QWORLD

Entanglement and basics of quantum protocols

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Monday, July 18

WOMANIUM QUANTUM 2022

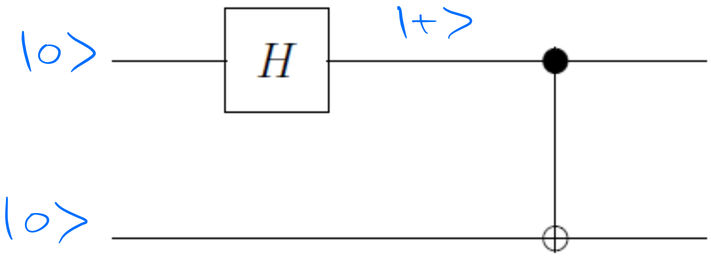
QBronze Summary

n-qubit Quantum State $|\psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$, $\sum_{i=0}^{2^n-1} \alpha_i^2 = 1$, $\alpha_i \in \mathbb{R}$

Unitary Evolution $U|\psi\rangle = |\varphi\rangle = \sum_{i=0}^{2^n-1} \beta_i |i\rangle$, $\sum_{i=0}^{2^n-1} \beta_i^2 = 1$, $\beta_i \in \mathbb{R}$

Measurement Probability to observe particular outcome i on measuring $|\psi\rangle$ is given by α_i^2

Preparing a Bell State



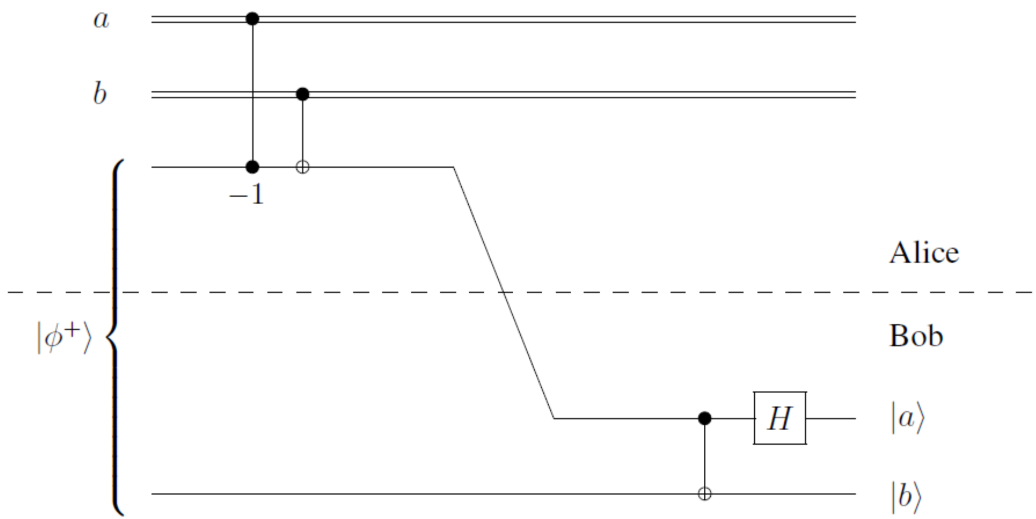
$$|+\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle+|10\rangle\right)$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Superdense Coding

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

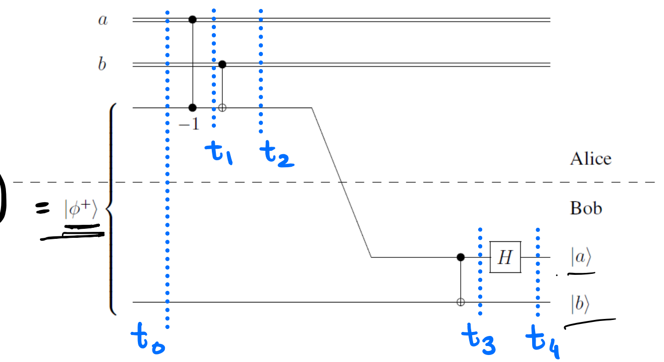


$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Holevo Bound

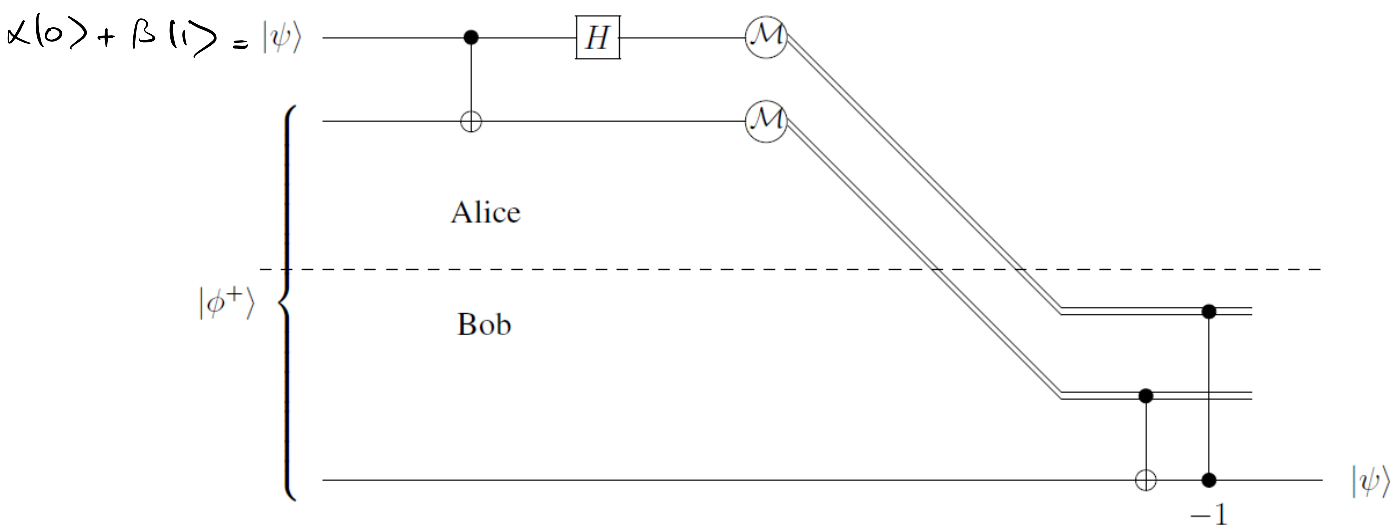
Superdense Coding

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\phi^+\rangle$



ab	State at t_1	State at t_2	State at t_3	State at t_4
00	$\|\varphi^+\rangle$	$\|\varphi^+\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|10\rangle) = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle) \otimes \|0\rangle$	$\|00\rangle$
01	$\|\varphi^+\rangle$	$\frac{1}{\sqrt{2}}(\|10\rangle + \|01\rangle)$	$\frac{1}{\sqrt{2}}(\|11\rangle + \|01\rangle) = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)\|1\rangle$	$\|01\rangle$
10	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle) = \|\varphi^-\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle - \|10\rangle) = \|-\rangle\|0\rangle$	$\|10\rangle$
11	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$	$\frac{1}{\sqrt{2}}(\|10\rangle - \|01\rangle)$	$\frac{1}{\sqrt{2}}(\|11\rangle - \|01\rangle) = -\|-\rangle\|1\rangle$	$-\|11\rangle$

Quantum Teleportation



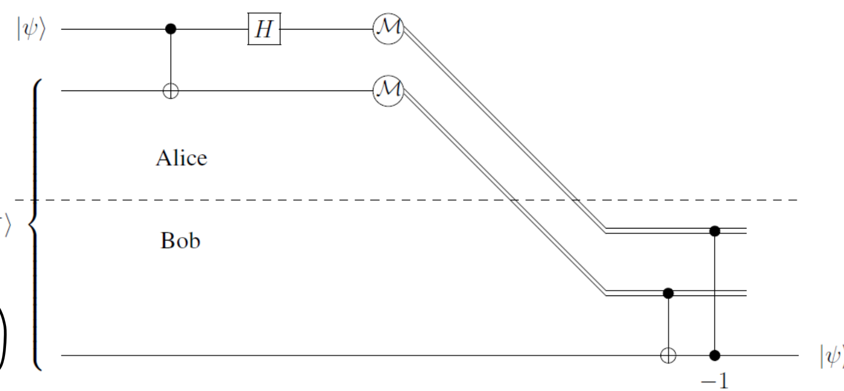
Quantum Teleportation

$$(\alpha|0\rangle+\beta|1\rangle)\otimes|\varphi^+\rangle$$

$$=(\alpha|0\rangle+\beta|1\rangle)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$=\frac{1}{\sqrt{2}}(\alpha|000\rangle+\alpha|011\rangle+\beta|100\rangle+\beta|111\rangle)$$

$$\xrightarrow{CNOT\otimes 1}\frac{1}{\sqrt{2}}(\alpha|000\rangle+\alpha|011\rangle+\beta|110\rangle+\beta|101\rangle)$$

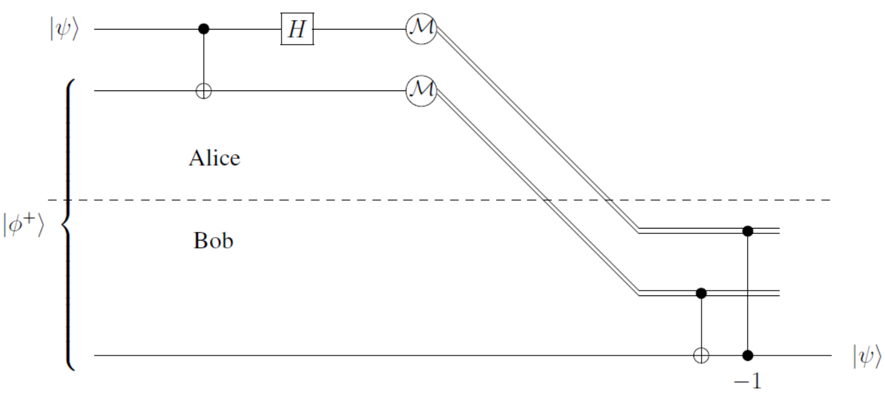


$$\xrightarrow{H\otimes 1\otimes 1}\frac{1}{2}(\alpha|000\rangle+\alpha|100\rangle+\alpha|011\rangle+\alpha|111\rangle+\beta|010\rangle-\beta|110\rangle+\beta|001\rangle-\beta|101\rangle)$$

$$\frac{1}{2}\Big(|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)\Big)$$

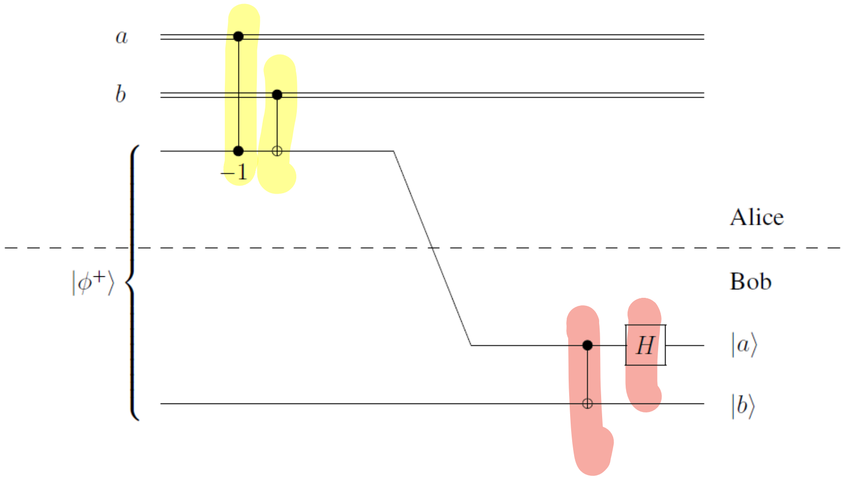
Quantum Teleportation

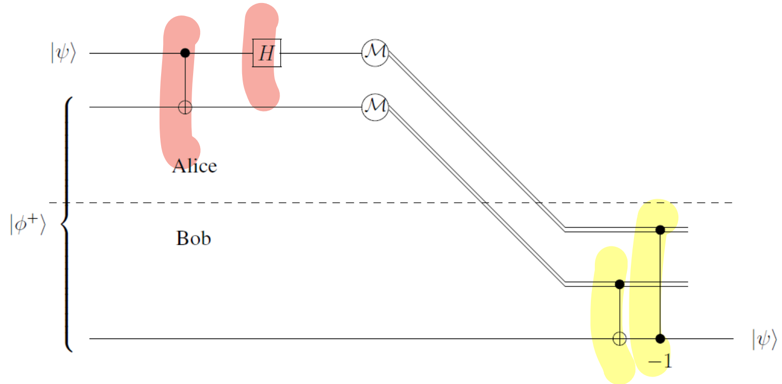
$$\frac{1}{2}|00\rangle(\alpha|0\rangle+\beta|1\rangle)$$
$$+\frac{1}{2}|01\rangle(\alpha|1\rangle+\beta|0\rangle)$$
$$+\frac{1}{2}|10\rangle(\alpha|0\rangle-\beta|1\rangle)$$
$$+\frac{1}{2}|11\rangle(\alpha|1\rangle-\beta|0\rangle)$$



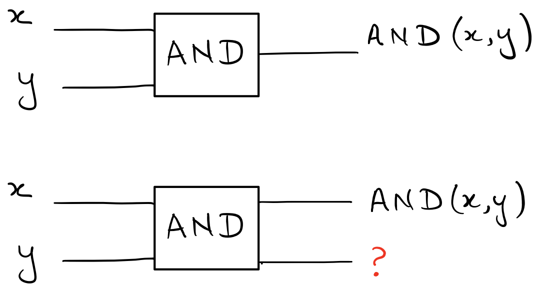
$$\alpha|1\rangle+\beta|0\rangle \xrightarrow{X} \alpha|0\rangle+\beta|1\rangle$$
$$\alpha|0\rangle-\beta|1\rangle \xrightarrow{Z} \alpha|0\rangle+\beta|1\rangle$$
$$\alpha|1\rangle-\beta|0\rangle \xrightarrow{X} \alpha|0\rangle-\beta|1\rangle \xrightarrow{Z} \alpha|0\rangle+\beta|1\rangle$$

Comparison


a
b
−1
Alice
Bob
$|\phi^+\rangle$
H
$|a\rangle$
$|b\rangle$


$|\psi\rangle$
H
$\mathcal{M}$
$\mathcal{M}$
Alice
Bob
$|\phi^+\rangle$
−1
$|\psi\rangle$

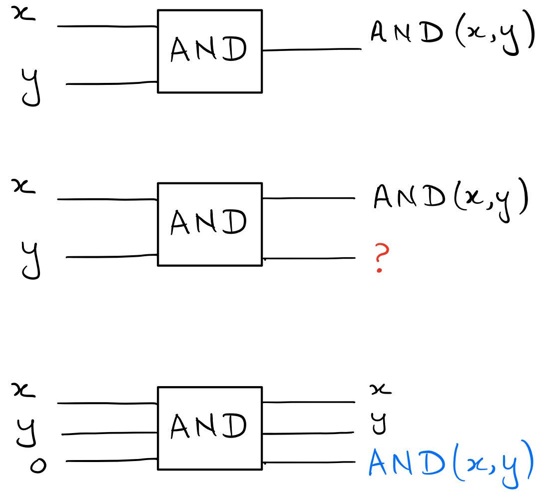
Reversible Transformations



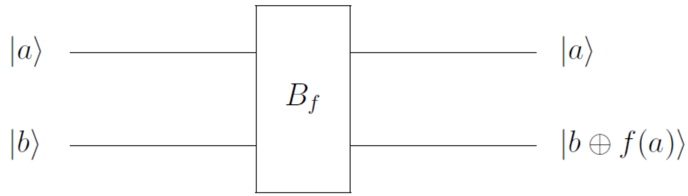
Reversible Transformations

$$U U^{\dagger} = \mathbb{1}$$

$$\underline{\underline{U^{\dagger}}} = U^{-1}$$



Classical Gates Via Unitaries



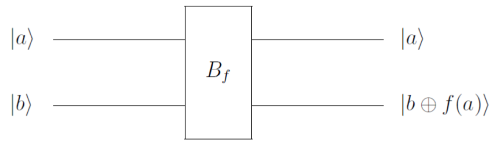
$$B_f : |a\rangle|b\rangle \longrightarrow |a\rangle|b \oplus f(a)\rangle$$

$$f : \{0,1\}^n \longrightarrow \{0,1\}$$

Phase Kickback

Recall

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$



Consider $B_f|a\rangle|-\rangle$

$$= B_f|a\rangle\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(B_f|a\rangle|0\rangle - B_f|a\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|a\rangle\underline{|0\oplus f(a)\rangle} - |a\rangle|1\oplus f(a)\rangle\right.$$

a	b	$a\oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

if $f(a)=0 \longrightarrow |0\rangle - |1\rangle$

$f(a)=1 \longrightarrow |1\rangle - |0\rangle$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} (-1)^{f(a)}\left(|0\rangle - |1\rangle\right)$$

$$= (-1)^{f(a)}|a\rangle\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$

Simple Search

f_{00}

input	output
00	1
01	0
10	0
11	0

f_{01}

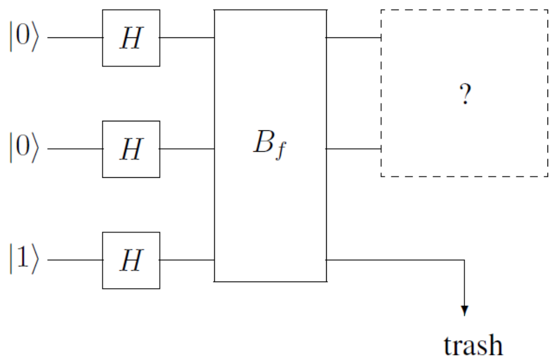
input	output
00	0
01	1
10	0
11	0

f_{10}

input	output
00	0
01	0
10	1
11	0

f_{11}

input	output
00	0
01	0
10	0
11	1



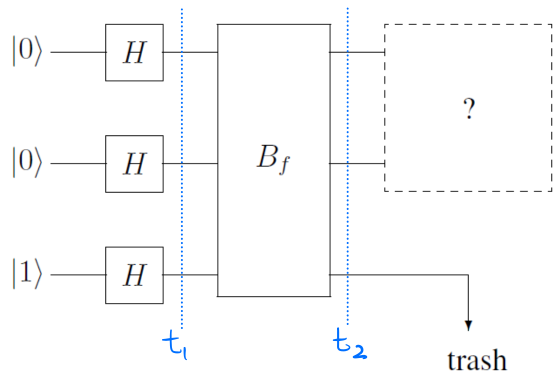
Simple Search

$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Determine $|\psi_2\rangle$ via phase kickback

$$B_f|\psi_1\rangle =$$

$$\frac{1}{2}\left((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\right)$$



Simple Search

$$\frac{1}{2}\left((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\right)$$

$$f = f_{00} \Rightarrow \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$f = f_{01} \Rightarrow \frac{1}{2}\left(+|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)$$

$$f = f_{10} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$

$$f = f_{11} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$