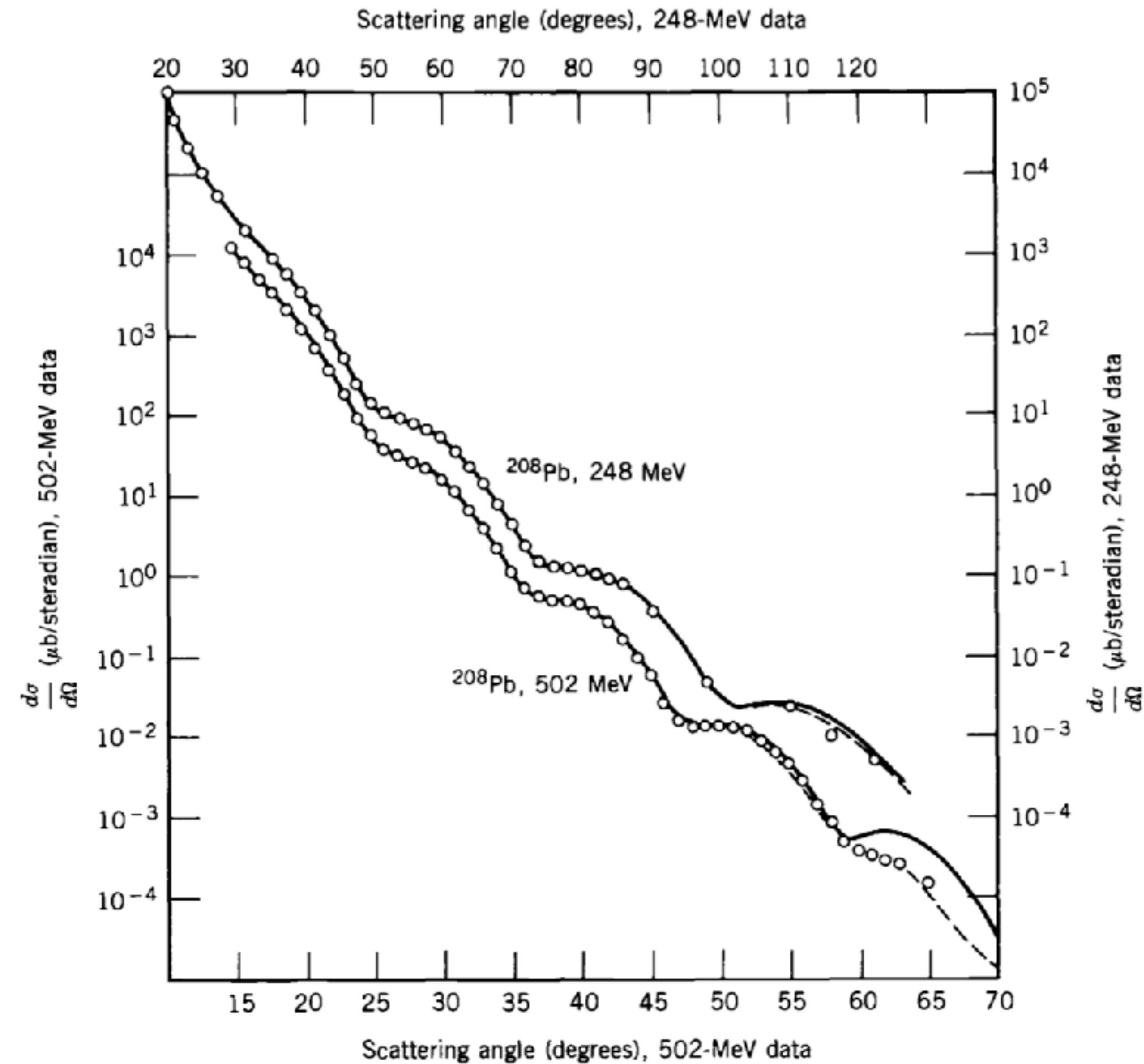
[hdenes@yachaytech.edu.ec](mailto:hdenes@yachaytech.edu.ec)

# Electron scattering



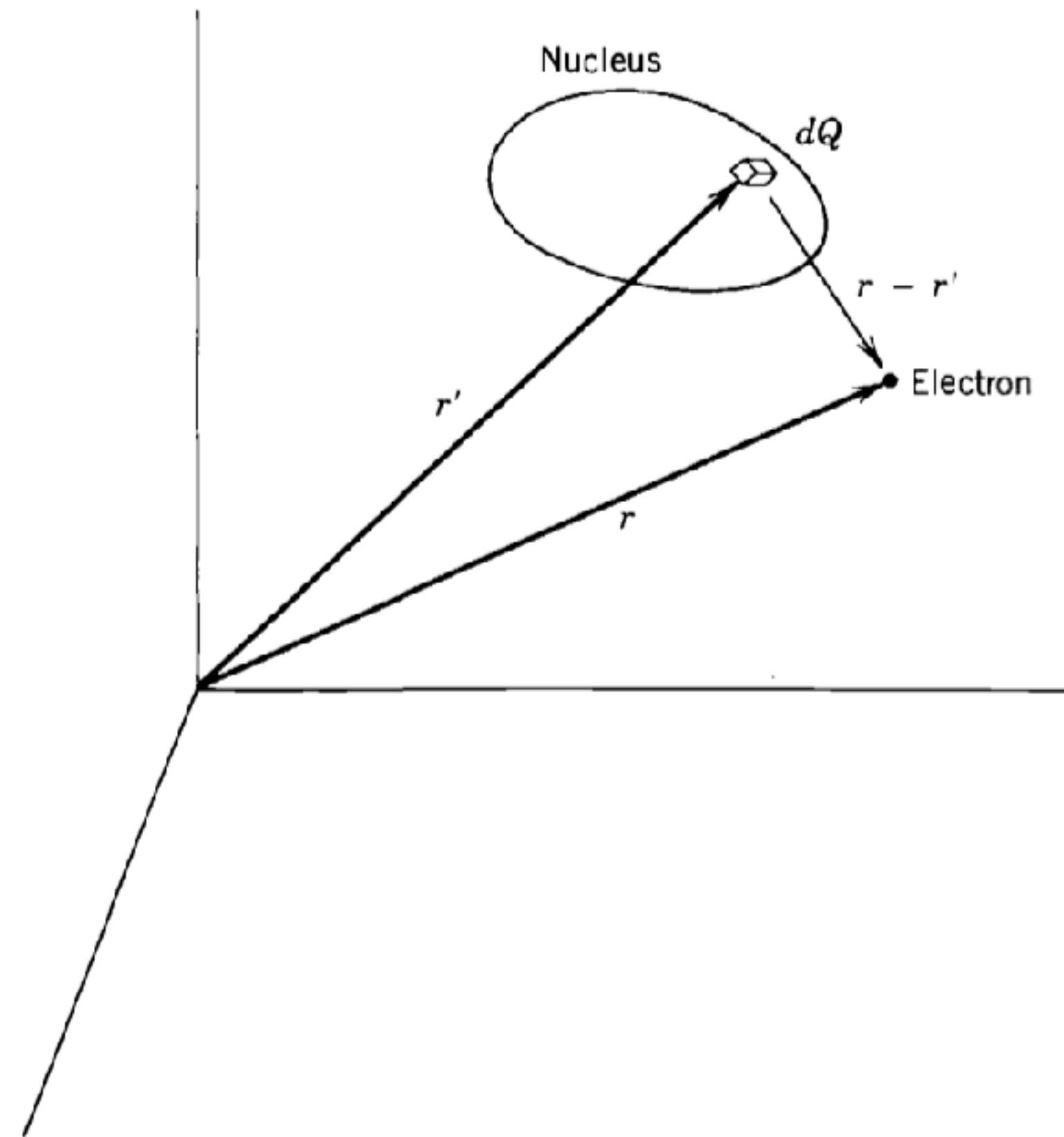
**Figure 3.1** Electron scattering from  $^{16}\text{O}$  and  $^{12}\text{C}$ . The shape of the cross section is somewhat similar to that of diffraction patterns obtained with light waves. The data come from early experiments at the Stanford Linear Accelerator Center (H. F. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

# Electron scattering



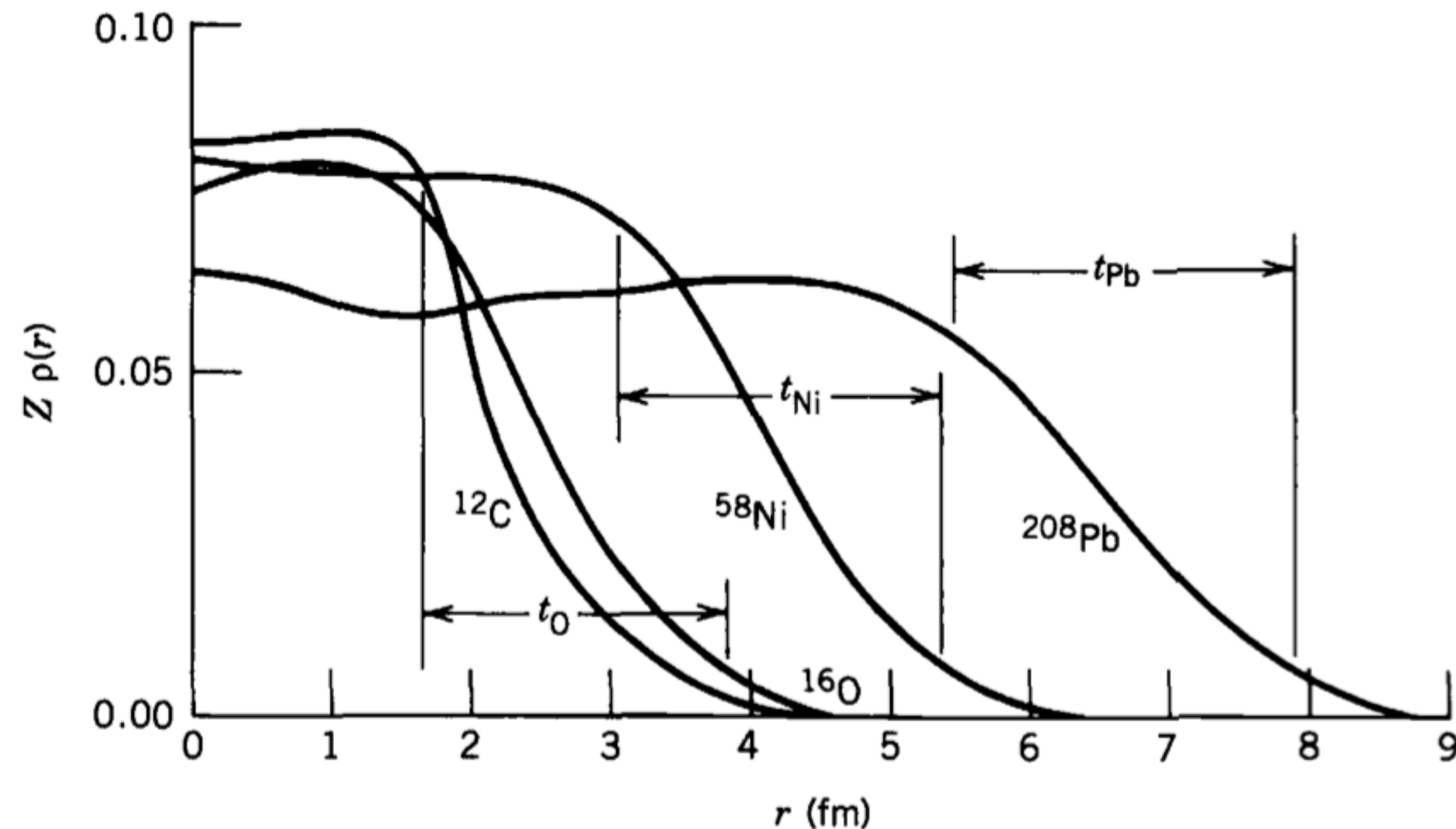
**Figure 3.2** Elastic scattering of electrons from  $^{208}\text{Pb}$ . Note the different vertical and horizontal scales for the two energies. This also shows diffractionlike behavior, but lacks sharp minima. (J. Heisenberg et al., *Phys. Rev. Lett.* **23**, 1402 (1969).)

# Electron scattering



**Figure 3.3** The geometry of scattering experiments. The origin of coordinates is located arbitrarily. The vector  $\mathbf{r}'$  locates an element of charge  $dQ$  within the nucleus, and the vector  $\mathbf{r}$  defines the position of the electron.

# Electron scattering



**Figure 3.4** The radial charge distribution of several nuclei determined from electron scattering. The skin thickness  $t$  is shown for O, Ni, and Pb; its value is roughly constant at 2.3 fm. The central density changes very little from the lightest nuclei to the heaviest. These distributions were adapted from R. C. Barrett and D. F. Jackson, *Nuclear Sizes and Structure* (Oxford: Clarendon, 1977), which gives more detail on methods of determining  $\rho(r)$ .

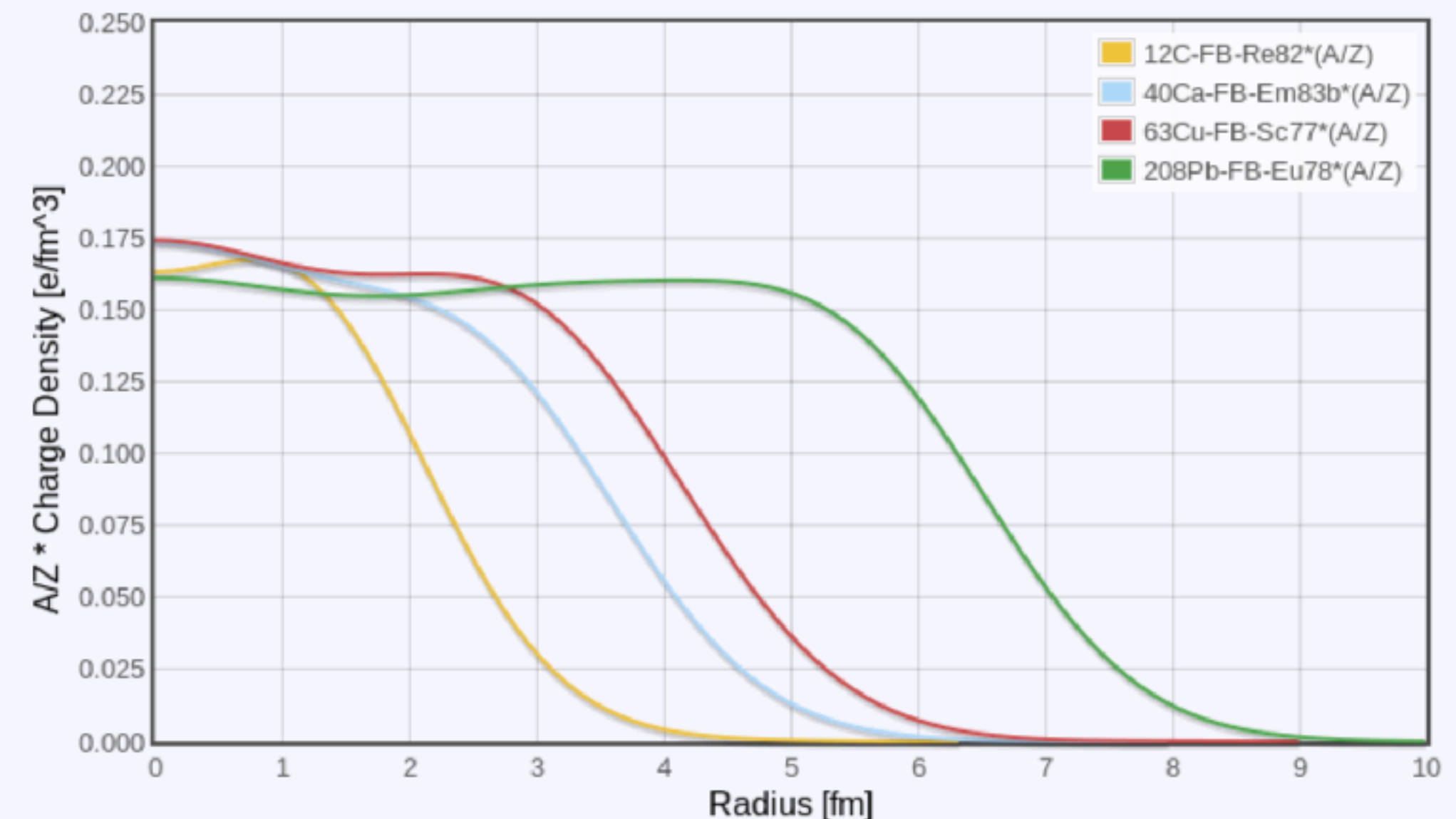
# Electron scattering

Plot and compare various nuclear charge densities with this online too:

The data for this plots was measured with **electron scattering experiments**.

<https://discovery.phys.virginia.edu/research/groups/ncd/index.html>

## Nuclei Charge Density Archive

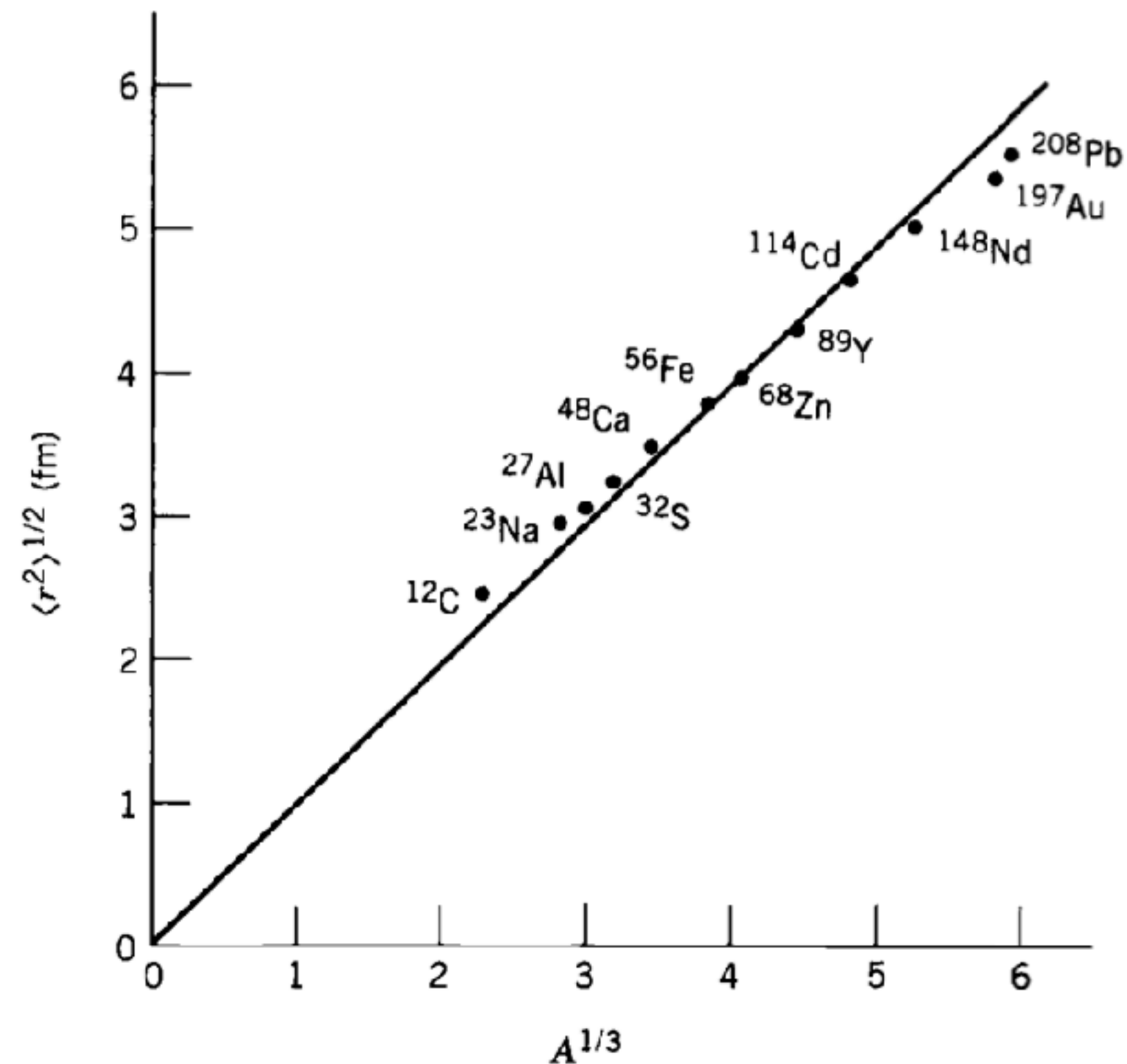


Welcome to the Nuclear Charge Density archive

We have collected here data from Atomic and Nuclear Data Tables, Volumes 14, 36 and 60, which provide a variety of fits for nuclear charge density extracted from elastic electron-nucleus scattering. This webpage was created in order to have a digital collection of raw data online that could then be used to calculate the charge density using Sum of Gaussian, Fourier Bessel, or Charge Density distribution formulas.

Currently this webpage provides data files along with C++ code to calculate charge densities  $\rho_{ch}$ , and adjusted charge densities  $(A/Z)*\rho_{ch}$

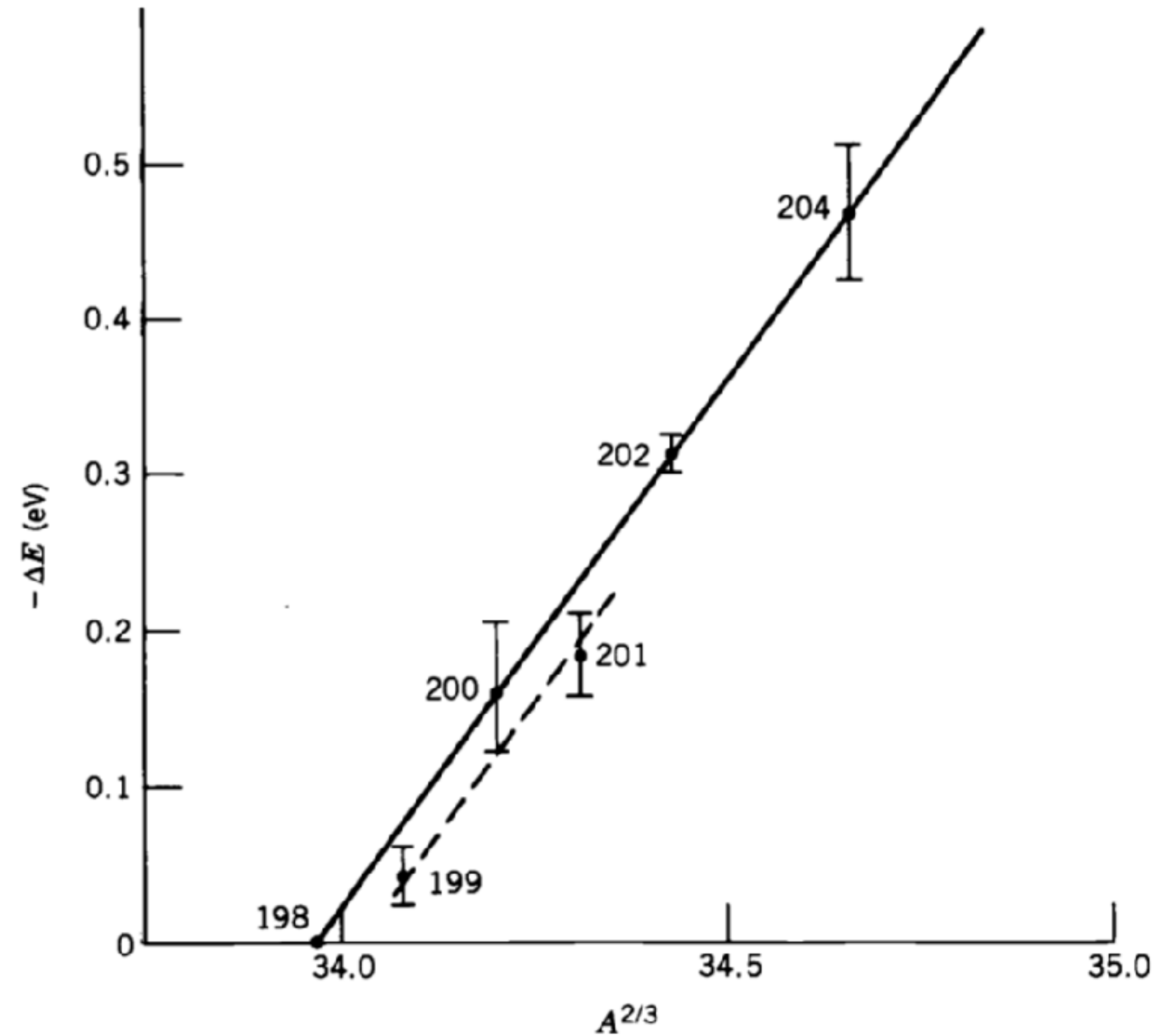
# Electron scattering



**Figure 3.5** The rms nuclear radius determined from electron scattering experiments. The slope of the straight line gives  $R_0 = 1.23$  fm. (The line is not a true fit to the data points, but is forced to go through the origin to satisfy the equation  $R = R_0 A^{1/3}$ .) The error bars are typically smaller than the size of the points ( $\pm 0.01$  fm). More complete listings of data and references can be found in the review of C. W. de Jager et al., *Atomic Data and Nuclear Data Tables* 14, 479 (1974).



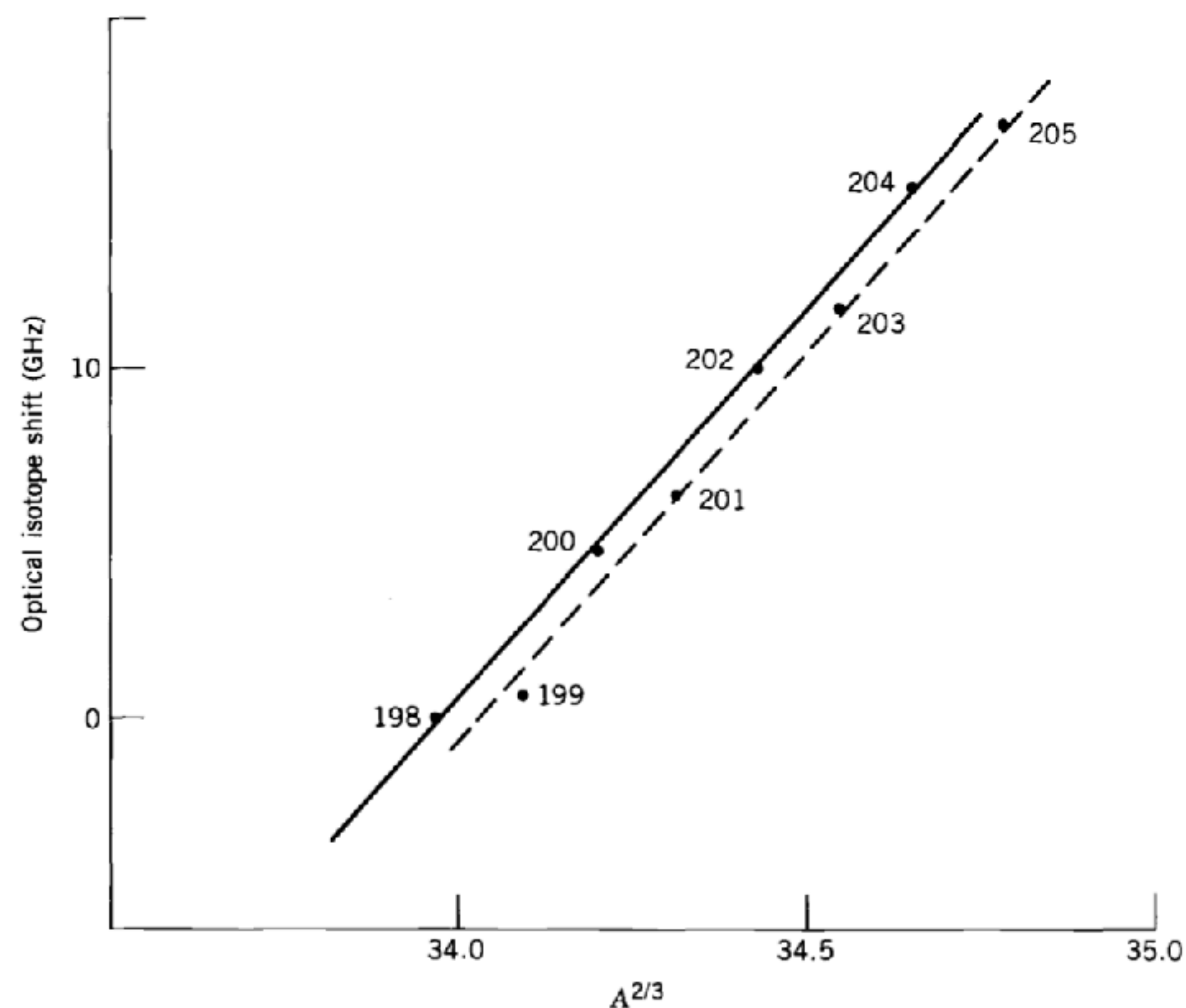
# Isotope shift



**Figure 3.6** K X-ray isotope shifts in Hg. The energy of the K X ray in Hg is about 100 keV, so the relative isotope shift is of the order of  $10^{-6}$ . The data show the predicted dependence on  $A^{2/3}$ . There is an “odd-even” shift in radius of odd-mass nuclei relative to their even- $A$  neighbors, brought about by the orbit of the odd particle. For this reason, odd- $A$  isotopes must be plotted separately from even- $A$  isotopes. Both groups, however, show the  $A^{2/3}$  dependence. The data are taken from P. L. Lee et al., *Phys. Rev. C* **17**, 1859 (1978).



# Isotope shift



**Figure 3.7** Optical isotope shifts in Hg isotopes from 198 to 205, measured relative to 198. These data were obtained through laser spectroscopy; the experimental uncertainties are about  $\pm 1\%$ . The optical transition used for these measurements has a wavelength of 253.7 nm, and the isotope shift is therefore about one part in  $10^7$ . Compare these results with Figure 3.6. Data taken from J. Bonn et al., *Z. Phys. A* **276**, 203 (1976).

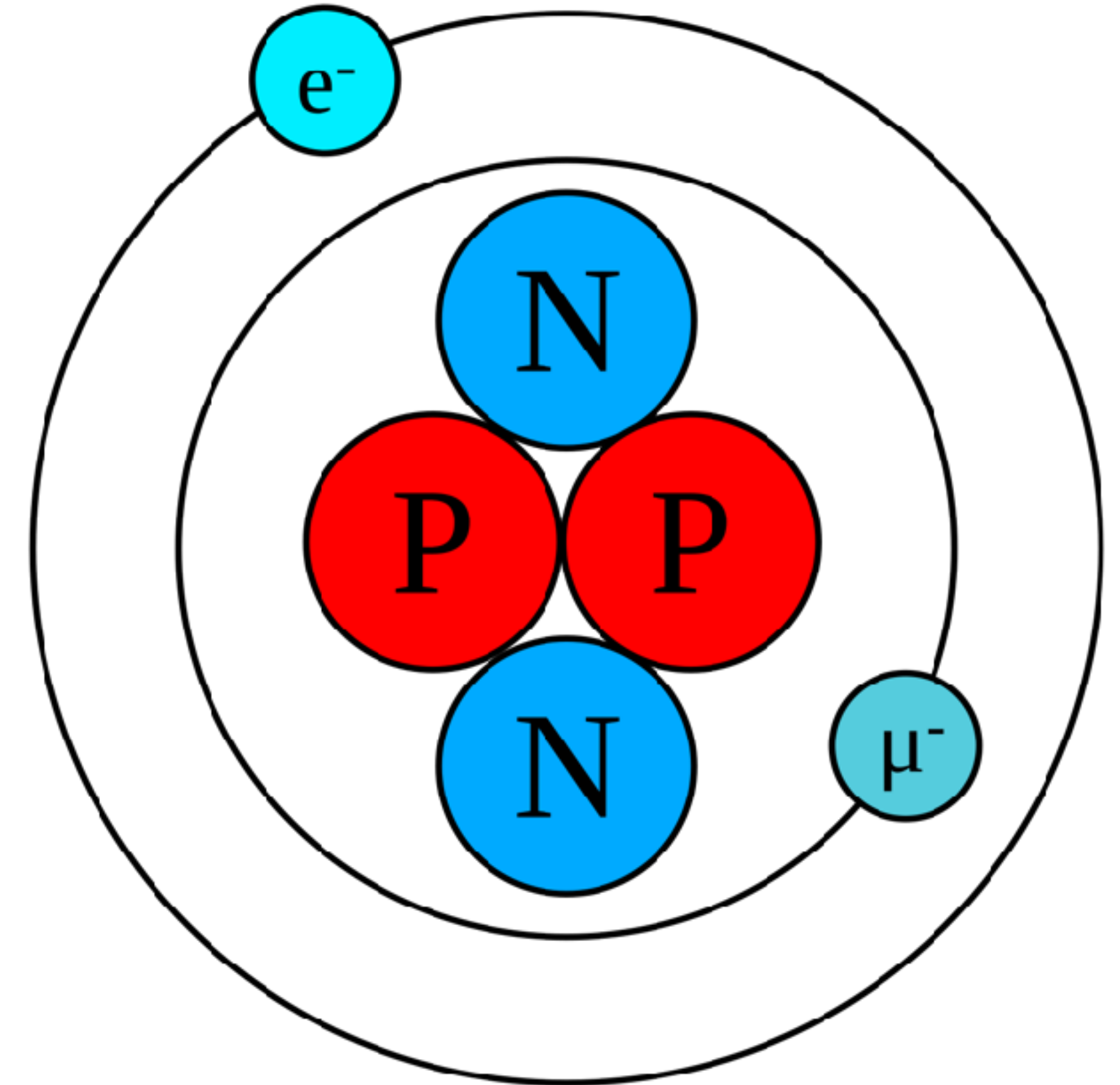
# Muonic atom

An **exotic atom** is an otherwise normal atom in which **one or more sub-atomic particles have been replaced by other particles of the same charge**.

For example, electrons may be replaced by other negatively charged particles such as **muons (muonic atoms)** or **pions (pionic atoms)**. Because these substitute particles are usually unstable, exotic atoms typically have **very short lifetimes** and no exotic atom observed so far can persist under normal conditions.

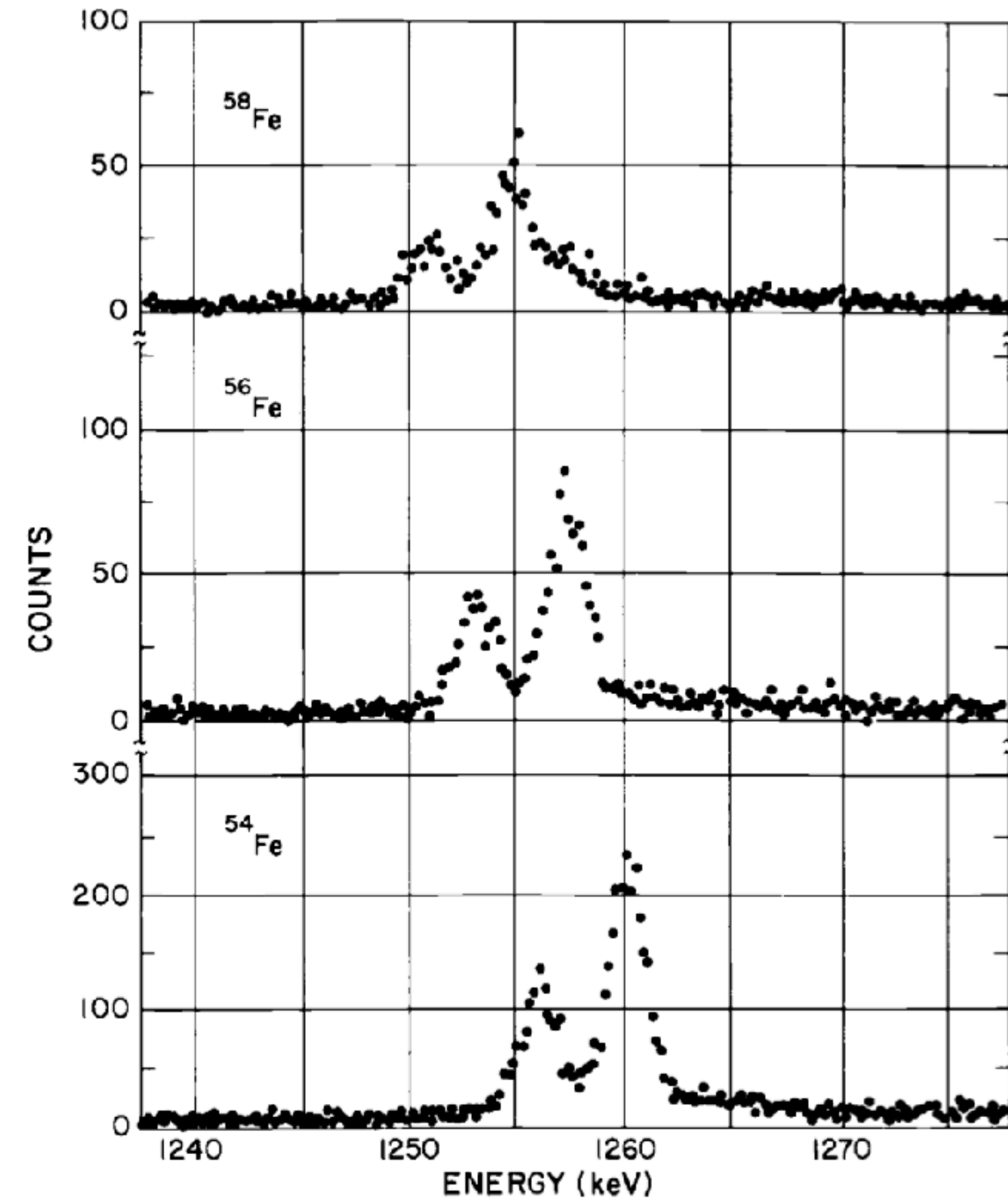
In a muonic atom an electron is replaced by a muon, which, like the electron, is a lepton.

Since a muon is more massive than an electron, the Bohr orbits are closer to the nucleus in a muonic atom than in an ordinary atom, and corrections due to quantum electrodynamics are more important. **Study of muonic atoms' energy levels as well as transition rates from excited states to the ground state therefore provide experimental tests of quantum electrodynamics.**



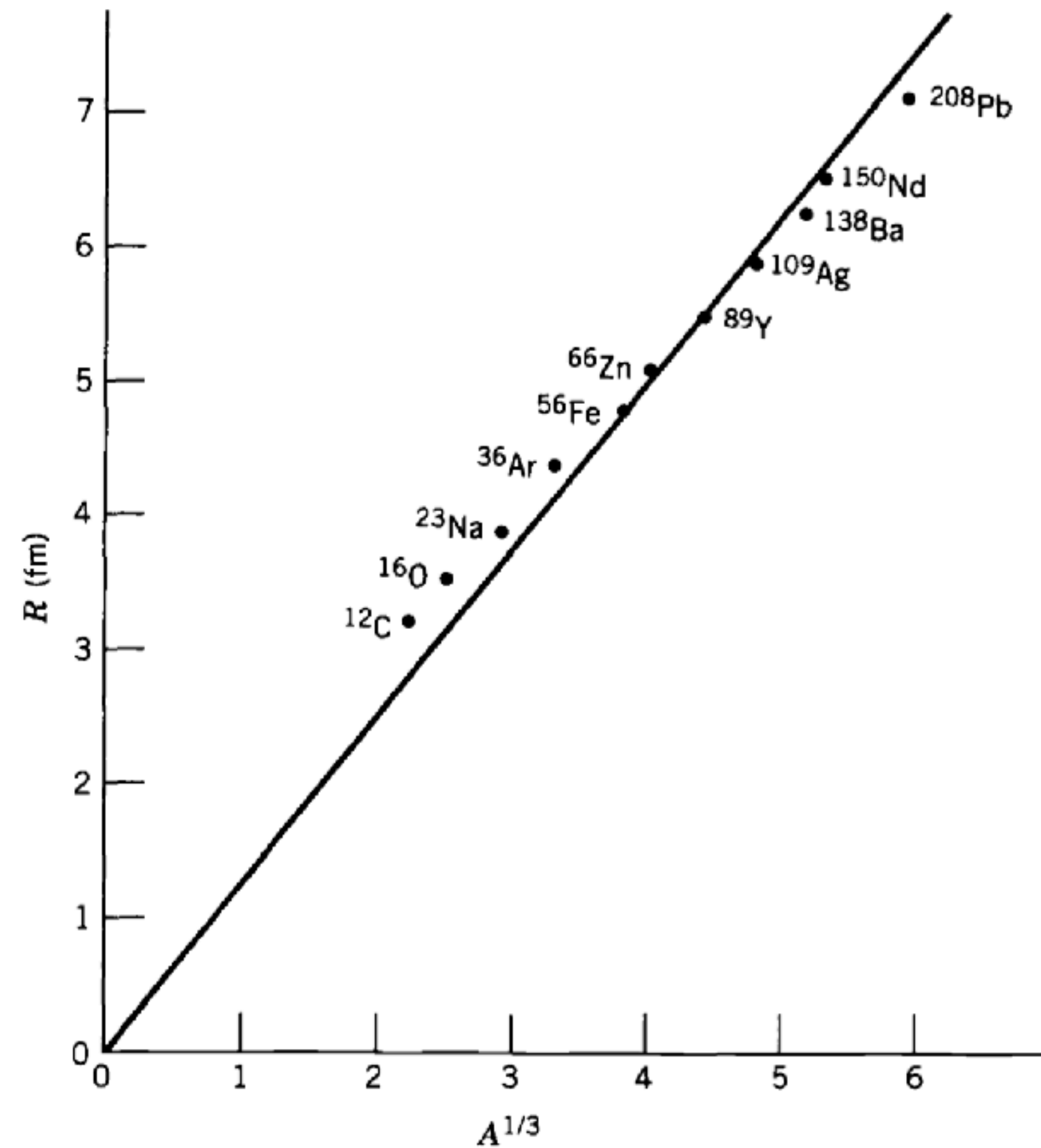
Muonic helium, made out of 2 protons, 2 neutrons, 1 muon and 1 electron.

# Isotope shift



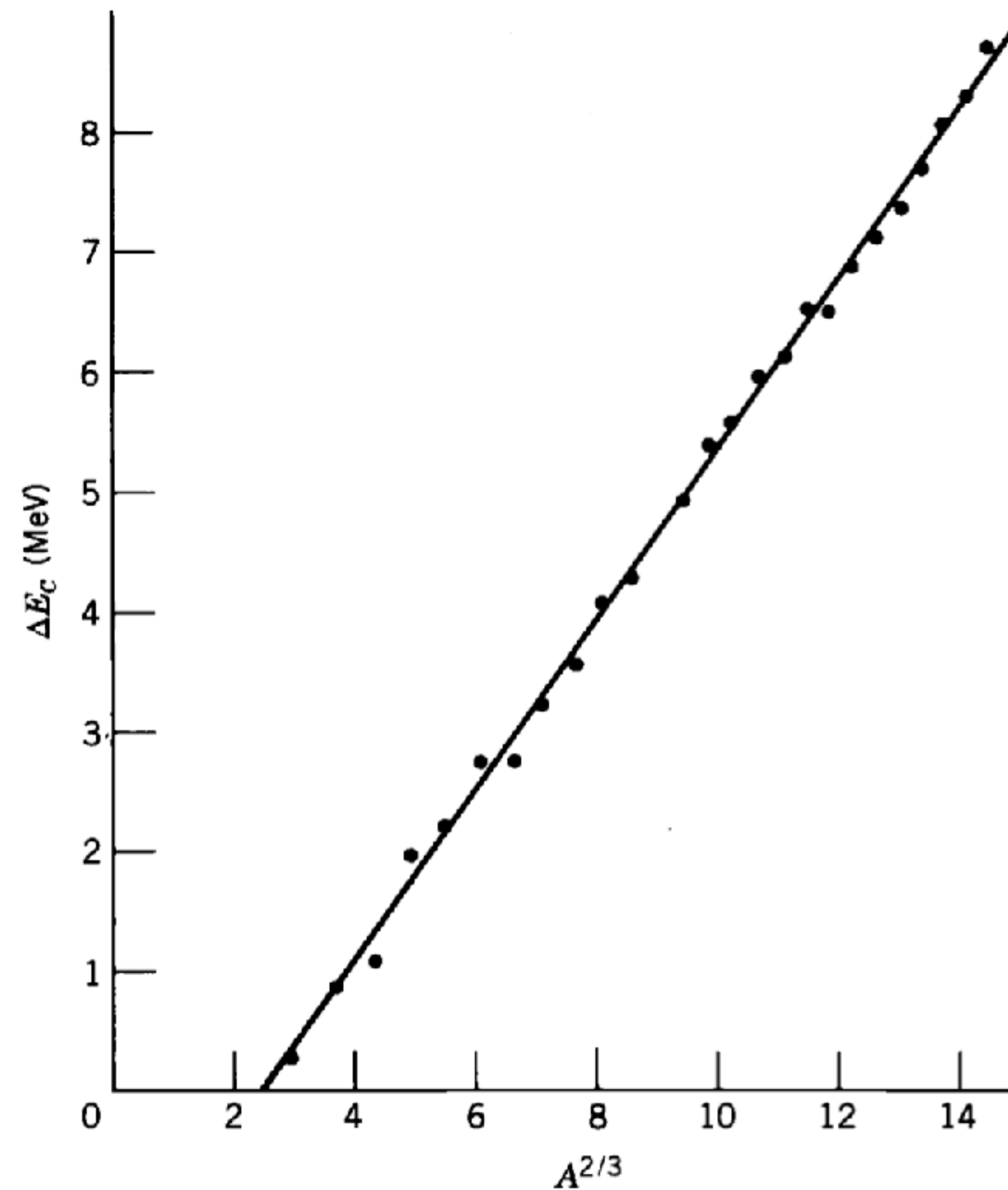
**Figure 3.8** The muonic K X rays in some Fe isotopes. The two peaks show the  $2p_{3/2}$  to  $1s_{1/2}$  and  $2p_{1/2}$  to  $1s_{1/2}$  transitions, which have relative intensities in the ratio 2:1 determined by the statistical weight ( $2j + 1$ ) of the initial state. The isotope shift can clearly be seen as the change in energy of the transitions. The effect is about 0.4%, which should be compared with the  $10^{-6}$  effect obtained with electronic K X rays (Figure 3.6). From E. B. Shera et al., *Phys. Rev. C* **14**, 731 (1976).

# Isotope shift



**Figure 3.9** The mean nuclear radius determined from muonic X-ray measurements. As in Figure 3.5, the data depend roughly linearly on  $A^{1/3}$  (again forcing the line to go through the origin). The slope of the line gives  $R_0 = 1.25 \text{ fm}$ . The data are taken from a review of muonic X-ray determinations of nuclear charge distributions by R. Engfer et al., *Atomic Data and Nuclear Data Tables* **14**, 509 (1974).

# Mirror nuclei

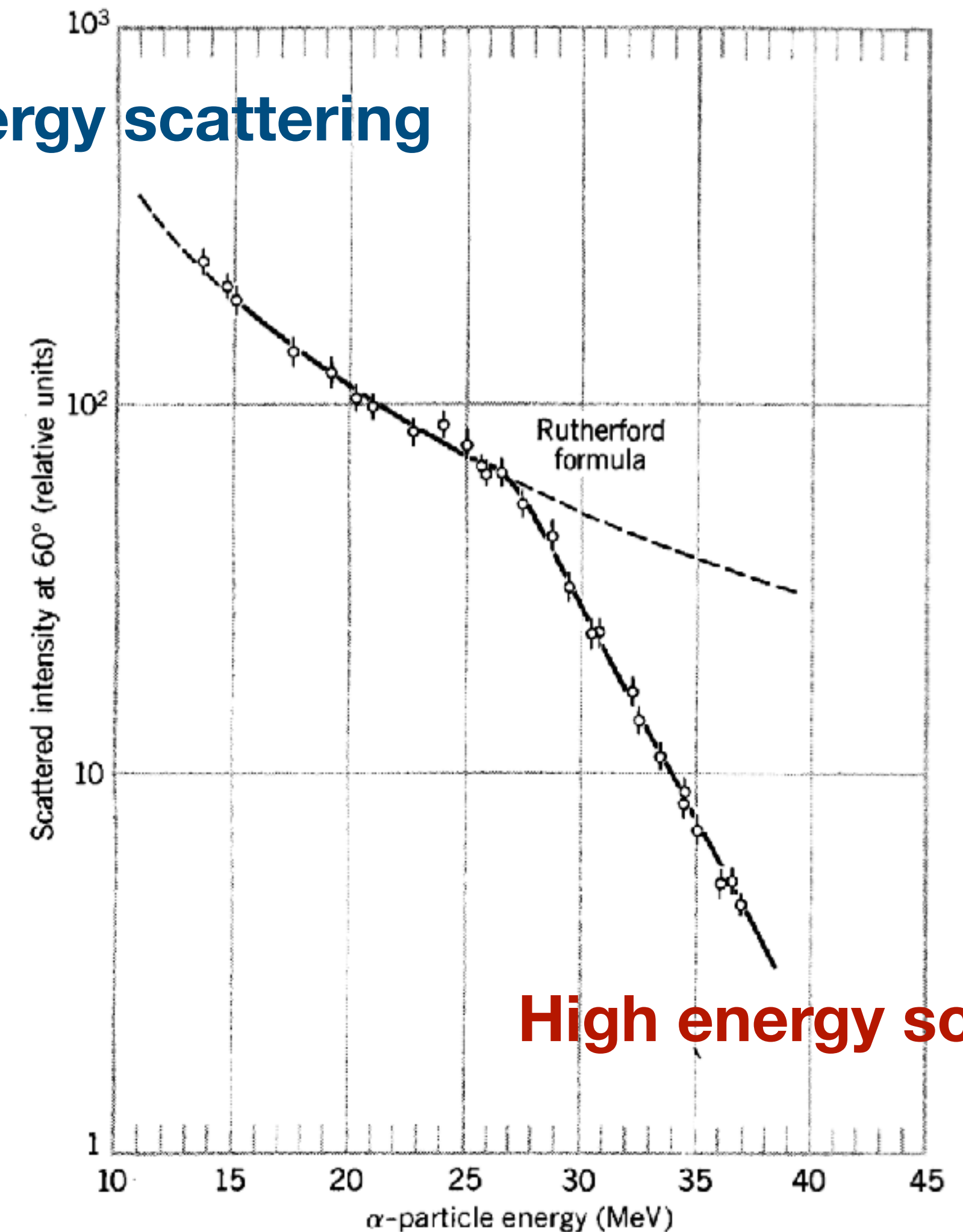


**Figure 3.10** Coulomb energy differences of mirror nuclei. The data show the expected  $A^{2/3}$  dependence, and the slope of the line gives  $R_0 = 1.22$  fm.



# He scattering

Low energy scattering



High energy scattering

**Figure 3.11** The breakdown of the Rutherford scattering formula. When the incident  $\alpha$  particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of  $\alpha$  particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

# Atomic mass table

```
1 a0dsskgw
0
0 *****
  * file : mass.mas20 *
  *****

This is one file out of a series of 3 files published in:
"The Ame2020 atomic mass evaluation (I)" by W.J.Huang, M.Wang, F.G.Kondev, G.Audi and S.Naimi
Chinese Physics C45, 030002, March 2021.
"The Ame2020 atomic mass evaluation (II)" by M.Wang, W.J.Huang, F.G.Kondev, G.Audi and S.Naimi
Chinese Physics C45, 030003, March 2021.
for files : mass.mas20 : atomic masses
            rct1.mas20 : react and sep energies, part 1
            rct2.mas20 : react and sep energies, part 2
A fourth file is the "Rounded" version of the atomic mass table (the first file)
            massround.mas20 atomic masses "Rounded" version

Values in files 1, 2 and 3 are unrounded version of the published ones
Values in file 4 are exact copy of the published ones

col 1 : Fortran character control: 1 = page feed 0 = line feed
format : a1,i3,i5,i5,i5,1x,a3,a4,1x,f14.6,f12.6,f13.5,1x,f10.5,1x,a2,f13.5,f11.5,1x,i3,1x,f13.6,f12.6
cc NZ N Z A el o mass unc binding unc B beta unc atomic_mass unc
Warnings : this format is not identical to that used in AME2016;
           one more digit is added to the "BINDING ENERGY/A", "BETA-DECAY ENERGY" and "ATOMIC-MASS" values and their uncertainties;
           # in a place of decimal point : estimated (non-experimental) value;
           * in a place of value : the not calculable quantity

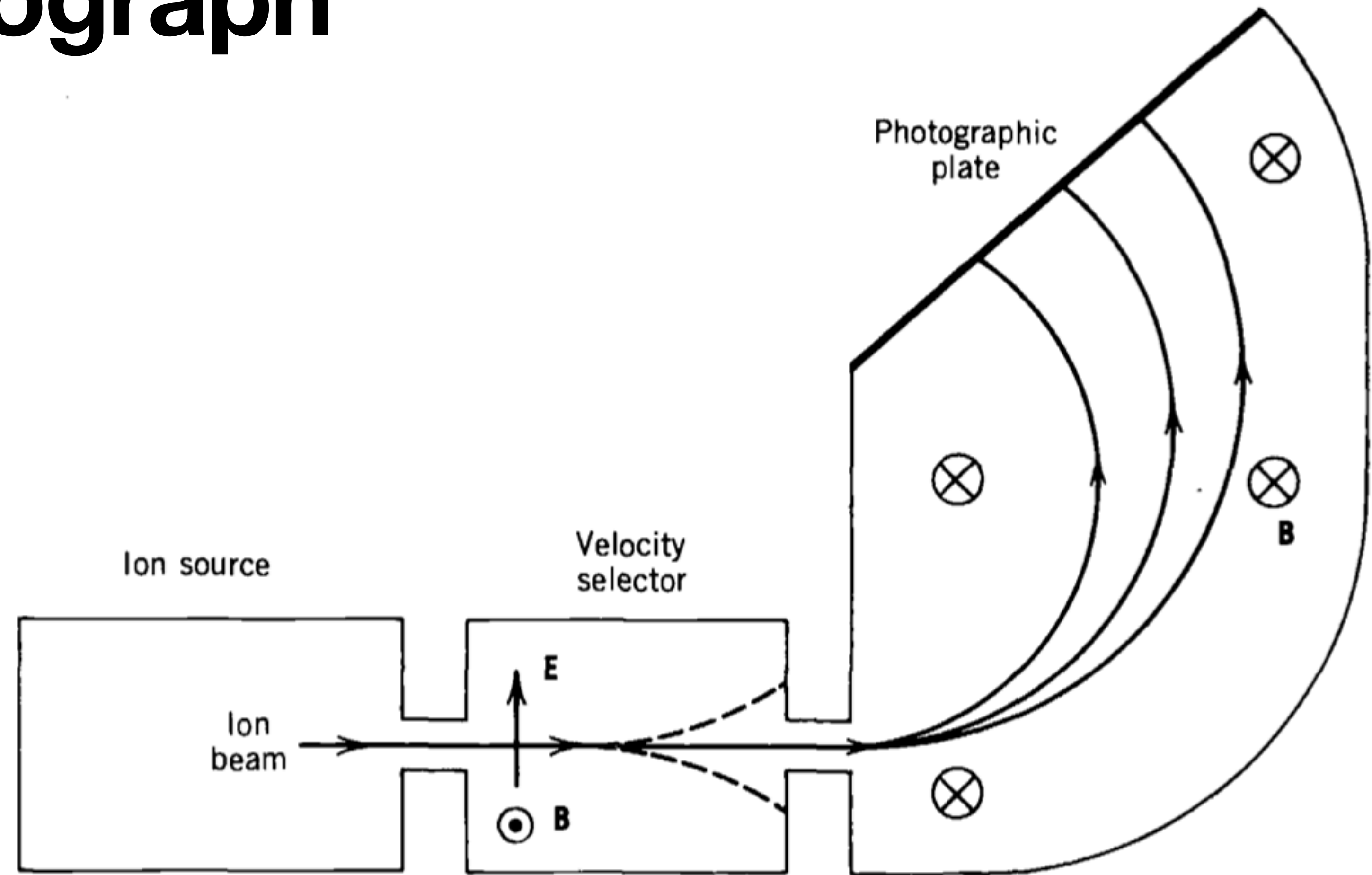
.....1.....2.....3.....4.....5.....6.....7.....8.....9.....10.....11.....12.....13

                                MASS LIST
                                for analysis

lN-Z  N  Z  A  EL  O  MASS EXCESS  BINDING ENERGY/A  BETA-DECAY ENERGY  ATOMIC MASS
      (keV)  (keV)  (keV)  (micro-u)
0 1 1 0 1 n 8071.31806 0.00044 0.0 0.0 B- 782.3470 0.0004 1 008664.91590 0.00047
-1 0 1 1 H 7288.971064 0.000013 0.0 0.0 B- * 1 007825.031898 0.000014
0 0 1 1 2 H 13135.722895 0.000015 1112.2831 0.0002 B- * 2 014101.777844 0.000015
0 1 2 1 3 H 14949.81090 0.00008 2827.2654 0.0003 B- 18.59202 0.00006 3 016049.28132 0.00008
-1 1 2 3 He 14931.21888 0.00006 2572.68044 0.00015 B- -13736# 2000# 3 016029.32197 0.00006
-3 0 3 3 Li -pp 28667# 2000# -2267# 667# B- * 3 030775# 2147#
0 2 3 1 4 H -n 24621.129 100.000 1720.4491 25.0000 B- 22196.2131 100.0000 4 026431.867 107.354
0 2 2 4 He 2424.91587 0.00015 7073.9156 0.0002 B- -22898.2740 212.1320 4 002603.25413 0.00016
-2 1 3 4 Li -p 25323.190 212.132 1153.7603 53.0330 B- * 4 027185.561 227.733
```

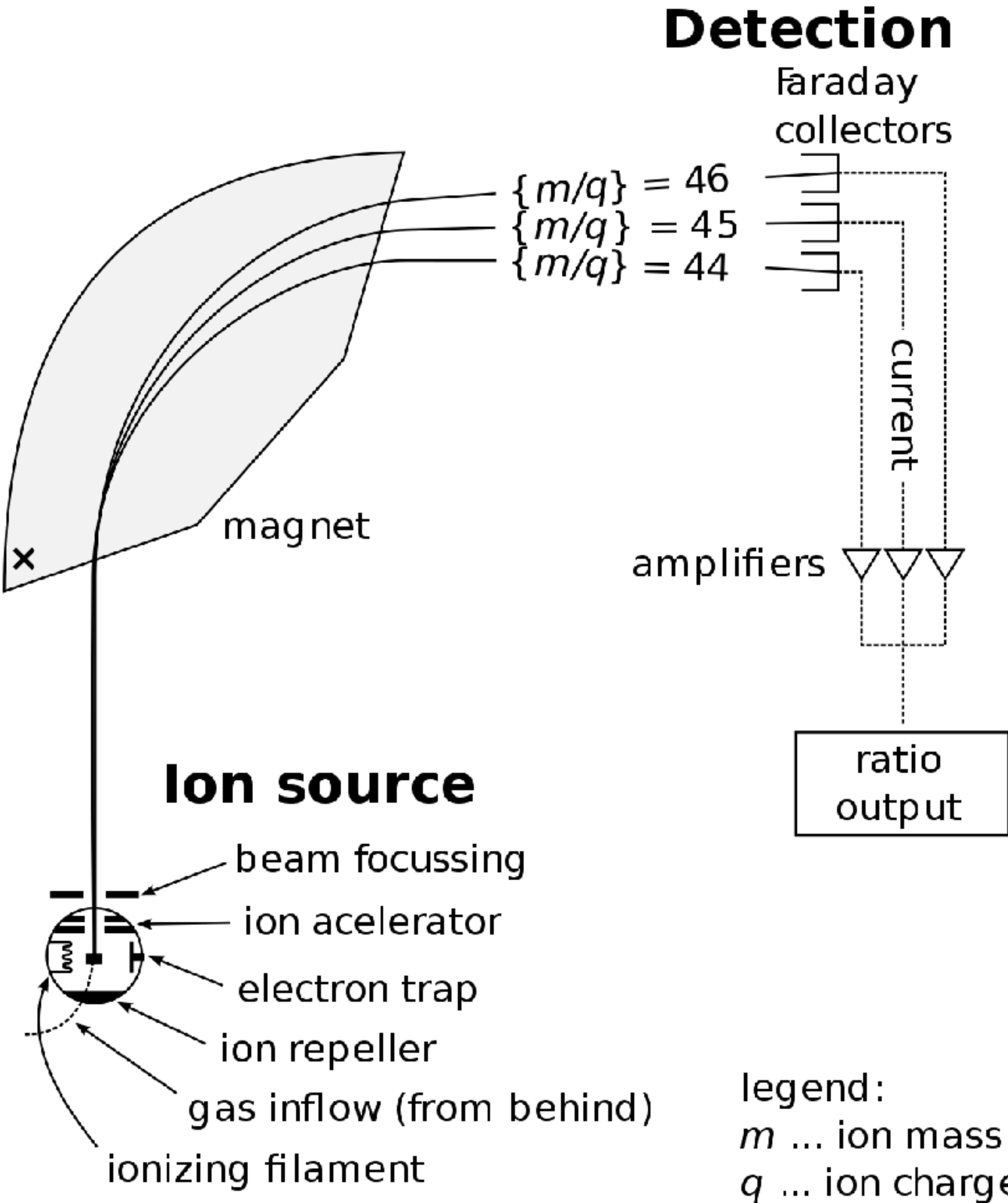


# Mass spectrograph



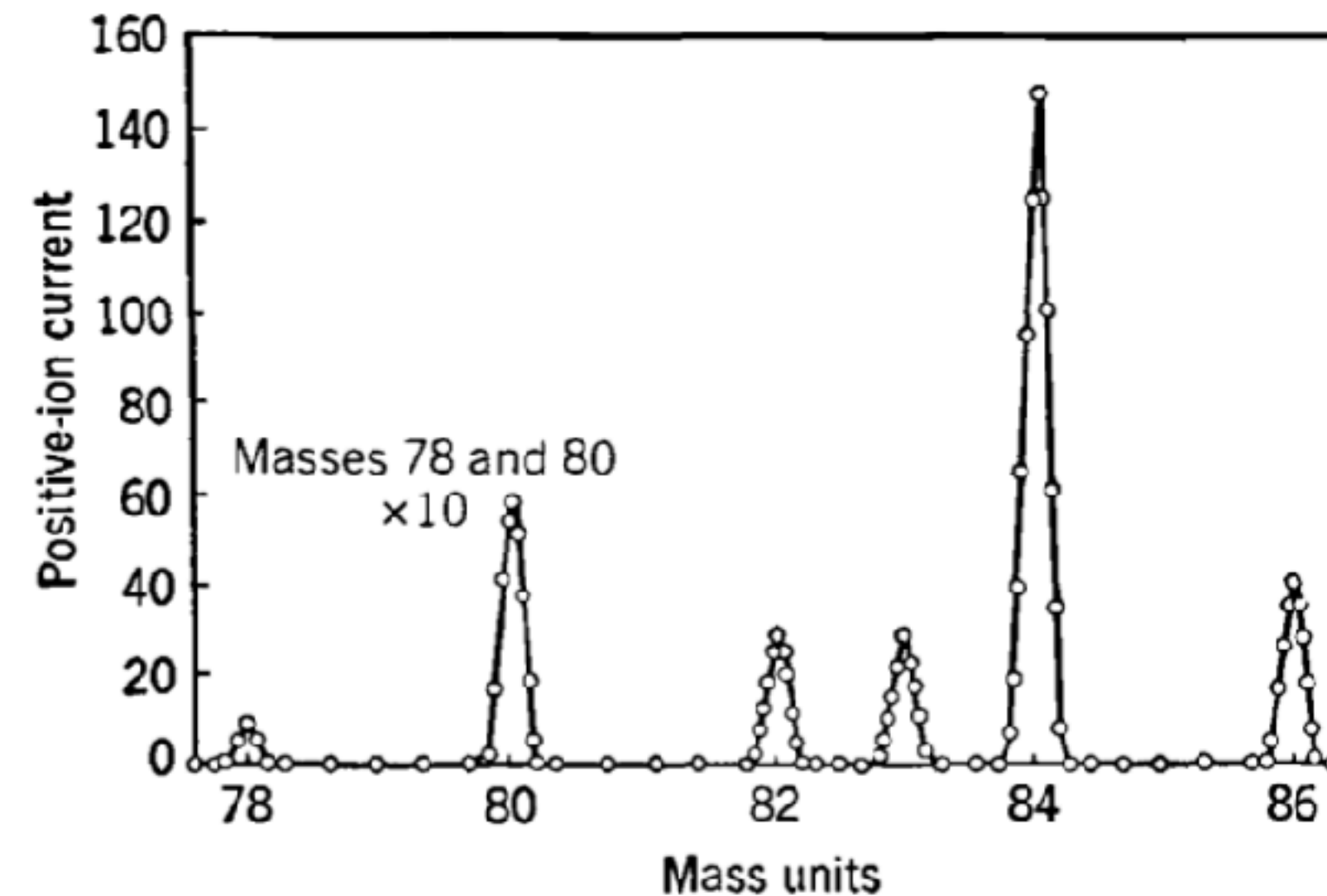
**Figure 3.13** Schematic diagram of mass spectrograph. An ion source produces a beam with a thermal distribution of velocities. A velocity selector passes only those ions with a particular velocity (others being deflected as shown), and momentum selection by a uniform magnetic field permits identification of individual masses.

# Mass spectrometer



# Mass-spectrum

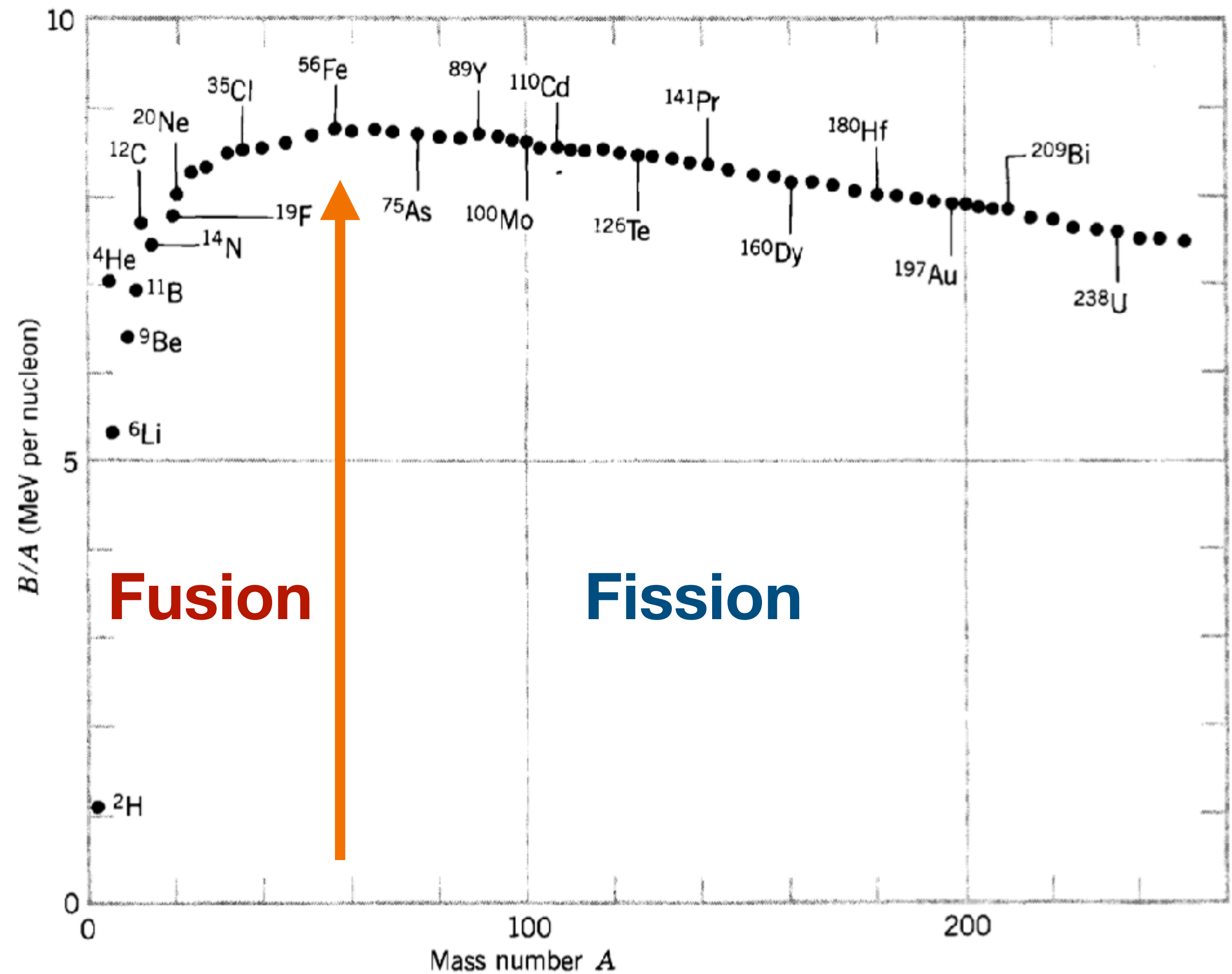
$^{78}\text{Kr}$	0.356%	$^{83}\text{Kr}$	11.5%
$^{80}\text{Kr}$	2.27%	$^{84}\text{Kr}$	57.0%
$^{82}\text{Kr}$	11.6%	$^{86}\text{Kr}$	17.3%



**Figure 3.14** A mass-spectrum analysis of krypton. The ordinates for the peaks at mass positions 78 and 80 should be divided by 10 to show these peaks in their true relation to the others.

Kr Isotopes that are not listed are too unstable to be measured with a mass spectrometer (e.g.  $^{79}\text{Kr}$ ,  $^{81}\text{Kr}$ ,  $^{85}\text{Kr}$ )

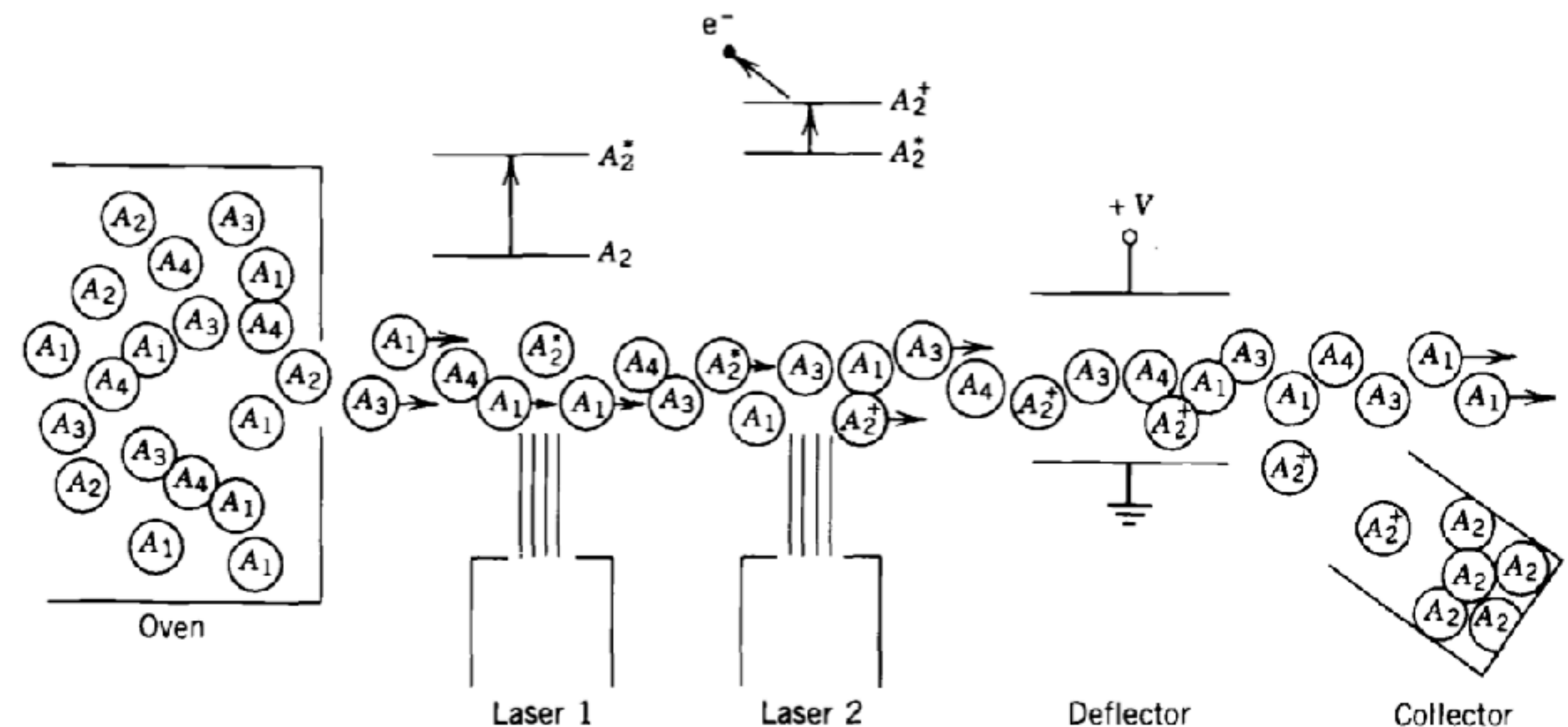
# Nuclear binding energy



**Figure 3.16** The binding energy per nucleon.

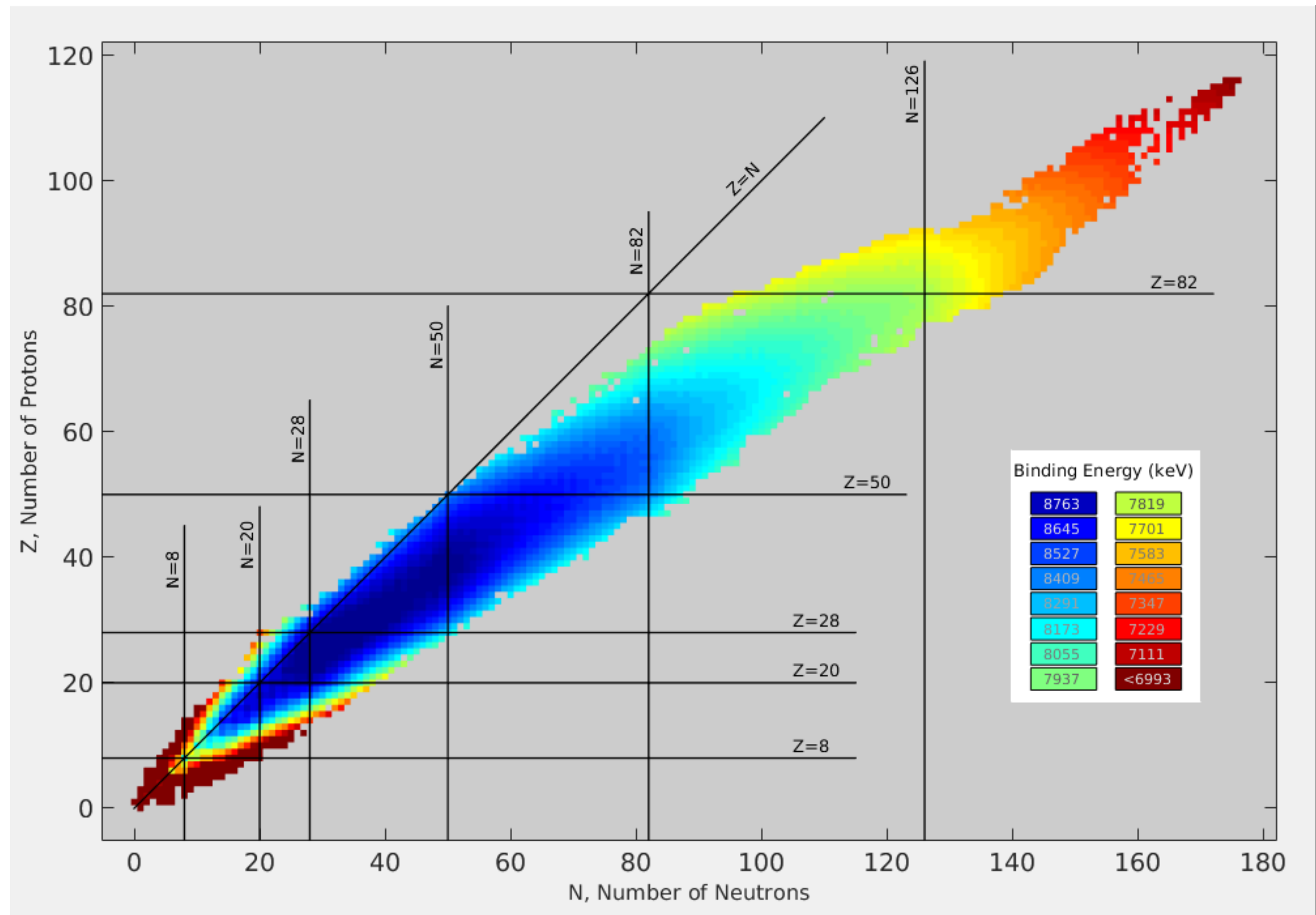
# Laser Isotop separation

- To separate different isotopes
- Isotopes have slightly different excitation energies  $\rightarrow$  isotope shift
- If we have a beam with different isotopes
  - Laser 1 has a specific frequency to excite one of the isotopes
  - Laser 2 ionises the already excited atoms
  - The ionised isotopes can be deflected and collected



**Figure 3.15** Laser isotope separation. The beam of neutral atoms from the oven is a mixture of four isotopes  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$ . The first laser is tuned to the transition corresponding to the resonant excitation of isotope  $A_2$  to a certain excited state; because of the sharpness of the laser energy and the isotope shift that gives that particular transition a different energy in the other isotopes, only  $A_2$  is excited. The second laser has a broad energy profile, so that many free-electron states can be reached in the ionization of the atoms; but because only the  $A_2$  isotopes are in the excited state, only the  $A_2$  atoms are ionized. The  $A_2$  ions are then deflected and collected.

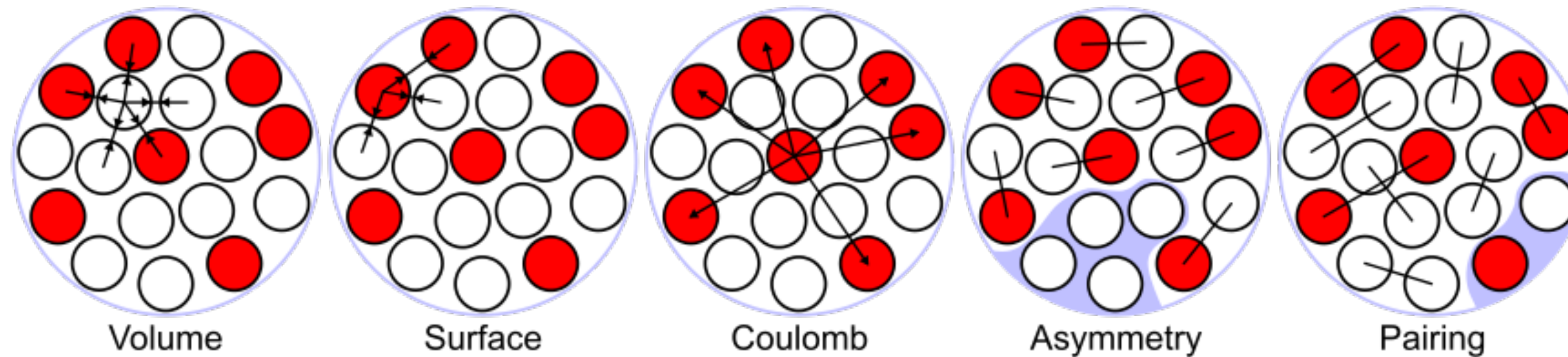
# Binding energy



- Chart of nuclides (isotopes) by **binding energy**, depicting the valley of stability.
- The diagonal line corresponds to equal numbers of neutrons and protons.
- Dark **blue squares represent nuclides with the greatest binding energy**, hence they correspond to the most stable nuclides.
- The binding energy is greatest along the floor of the valley of stability.



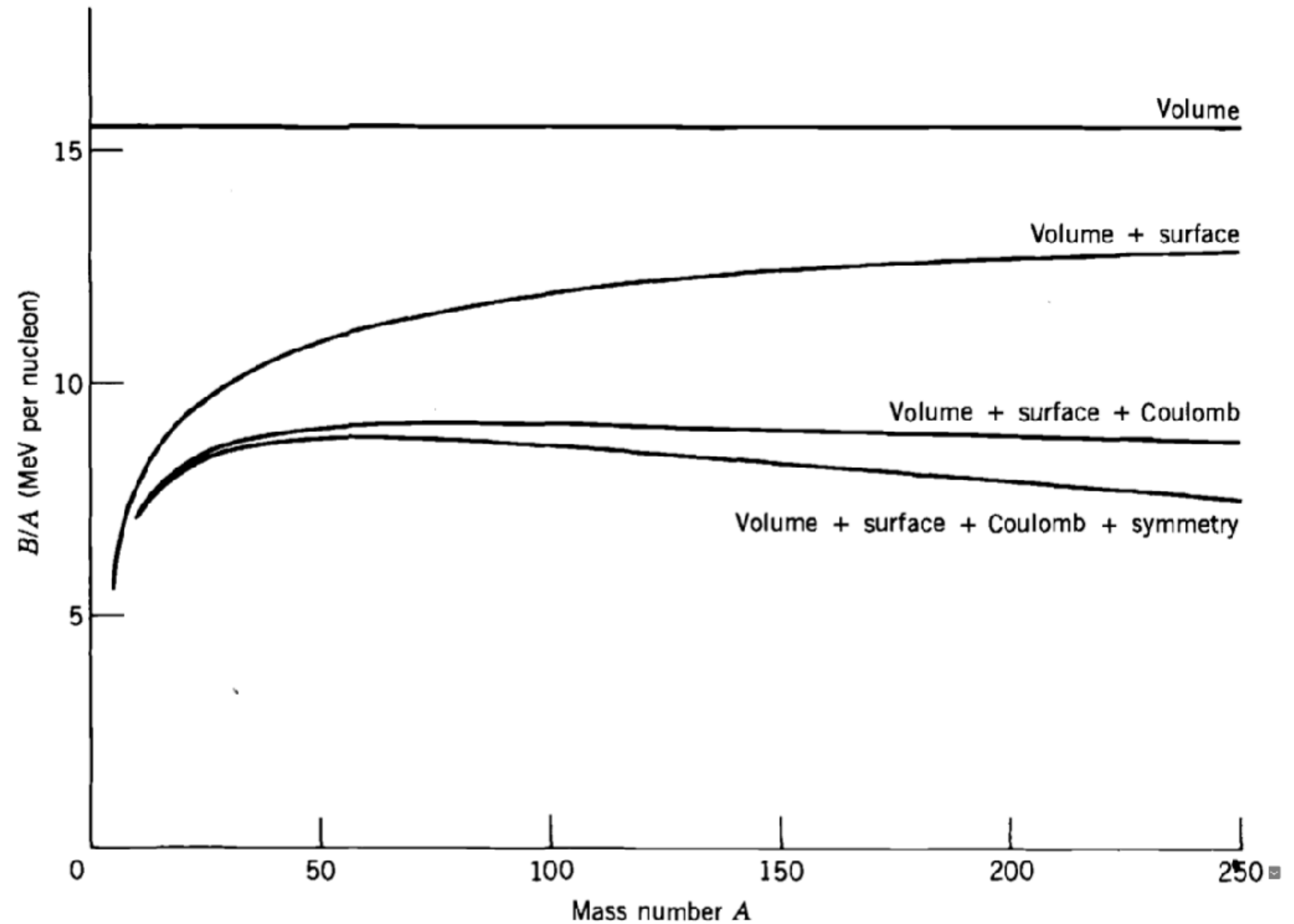
# Semi empirical mass formula - binding energy



1. **Volume energy**, when an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain number of other nucleons in contact with it. So, this nuclear energy is proportional to the volume.
2. **Surface energy** corrects for the previous assumption made that every nucleon interacts with the same number of other nucleons. This term is negative and proportional to the surface area, and is therefore roughly equivalent to liquid surface tension.
3. **Coulomb energy**, the potential energy from each pair of protons. As this is a repulsive force, the binding energy is reduced.
4. **Asymmetry energy** (also called Pauli energy), which accounts for the Pauli exclusion principle. Unequal numbers of neutrons and protons imply filling higher energy levels for one type of particle, while leaving lower energy levels vacant for the other type.
5. **Pairing energy**, which accounts for the tendency of proton pairs and neutron pairs to occur. An even number of particles is more stable than an odd number due to spin coupling.



# Nuclear binding energy




**Figure 3.17** The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.


# Nuclear binding energy

Comparison between the semi empirical mass formula (orange) and the measured binding energy (blue symbols).

Example code on the GitHub repository to calculate the binding energy per nucleon with the semi empirical mass formula.










 jupyter














































Binding\_energy (autosaved)

 Logout

File Edit View Insert Cell Kernel Widgets Help

Not Trained Python 3 (ipykernel)

Run                                             

Markdown

### Binding energy

Source of the original code: <https://scipython.com/blog/nuclear-binding-energies-1/>

The **binding energy** of a nucleus is the energy that would be required to split apart each of its constituent protons and neutrons (nucleons). This energy is due to the forces that hold the nucleus together, which may be thought of as a balance between the attractive strong nuclear force and the electromagnetic force (which is repulsive between the positively-charged protons). The binding energy manifests itself in a mass difference between the nucleus and the individual nucleons ( $E = mc^2$ ) and the difference in binding energy between different nuclei is the energy released in the process of nuclear fusion and fission.

In nuclear physics, the **semi-empirical mass formula (SEMF)** (sometimes also called the Weizsäcker formula, Bethe–Weizsäcker formula, or Bethe–Weizsäcker mass formula to distinguish it from the Bethe–Weizsäcker process) is used to approximate the mass of an atomic nucleus from its number of protons and neutrons. As the name suggests, it is based partly on theory and partly on empirical measurements. The formula represents the liquid-drop model proposed by George Gamow, which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker, and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today.

The formula gives a good approximation for atomic masses and thereby other effects. However, it fails to explain the existence of lines of greater binding energy at certain numbers of protons and neutrons. These numbers, known as magic numbers, are the foundation of the nuclear shell model.

[https://en.wikipedia.org/wiki/Semi-empirical\\_mass\\_formula](https://en.wikipedia.org/wiki/Semi-empirical_mass_formula)

```
In [1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

In [2]: def SEMF(Z, N):
    """Calculate the average binding energy per nucleon for nucleus Z, N.

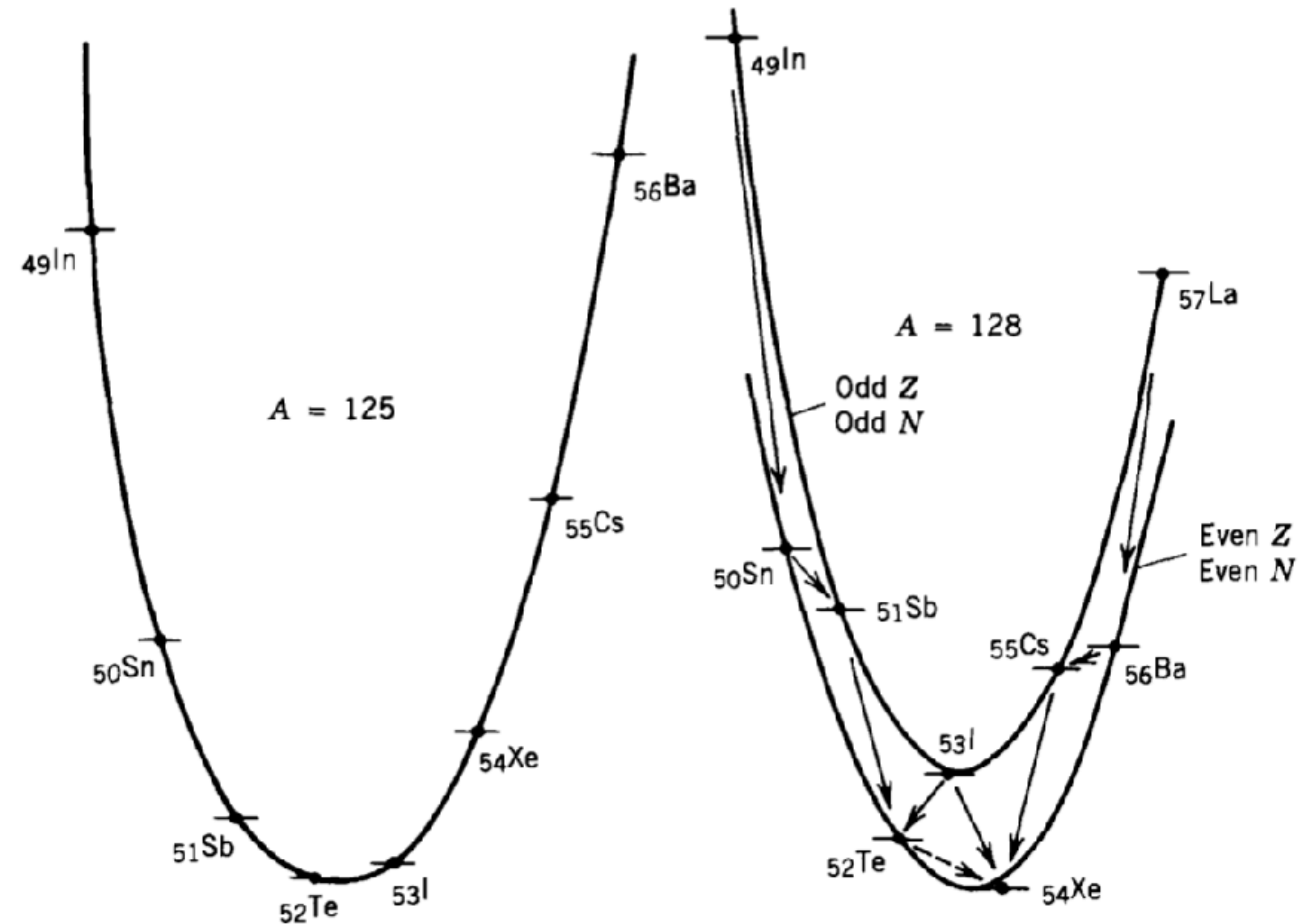
    Calculate the average nuclear binding energy per nucleon for a nucleus
    with Z protons and N neutrons, using the semi-empirical mass formula and
    parameters of J. M. Rohlif, "Modern Physics from alpha to Z", Wiley (1994).
```

The graph plots the binding energy  $B$  in MeV against the mass number  $A = N + Z$ . The y-axis ranges from 0 to 9 MeV, and the x-axis ranges from 0 to 250. The blue dots represent experimental data from Ame2003, and the orange line represents the SEMF calculation. The binding energy per nucleon increases rapidly for small  $A$ , reaches a maximum of about 8.8 MeV at  $A \approx 60$ , and then slowly decreases for larger  $A$ . The SEMF curve provides a very good fit to the experimental data points.

$A = N + Z$	$B$ [MeV] (Ame2003)	$B$ [MeV] (SEMF)
2	1.1	1.1
4	7.1	7.1
12	7.7	7.7
20	8.0	8.0
40	8.7	8.7
60	8.8	8.8
100	8.6	8.6
150	8.3	8.3
200	8.0	8.0
250	7.5	7.5

# Nuclear binding energy

## Mass parabolas for constant $A$



**Figure 3.18** Mass chains for  $A = 125$  and  $A = 128$ . For  $A = 125$ , note how the energy differences between neighboring isotopes increase as we go further from the stable member at the energy minimum. For  $A = 128$ , note the effect of the pairing term; in particular,  $^{128}\text{I}$  can decay in either direction, and it is energetically possible for  $^{128}\text{Te}$  to decay directly to  $^{128}\text{Xe}$  by the process known as double  $\beta$  decay.



# Nuclear binding energy

## Mass parabolas for constant A

Example code on the GitHub repository to produce mass parabolas for any given A.

```
jupyter Mass_parabola (autosaved)
File Edit View Insert Cell Kernel Widgets Help Not Trusted Python 3 (ipykernel)
In [1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

In [2]: # 1 u = 931.49432 MeV/c^2
f = 931.49432
# Hydrogen atom mass (MeV)
mH = 1.007825031898 * f
# Neutron mass (MeV)
mn = 1.008664904 * f

# Liquid drop model parameters, MeV: volume, surface and pairing terms.
aV, aS, delta = 15.75, 17.8, 11.18
# Liquid drop model parameters, MeV: asymmetry term (aA) and Coulomb term (aC).
aA, aC = 23.7, 0.711

In [3]: def get_SEMF_mass_excess(A, Z):
    """Return the mass excess calculated using the SEMF model."""

    # The pairing term is -delta for Z and N both odd, +delta for Z and N both
    # even, and 0 otherwise. Create an array of the sign of this term so that
    # we can vectorize the calculation across the arrays Z and N.
    N = A - Z
    sgn = np.zeros(Z.shape)
    sgn[(Z%2) & (N%2)] = -1
```

