Mathematical models Assessment of learning

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1)

a) $27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

So, the final answer of $27^{\frac{1}{3}}$ is 3.

b) $32^{\frac{3}{5}}$

$= 32^{\frac{3}{5}}$

$= (\,2^5)\,\frac{3}{5}$

$= (\,2^5) \times \frac{3}{5}$

$= 2^{5\,\cdot\,\frac{3}{5}}$

$= 2^3$

$= 8$

Finally, the answer to $32^{\frac{3}{5}}$ is 8.

c) $8^{\frac{5}{3}}$

$= \ (2^3)^{\frac{5}{3}}$

$= (2)^5$

$= 2^5$

$= 32$

Therefore, $8^{\frac{5}{3}} \ = 32$.

d) $25^{-\frac{1}{2}}$

Property of exponents states that $a^{-n} = \frac{1}{a^n}$

$= \frac{1}{25^{\frac{1}{2}}}$

The square root of 25 is 5.

So, the inverse of 5 is $\frac{1}{5}$.

$So,\ 25^{-\frac{1}{2}} = \frac{1}{5}$

Therefore, $25^{-\frac{1}{2}} = \frac{1}{5}$

e) $\left(\frac{64}{27}\right)^{-\frac{1}{3}}$

$The\ cube\ root\ of\ \left(\frac{64}{27}\right)^{-\frac{1}{3}} = \sqrt[3]{\frac{64}{27}}$

$= \frac{\sqrt[3]{64}}{\sqrt[3]{27}}$

$= \frac{4}{3}$

$So,$ the reciprocal is $\frac{3}{4}$

Therefore, $\left(\frac{64}{27}\right)^{-\frac{1}{3}} = \frac{3}{4}$

f) $250^{\frac{1}{4}}$

$= \sqrt[4]{250}$

$= 3.9763$

$= 3.98$

Therefore, $250^{\frac{1}{4}} = 3.98$

g) $1.28^{3.7}$

$= 2.49$

Therefore, $1.28^{3.7} = 2.49$

h) $\left(\frac{3}{5}\right)^{-5}$

$= \left(\frac{3}{5}\right)^{-5}$

$= \left(\frac{5}{3}\right)^{5}$

$= \left(\frac{5^5}{3^5}\right)$

$= \frac{3125}{243}$

Therefore, $\left(\frac{3}{5}\right)^{-5} = \frac{3125}{243}$

i) $\sqrt[6]{28}$

So $\sqrt[6]{28}$

$= (28)^{\frac{1}{6}}$

$=1.7425$

So that, the final answer is, $\sqrt[6]{28} = 1.7425$

2.

a) $(a^{3})^{2}$

$= a^{3\times 2}$

$= a^{6}$

So, a^3 *to the power of* 2 *equals to* a^{6}

b) $\frac{n^5}{n^3}$

$= n^{5-3}$

$= n^2$

The last answer of, $\frac{n^5}{n^3} = n^2$

c) $(3x^{\,2})^3$

$= 3^3 \cdot (x^2)^3$

$3^3 = 27$

$= 27 \cdot x^6$

$= 27x^6$.

Finalist answer of, $(3x^2)^3 = 27x^6$.

d) $x(x^2)(x^4)$

$= \ x^{1+2+4}$

$= x^{\,7}$.

So finally, $x(x^2)(x^4) = x^{\,7}$

e) $\frac{t^{\,2}(t^{\,5})}{(t^{\,3})2}$

$t^2 \cdot t^5 = t^{\,2+5} = \ t^7$

$= \frac{t^{\,2+5}}{t^{\,3\times 2}}$

$= \frac{t^{\,7}}{t^{\,6}}$

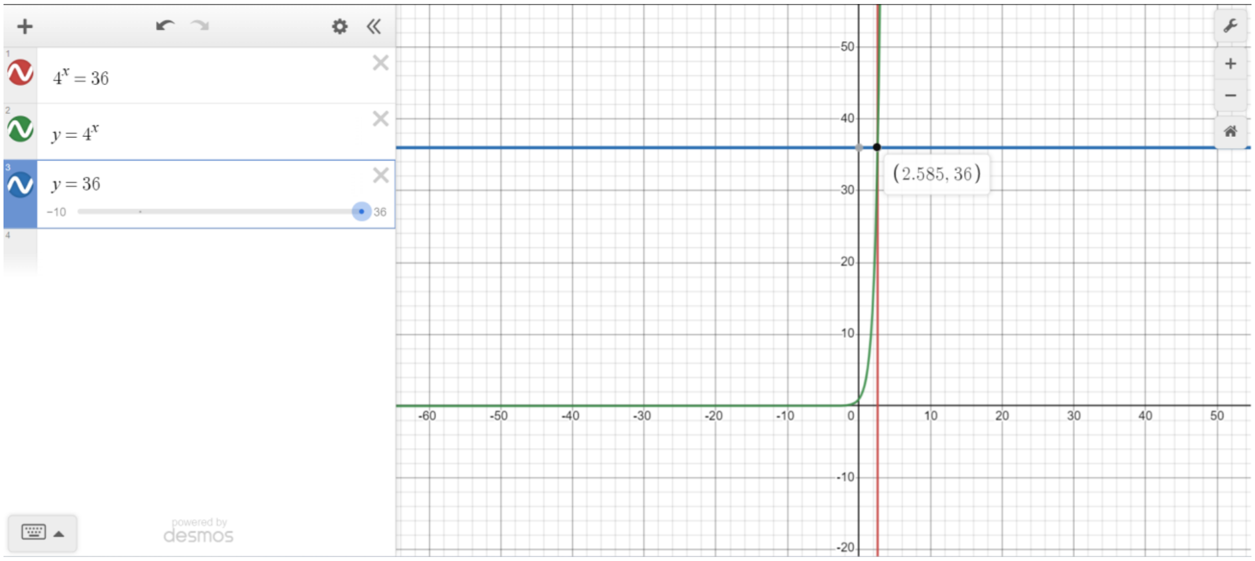
$= t^{\,7-6}$

$= t^{\,1}$

$= t$

3)

In accordance with the given equation $4^x = 36$, we can utilize a graphical approach, specifically the Demos method to illustrate the functions represented by y= 4^x and y= 36.



Based on the graph, the point of intersection between the two functions occurs at approximately (2.585,36).

Consequently, the value of $x = 2.585$.

4)

a) $2^x = 64$

We know that 64 can be expressed as 2^6.

Therefore, we can rewrite the equation as:

$2^x = 2^6$

Since the bases are the same, the exponents must be equal. So, we have:

$x = 6$

Hence, the solution to the equation $2^x = 64$ is $x = 6$.

Therefore, x = 6.

b) $3^{n+4} = 27^{2n}$

$Here,$ we have to rewrite 27^{2n} as 3^3

$3^{n+4} = (3^3)^{2n}$

Substitute into the equation

$3^{n+4} = 3^{3\times 2n}$

$3^{n+4} = 3^{6n}$

Since the bases are equal and the exponent must also be equal.

$n + 4 = 6n$

Solve the equation for n:

$4 = 6n - n$

$4 = 5n$

$$n = \frac{4}{5}$$

$so\ n = \frac{4}{5}$ is the solution.

c) $4^{2(x+5)} - 11 = 245$

First, we have to simplify the expression

$4^{2\,(x+5)} = 11 + 245$

$4^{2(x+5)} = 256$

Take the logarithm of both sides

Bring bases that are common to two side of the equation

$\log 4(4^{2(x+5)} = \log 4\ \ 256$

Apply the logarithm property

$\log_b(bx) = x$

$2^{(x+5)} = \log 4\ \ 256$

$\log 4\ \ 256 = 4$

Since the bases are now identical the exponents must also be equal.

$2\,(x + 5\,) = 4$

$2x + 10 = 4$

$2x = 4 - 10$

$2x = -6$

$x = -3$

Solution to the equation is, $x = -3$.

5)

The initial quantity is denoted as 256g, the overall weight measure 0.25 grams.

Represented M = 0.25 and I= 256g

These quantities are replaced within the formula

$M = I \times (0.5)^n$

$0.25 = 256 \times (0.5)^n$

Dividing both by 256 yields

$\frac{0.25}{256} = \frac{256(0.5)}{256}^n$

Which simplifies to,

$0.0009765625 = 0.5^n$

Recognize that both sides share a common base, reformulate the mathematical expression,

$(0.5)^n = (0.5)^{10}$

As the bases are identical, the exponents must also be identical.

Consequently, $n = 10$

It will take 10 days for 256 grams of tungsten -187 to decay and reach a mass of 0.25 grams.

6)

The final amount of the investment is $10,000 here

The value represented $A = 2500\ (1.06)^t$

Solve the equation

$10{,}000 = 2500\ (1.06)^t$

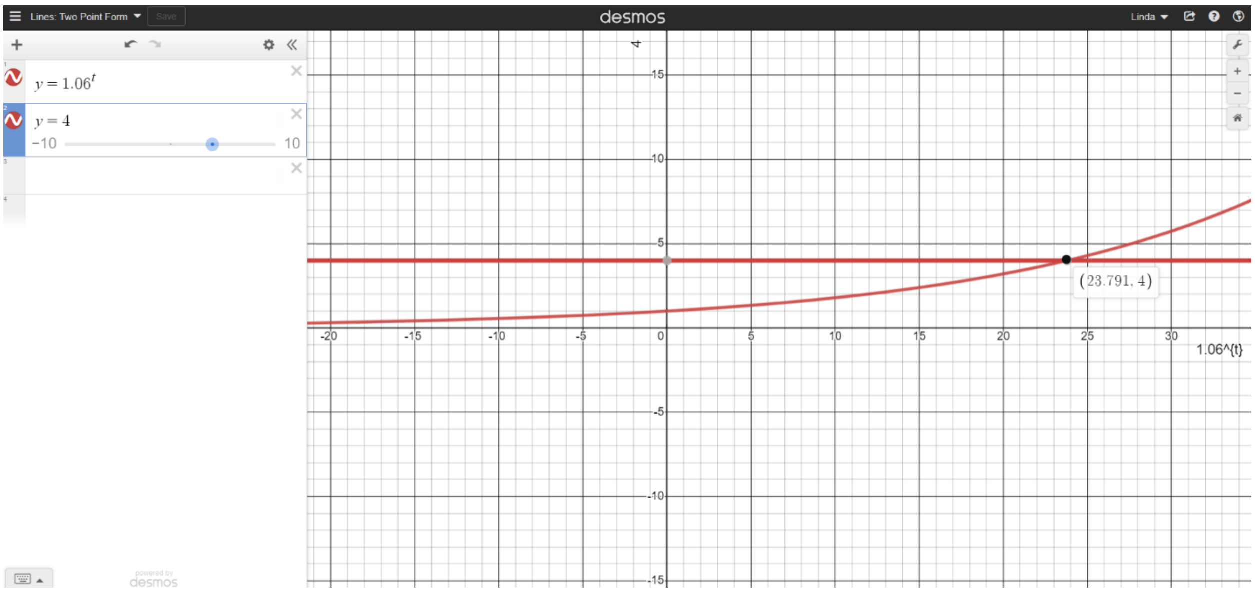
Divide both sides by 2500

$\frac{10{,}000}{2500} = \frac{2500(1.06)t}{2500}$

$4 = (1.06)^t$

Using demos, we have the capability to graph the functions

$y = (1.06)^t$ and $y = 4$



Within the graph, both functions meet at the coordinate (23.791, 4).

Indicating, it will take 23.8 years to reach a final sum of $10,000.

7)

The formula for surface area of cylinder $SA = 2\pi r^2 + 2\pi rh$

To isolate the variable h the formula is rearranged as follows:

$SA - 2\pi r^2 = 2\pi rh$

Both sides of the equation are then divided by $2\pi r$

$$\frac{SA - 2\pi r2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

So, h= $\frac{SA - 2\pi r2}{2\pi r}$

The height of cylinder A:

$h= \frac{2500 - 2\pi \times 8^2}{2\pi \times 8}$

$$h = \frac{2500 - 2\pi \times 64}{16\pi}$$

$$h = \frac{2500 - 128\pi}{16\pi}$$

$$h = \frac{2500 - 128 \times 3.14}{16 \times 3.14}$$

$$h = \frac{2500 - 401.92}{50.24}$$

$$h = \frac{2098.08}{50.24}$$

$$h = 41.76$$

The vertical dimension of cylinder B:

$h= \frac{5000 - 2\pi \times 6^2}{2\pi \times 6}$

$$h = \frac{5000 - 2\pi \times 36}{12\pi}$$

$$h = \frac{5000 - 72\pi}{12\pi}$$

$$h = \frac{5000 - 72 \times 3.14}{12 \times 3.14}$$

$$h = \frac{5000 - 226.08}{37.68}$$

$$h = \frac{4773.92}{37.68}$$

$$h = 126.69$$

The measurement of cylinder C:

$h= \frac{4275 - 2\pi \times 15^2}{2\pi \times 15}$

$$h = \frac{4275 - 2\pi \times 225}{30\pi}$$

$$h = \frac{4275 - 450\pi}{30\pi}$$

$$h = \frac{4275 - 450 \times 3.14}{30 \times 3.14}$$

$$h = \frac{4275 - 1413}{94.2}$$

$$h = \frac{2862}{94.2}$$

$$h = 30.38$$

8)

a) Find the value of K given m = 20 kg and v = 18 m/s

Using the formula for kinetic energy K = $\frac{1}{2}$ mv^2

We substitute the provided values:

$$k = \frac{1}{2} \times 20 \times (18)^2$$

$$k = \frac{1}{2} \times 20 \times 324$$

$$k = 10 \times 324$$

$$k = 3240$$

Hence, the kinetic energy K is 3240 kg m^2/s^2

b)

Determine the value of m, given K = 486 and v = 9 m/s.

We use the formula for kinetic energy K = $\frac{1}{2}$ mv^2

$$486 = \frac{1}{2} m \times 9^2$$

$$486 = \frac{1}{2} m \times 81$$

$$486 = 40.5m$$

So, to calculate m, divide both sides by 40.5

$\frac{486}{40.5} = \frac{40.5}{40.5}$m

$m = \frac{486}{40.5}$

$m = 12$kg

So, m is equivalent to 12 kg.

Thus, denoting the object mass as 12 kg.

c)

Determine the value of v given K = 345 and m = 7 kg.

To find v, we use the formula for kinetic energy $K = \frac{1}{2} mv^2$

$345 = \frac{1}{2} \times 7 \times v^2$

Finding velocity v:

$345 = \frac{7}{2} \times v^2$

$v^2 = \frac{345\times2}{7}$

$v^2 = \frac{690}{7}$

$v^2 = 98.57$

$v = \sqrt{98.57}$

$v = 9.93$ m/s

Therefore, velocity v= 9.93 m/s

9)

In question #8, we used to find the kinetic energy of an object, it can be calculated using the equation $K = \frac{1}{2} mv^2$, where m represents the mass of the object and v represents its velocity.

So, in part b), we solve the mass m with the given value's kinetic energy k and velocity v using the equation $K = \frac{1}{2} mv^2$

This equation is quadratic in the base of power with degree 2

In part c) we solve the velocity v with the given value's kinetic energy k and mass m using the equation $K = \frac{1}{2} mv^2$

So, we rearrange the equation to solve for v

$$v^2 = \frac{2k}{m}$$

This is a quadratic equation with a degree of 2 in terms of the base of power.

Both parts b) and c) remain quadratic equations.

10)

a) In a linear graph rate of change equals zero, indicating that the rate of change at point C is Zero.

b) When the plot curves, it signifies a fluctuating rate of change. therefore, since the graph curves at point D, the rate of change is variable at that point.

c) If the graph slopes upwards to the right, it indicates a positive rate of change.

Therefore, at point 'A' and 'B' rate of change is positive

d) A steeper graph means a greater rate of change,

Therefore, at the point 'B' rate of change is greatest.

Rate of change at B, $= \frac{30-60}{30-20} = \frac{30}{10} = \frac{3}{10} = 3$

11)

a)

The general trend observed in the graph of winning discus throws from the Olympics Indicates an overall increase in the distance achieved over time.

b)

The connection seems to follow a straight-line pattern, showing a consistent rise in the winning distance, the rate of increase is not consistent, and there are periods of decreased progress. Therefore, we can define the relationship is close to being linear.

c)

There has been improvement in the training methods and equipment, leading to enhanced performances. Rules and techniques from coaches with a greater focus on practice aim to improve discus throw performance. Another factor that could explain the shape of the graph is increased competition and participation, contributing to the growing popularity of the sport of discus throw.

d)

By extrapolating the increasing winning distance over time, we can estimate the prediction for the upcoming Olympics. If the pattern persists, the winning distance is likely to reach approximately 71 minutes.

12)

a)

Both individuals experience a positive rate of change in their investments, with the value increasing over time. The total amount of investment for Bethany is increasing more rapidly, and the investment amount is not constant. The graphs appear to be more curved. Conversely, the total amount of investment for Dana remains fairly constant.

b)

Bethany	Dana
1. Starting with \$2500, Bethany's Initial investment is depicted on the graph, commencing from zero. 2. Bethany's investment, illustrated in the curved graph above, grows rapidly over time due to compounding at an 8% interest rate. 3. For the initial two years, Bethany's graph shows no growth, indicating that she kept her money stored under the mattress before deciding to invest it, Subsequently the graph demonstrates a rapid increase.	1. Starting with \$5000, Dana's initial investment is depicted on the graph, commencing from zero. 2. The graph of Dana's investment illustrates a consistent increase over time, reflecting a linear pattern, as she invested at an 8% simple interest rate. 3. After 20 years there is a significant increase in the investment's value due to a change to an 11% simple interest rate.

13)

The given equation for investment is:

$y = 65000(1.07)^x$

want to retire with \$500,000

Substitute the equation

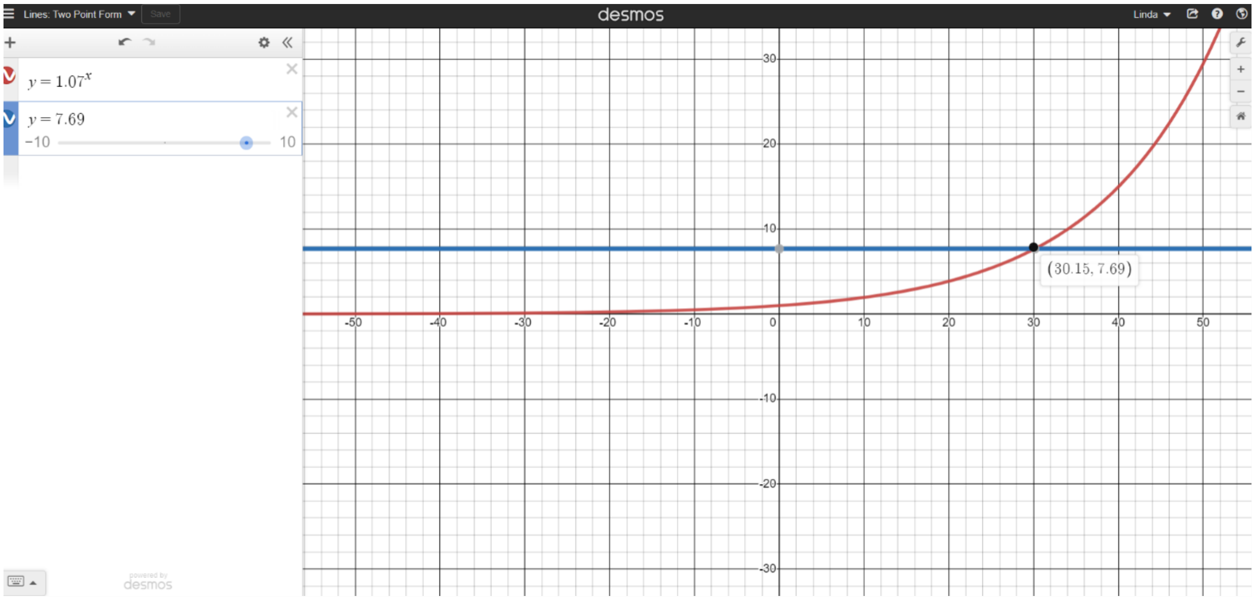
$500{,}000 = 65000(1.07)^x$

we need to find the value of x, divide both sides by 65,000 in the equation

$$\frac{500{,}000}{65{,}000} = \frac{65000(1.07)x}{65000}$$

$7.69230769 = (1.07)^x$

Using demos, we can plot the graph of the functions $y = (1.07)^x$ and $y = 7.69$, and then determine the point at which they intersect.



According to the graph, the point where the two lines intersect is approximately (30.15, 7.69). This indicates that x= 30.15. Thus, it will require approximately 30.15 years to accumulate $500,000.

14)

We need to determine the number of passes required to achieve a 1.5 mm thickness for the aluminum sheet.

This can be calculated using the equation,

$A = 80 \times (0.75)^x$

Where A represents the thickness of the aluminum and x denotes the number of passes through the rollers.

After rearranging the mathematical expression,

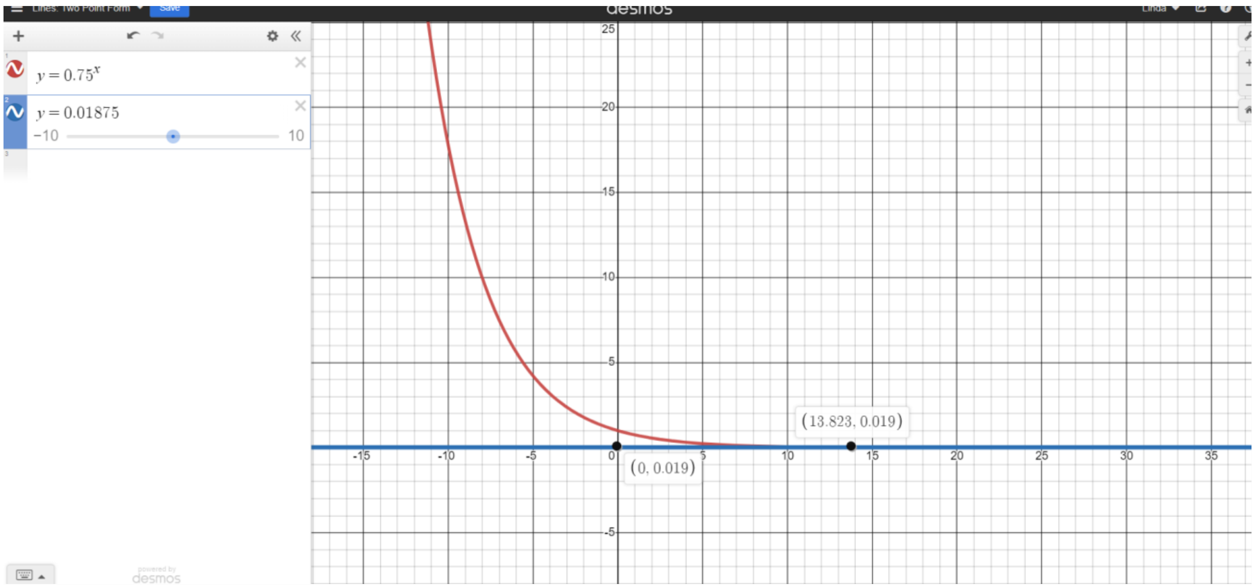
$1.5 = 80 \times (0.75)^x$

We aim to find the value of x, divide both sides of the equation by 80, we obtain,

$\frac{1.5}{80} = (0.75)^x$

$\ln 0.01875 = \ln(0.75)^x$

To find the value of x, we graph y= $(0.75)^x$ and y= 0.01875 and locate their point of intersection.



Upon examination of the graph, we identify the point of intersection as approximately $(13.823, 0.01875)$. Thus, $x = 13.823$, which we approximate to $x = 13.82$.

Therefore, approximately 13.82 passes are necessary.

15)

The volume of the cylinder, $v = \pi r^2 h$

Total volume of the cylinder= $20m^3$

The radius of the cylinder= $1.2m$

We have to find the height h of the one cylinder

We can set up an equation:

$2 \times \pi \times (1.2)^2 \times h = 20$

Now let's solve for h:

$2 \times \pi \times 1.44 \times h = 20$

$2 \times 1.44 \times \pi \times h = 20$

$2.88 \times \pi \times h = 20$

Divide both sides by $2.88 \times \pi$ to solve for h:

$10 = 1.44\pi h$

$10 = 1.44\pi h$

$h = \frac{20}{2.88 \times \pi}$

Now let's calculate the value for h:

$h = \frac{20}{9.05}$

$h \approx 2.21$m

Hence, the height of each cylinder should be approximately 2.21m to achieve a combined total volume of 20 m^3.

16)

Brent's final amount is calculated using the formula $A = 10000(1.06)^8$

While Lincoln's final amount is determined by the formula $A = 8000(1 + i)^8$

Despite the different formulas, they both yield the same final amount.

$10000(1.06)^8 = 8000(1 + i)^8$

We have to find the value of i:

First, let's simplify the equation divide both sides by 8000

$\frac{10000(1.06)^8}{8000} = \frac{8000(1+i)^8}{8000}$

$$\frac{10000(1.06)^8}{8000} = (1+i)^8$$

$$\frac{5}{4}(1.06)^8 = (1+i)^8$$

$$\sqrt[8]{\frac{5}{4}(1.06)^8} = (1+i)$$

$$\sqrt[8]{\frac{5}{4}}(1.06) = (1+i)$$

$$\sqrt[8]{\frac{5}{4}}(1.06) - 1 \; = \; i$$

$$i = \sqrt[8]{\frac{5}{4}}\,(1.06) - 1$$

$$i = \sqrt[8]{1.25}\,(1.06) - 1$$

$$i = 1.028285595 \times 1.06 - 1$$

$$i = 1.089982731 - 1$$

$$i = 0.08998273101 \cong 0.09$$

$$i = 0.09\%$$

So, Lincoln invests \$8000 at 9% per year.

17)

a) The rate of change experiences an increase at the coordinate (40, 1200).

b) The graph spans from the origin (0,0) to (40, 1200),

Depicting the rate of change= $\frac{y2-y1}{\text{x2}-\text{x1}}$

$$= \frac{1200-0}{40-0}$$

$$= \frac{1200}{40}$$

$$= 30$$

The points on the graphs range from (40,1200) to (50, 1700)

Rate of change equals the difference in y values divided by the difference in x values.

$$= \frac{1700 - 1200}{50 - 40}$$

$$= 500 \div 10$$

$$= 50$$

Hence, the rate of growth in the lower side is \$30 per hour, whereas on the upper side, it is \$50 per hour.

c) According to the graph, the rate of increase on the lower side of point (40, 1200) is \$30 per hour, while on the upper side, it is \$50 per hour. The hourly wage shifts after reaching 40 hours of work.