QWORLD

Quantum Operators on a Qubit

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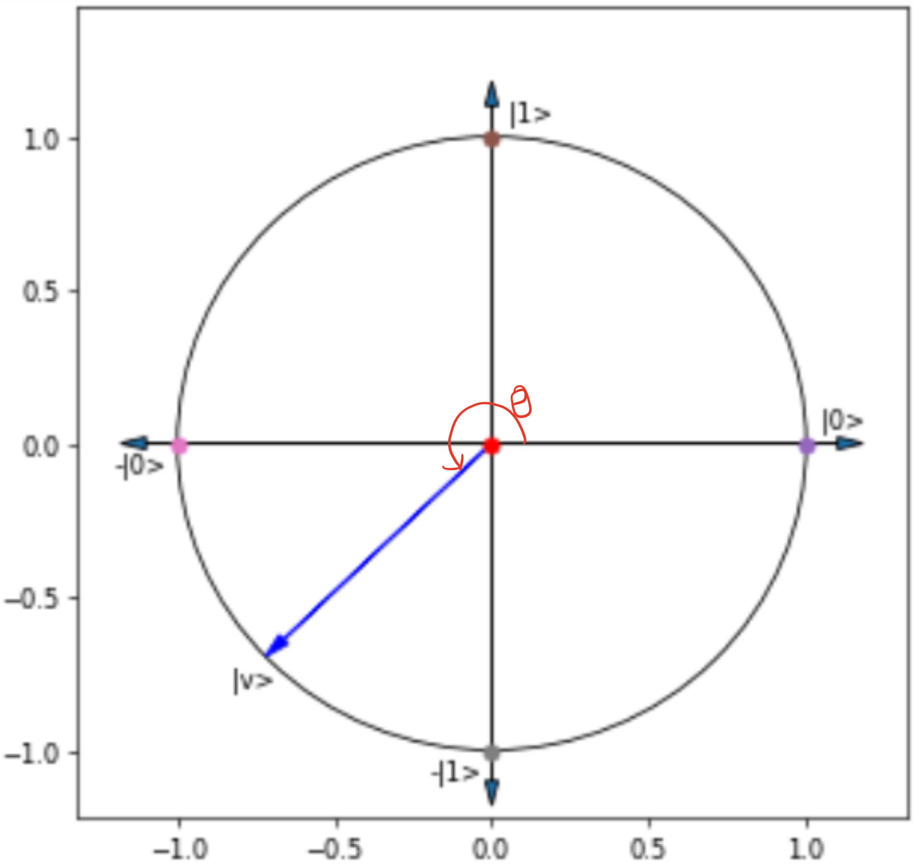
WOMANIUM QUANTUM 2022

Qubit on a Unit Circle

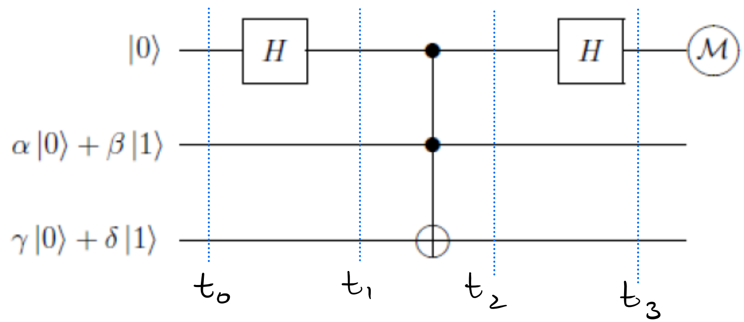
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\psi\rangle = \cos\theta\, |0\rangle + \sin\theta\, |1\rangle$$



Circuit Evaluation



$$|0\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$$

$$\xrightarrow{H\otimes 1\otimes 1} \underbrace{\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\otimes(\alpha|0\rangle+\beta|1\rangle)}\otimes(\gamma|0\rangle+\delta|1\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{1}{\sqrt{2}}\beta|11\rangle(\gamma|0\rangle+\delta|1\rangle)$$

$$\xrightarrow{\text{Toffoli}} \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{\beta|11\rangle}{\sqrt{2}}\underline{(\gamma|1\rangle+\delta|0\rangle)}$$

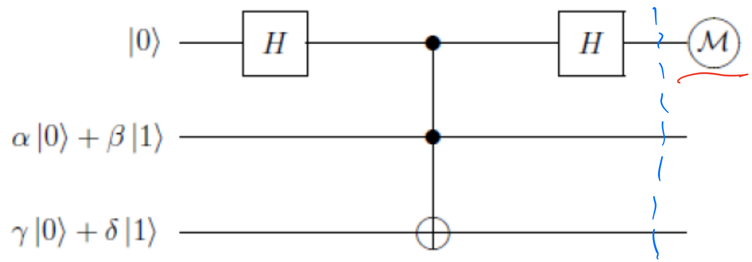
$$\xrightarrow{H\otimes 1\otimes 1} \frac{1}{2}\Big[(\alpha|00\rangle+\alpha|10\rangle + \beta|01\rangle+\beta|11\rangle+\alpha|00\rangle-\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + (\beta|01\rangle-\beta|11\rangle)(\gamma|1\rangle+\delta|0\rangle)$$

$$= \frac{1}{2}\Big[2\alpha\gamma|000\rangle+2\alpha\delta|001\rangle+\beta(\gamma+\delta)|010\rangle+\beta(\gamma+\delta)|011\rangle + \beta(\gamma-\delta)|110\rangle+\beta(\delta-\gamma)|111\rangle\Big]$$

Circuit Evaluation

Prob for first qubit to be in state $|1\rangle$

$$= \frac{1}{2}\beta^2(\gamma-\delta)^2$$



Given that first qubit is measured in state 1, what is the probability distribution for 2nd qubit?

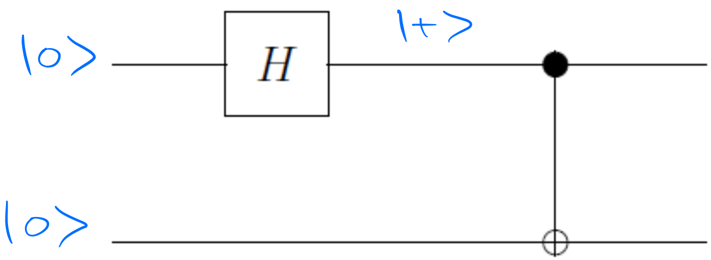
second qubit is always $|1\rangle$.

Does there exist a choice of $\alpha, \beta, \gamma, \delta$ for which first qubit is measured in state $|1\rangle$ with probability 1.

$$\alpha=0,\ \beta=1,\ \gamma=\frac{1}{\sqrt{2}},\ \delta=-\frac{1}{\sqrt{2}} \longrightarrow \frac{1}{2}(1)^2\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)^2 = 1.$$

$$\longrightarrow \frac{1}{2}\Big[2\alpha\gamma|000\rangle + 2\alpha\delta|001\rangle + \beta(\gamma+\delta)|010\rangle + \beta(\gamma+\delta)|011\rangle + \beta(\gamma-\delta)|110\rangle + \beta(\delta-\gamma)|111\rangle\Big]$$

Preparing a Bell State



$$|+\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle+|10\rangle\right)$$

$$\xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Entanglement vs Perfect Correlation

Correlation

$$\frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Entanglement

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Bell state